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sum_arith  THM

⊢  ∀n:ℕ, a,b:ℤ. sum(a+b·i | i < n) = ((n·(a+a+b·(n-1))) ÷ 2)
| by Auto
|    THEN
|    (Inst
|     Thm* ∀n:ℕ, a,b:ℤ. sum(a+b·i | i < n)·2 = n·(a+a+b·(n-1))
|     {sum_arith1}
|     [n;a;b])
|
1. n : ℕ
2. a : ℤ
3. b : ℤ
4. sum(a+b·i | i < n)·2 = n·(a+a+b·(n-1))
⊢  sum(a+b·i | i < n) = ((n·(a+a+b·(n-1))) ÷ 2)
| by Using [‘n’,2]
|    (BackThru
|     Thm* ∀a,b:ℤ, n:ℤ^-0. n·a = n·b ⇒ a = b    {mul_cancel_in_eq})
|
1. n : ℕ
2. a : ℤ
3. b : ℤ
4. sum(a+b·i | i < n)·2 = n·(a+a+b·(n-1))
⊢  2·sum(a+b·i | i < n) = 2·((n·(a+a+b·(n-1))) ÷ 2)
| by (Inst Thm* ∀a:ℤ, n:ℤ^-0. a = (a ÷ n)·n+(a rem n)    {div_rem_sum}
|     [n·(a+a+b·(n-1));2])
|    THEN
|    (Assert (((n·(a+a+b·(n-1))) rem 2) = 0))
|
.....assertion .....
1. n : ℕ
2. a : ℤ
3. b : ℤ
4. sum(a+b·i | i < n)·2 = n·(a+a+b·(n-1))
5. n·(a+a+b·(n-1))
|  =
|  ((n·(a+a+b·(n-1))) ÷ 2)·2+((n·(a+a+b·(n-1))) rem 2)
⊢  ((n·(a+a+b·(n-1))) rem 2) = 0  by RevHypSubst -2 0
|
1. n : ℕ
2. a : ℤ
3. b : ℤ
4. sum(a+b·i | i < n)·2 = n·(a+a+b·(n-1))
5. n·(a+a+b·(n-1))
|  =
|  ((n·(a+a+b·(n-1))) ÷ 2)·2+((n·(a+a+b·(n-1))) rem 2)
⊢  ((sum(a+b·i | i < n)·2) rem 2) = 0
| by (GenConclAtAddr [2;1;1]) THEN (Lemmaize [])
|
⊢  ∀z:ℤ. ((z·2) rem 2) = 0  by Auto THEN (Decide (0≤z))
|
|\
| 1. z : ℤ
| 2. 0≤z
| ⊢  ((z·2) rem 2) = 0
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|   by (Inst
|       Thm* ∀a,b:ℕ, n:ℕ+. ((a+b·n) rem n) = (a rem n)
|       {rem_invariant}
|       [0;z;2])
|      THEN
|      (Reduce -1)
|
.\
. 1. z : ℤ
. 2. ¬0≤z
. ⊢  ((z·2) rem 2) = 0
.   by (RWO
.       Thm* ∀a:{...0}, n:ℕ+. (a rem n) = -((-a) rem n)
.       {rem_2_to_1}
.       0)
.      THEN
.      (Inst
.       Thm* ∀a,b:ℕ, n:ℕ+. ((a+b·n) rem n) = (a rem n)
.       {rem_invariant}
.       [0;-z;2])
.      THEN
.      (Reduce -1)
END
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