```
rem_minus  THM

⊢  ∀x:ℤ, n:ℤ⁻⁰. ((-x) rem n) = -(x rem n)
| by Auto
|    THEN
|    (Inst
|     Thm* ∀a:ℤ, n:ℤ⁻⁰.
|          a = (a ÷ n)·n+(a rem n)
|          & |a rem n|<|n|
|          & ((a rem n)<0 ⇒ a<0)
|          & ((a rem n)>0 ⇒ a>0)
|     {div_rem_properties}
|     [x;n])
|    THEN
|    (Inst
|     Thm* ∀a:ℤ, n:ℤ⁻⁰, q,r:ℤ.
|          a = q·n+r
|          ⇒
|          |r|<|n|
|          ⇒
|          (r<0 ⇒ a<0) ⇒ (r>0 ⇒ a>0) ⇒ q = (a ÷ n) & r = (a rem n)
|     {div_rem_unique}
|     [-x;n;-(x ÷ n);-(x rem n)])
|
|\
| 1. x : ℤ
| 2. n : ℤ⁻⁰
| 3. x = (x ÷ n)·n+(x rem n)
| 4. |x rem n|<|n|
| 5. (x rem n)<0 ⇒ x<0
| 6. (x rem n)>0 ⇒ x>0
| ⊢  |-(x rem n)|<|n|  by Subst (|-(x rem n)| = |x rem n|) 0
| |
| .....equality .....
| 1. x : ℤ
| 2. n : ℤ⁻⁰
| 3. x = (x ÷ n)·n+(x rem n)
| 4. |x rem n|<|n|
| 5. (x rem n)<0 ⇒ x<0
| 6. (x rem n)>0 ⇒ x>0
| ⊢  |-(x rem n)| = |x rem n|
|   by (Unfold `absval` 0) THEN (Repeat SplitOnConclITE)
|
|\
| 1. x : ℤ
| 2. n : ℤ⁻⁰
| 3. x = (x ÷ n)·n+(x rem n)
| 4. |x rem n|<|n|
| 5. (x rem n)<0 ⇒ x<0
| 6. (x rem n)>0 ⇒ x>0
| 7. -(x rem n)<0
| ⊢  -x<0  by Analyze -2
|
.\
. 1. x : ℤ
```

```
. 2. n : ℤ⁻⁰
. 3. x = (x ÷ n)·n+(x rem n)
. 4. |x rem n|<|n|
. 5. (x rem n)<0 ⇒ x<0
. 6. (x rem n)>0 ⇒ x>0
. 7. -(x rem n)>0
. ⊢  -x>0  by Analyze -3
END
```