```
Ramsey-base-case  THM

⊢  ∀L:ℕ List. sum(L[i]-1 | i < ||L||)+1->L^1
| by Auto THEN (Analyze 0)
|
1. L : ℕ List
2. n : ℕ
3. sum(L[i]-1 | i < ||L||)+1≤n
4. G :
|  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→ℕ||L||
⊢  ∃c:ℕ||L||, f:(ℕL[c]→ℕn).
|  increasing(f;L[c])
|  & (∀s:ℕL[c] List.
|      ||s|| = 1 ⇒ (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) ⇒ G(map(f;s)) = c)
| by Assert
|    (∃c:ℕ||L||, f:(ℕL[c]→ℕn).
|     increasing(f;L[c]) & (∀s:ℕL[c]. G([(f(s))]) = c))
|
|\
| .....assertion .....
| 1. L : ℕ List
| 2. n : ℕ
| 3. sum(L[i]-1 | i < ||L||)+1≤n
| 4. G :
| |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
| |  ℕ||L||
| ⊢  ∃c:ℕ||L||, f:(ℕL[c]→ℕn).
| |  increasing(f;L[c]) & (∀s:ℕL[c]. G([(f(s))]) = c)
| | by ((Inst Thm: finite_DASH_partition [n;||L||;λx.G([x])])
| |     THENA
| |     (Auto THEN Analyze THEN (Reduce 0)))
| |    THEN
| |    ExRepD
| |
| 1. L : ℕ List
| 2. n : ℕ
| 3. sum(L[i]-1 | i < ||L||)+1≤n
| 4. G :
| |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
| |  ℕ||L||
| 5. p : ℕ||L||→(ℕ List)
| 6. sum(||p(j)|| | j < ||L||) = n
| 7. ∀j:ℕ||L||, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| 8. ∀j:ℕ||L||, x:ℕ||p(j)||. (p(j))[x]<n & (λx.G([x]))((p(j))[x]) = j
| ⊢  ∃c:ℕ||L||, f:(ℕL[c]→ℕn).
| |  increasing(f;L[c]) & (∀s:ℕL[c]. G([(f(s))]) = c)
| | by Decide (∃c:ℕ||L||. L[c]≤||p(c)||)
| |
| |\
| | 1. L : ℕ List
| | 2. n : ℕ
| | 3. sum(L[i]-1 | i < ||L||)+1≤n
| | 4. G :
| | |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
| | |  ℕ||L||
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| | 5. p : ℕ||L||→(ℕ List)
| | 6. sum(||p(j)|| | j < ||L||) = n
| | 7. ∀j:ℕ||L||, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| | 8. ∀j:ℕ||L||, x:ℕ||p(j)||.
| | |  (p(j))[x]<n & (λx.G([x]))((p(j))[x]) = j
| | 9. ∃c:ℕ||L||. L[c]≤||p(c)||
| | ⊢  ∃c:ℕ||L||, f:(ℕL[c]→ℕn).
| | |  increasing(f;L[c]) & (∀s:ℕL[c]. G([(f(s))]) = c)
| | | by ParallelOp -1
| | |
| | |\
| | | 1. L : ℕ List
| | | 2. n : ℕ
| | | 3. sum(L[i]-1 | i < ||L||)+1≤n
| | | 4. G :
| | | |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
| | | |  ℕ||L||
| | | 5. p : ℕ||L||→(ℕ List)
| | | 6. sum(||p(j)|| | j < ||L||) = n
| | | 7. ∀j:ℕ||L||, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| | | 8. ∀j:ℕ||L||, x:ℕ||p(j)||.
| | | |  (p(j))[x]<n & (λx.G([x]))((p(j))[x]) = j
| | | 9. c : ℕ||L||
| | | 10. L[c]≤||p(c)||
| | | ⊢  ∃f:(ℕL[c]→ℕn).
| | | |  increasing(f;L[c]) & (∀s:ℕL[c]. G([(f(s))]) = c)
| | | | by (AllHyps (InstHyp [c])) THEN (RepeatFor 3 (MoveToConcl -1))
| | | |    THEN
| | | |    (GenConcl (p(c) = A))
| | | |    THEN
| | | |    (GenConcl (L[c] = m))
| | | |    THEN
| | | |    (AllHyps Thin)
| | | |    THEN
| | | |    (Reduce 0)
| | | |
| | | 1. L : ℕ List
| | | 2. n : ℕ
| | | 3. G :
| | | |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
| | | |  ℕ||L||
| | | 4. c : ℕ||L||
| | | 5. A : ℕ List
| | | 6. m : ℕ
| | | ⊢  m≤||A||
| | | |  ⇒
| | | |  (∀x:ℕ||A||. A[x]<n & G([A[x]]) = c)
| | | |  ⇒
| | | |  (∀x,y:ℕ||A||. x<y ⇒ A[x]>A[y])
| | | |  ⇒
| | | |  (∃f:(ℕm→ℕn). increasing(f;m) & (∀s:ℕm. G([(f(s))]) = c))
| | | | by Auto
| | | |
| | | |\
| | | | 1. L : ℕ List
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| | | | 2. n : ℕ
| | | | 3. G :
| | | | |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
| | | | |  →ℕ||L||
| | | | 4. c : ℕ||L||
| | | | 5. A : ℕ List
| | | | 6. m : ℕ
| | | | 7. m≤||A||
| | | | 8. ∀x:ℕ||A||. A[x]<n & G([A[x]]) = c
| | | | 9. ∀x,y:ℕ||A||. x<y ⇒ A[x]>A[y]
| | | | ⊢  ∃f:(ℕm→ℕn). increasing(f;m) & (∀s:ℕm. G([(f(s))]) = c)
| | | | | by (InstConcl [λx.A[(||A||-(x+1))]])
| | | | |    THEN
| | | | |    (Try (Complete Auto))
| | | | |
| | | | |\
| | | | | .....wf .....
| | | | | 1. L : ℕ List
| | | | | 2. n : ℕ
| | | | | 3. G :
| | | | | |  {s:(ℕn List)
| | | | | |   | ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
| | | | | |  →ℕ||L||
| | | | | 4. c : ℕ||L||
| | | | | 5. A : ℕ List
| | | | | 6. m : ℕ
| | | | | 7. m≤||A||
| | | | | 8. ∀x:ℕ||A||. A[x]<n & G([A[x]]) = c
| | | | | 9. ∀x,y:ℕ||A||. x<y ⇒ A[x]>A[y]
| | | | | ⊢  (λx.A[(||A||-(x+1))]) ∈ ℕm→ℕn
| | | | | | by Analyze THEN (Try Analyze)
| | | | | |
| | | | | 1. L : ℕ List
| | | | | 2. n : ℕ
| | | | | 3. G :
| | | | |    {s:(ℕn List)
| | | | |     | ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
| | | | |    →ℕ||L||
| | | | | 4. c : ℕ||L||
| | | | | 5. A : ℕ List
| | | | | 6. m : ℕ
| | | | | 7. m≤||A||
| | | | | 8. ∀x:ℕ||A||. A[x]<n & G([A[x]]) = c
| | | | | 9. ∀x,y:ℕ||A||. x<y ⇒ A[x]>A[y]
| | | | | 10. x : ℕm
| | | | | ⊢  A[(||A||-(x+1))]<n
| | | | |   by (InstHyp [||A||-(x+1)] -3) THEN (Analyze -1)
| | | | |
| | | | |\
| | | | | 1. L : ℕ List
| | | | | 2. n : ℕ
| | | | | 3. G :
| | | | | |  {s:(ℕn List)
| | | | | |   | ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
| | | | | |  →ℕ||L||
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| | | | | 4. c : ℕ||L||
| | | | | 5. A : ℕ List
| | | | | 6. m : ℕ
| | | | | 7. m≤||A||
| | | | | 8. ∀x:ℕ||A||. A[x]<n & G([A[x]]) = c
| | | | | 9. ∀x,y:ℕ||A||. x<y ⇒ A[x]>A[y]
| | | | | ⊢  increasing(λx.A[(||A||-(x+1))];m)
| | | | | |  & (∀s:ℕm. G([((λx.A[(||A||-(x+1))])(s))]) = c)
| | | | | | by (Reduce 0) THEN (Analyze 0)
| | | | | |
| | | | | |\
| | | | | | 1. L : ℕ List
| | | | | | 2. n : ℕ
| | | | | | 3. G :
| | | | | |    {s:(ℕn List)
| | | | | |     | ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
| | | | | |    →ℕ||L||
| | | | | | 4. c : ℕ||L||
| | | | | | 5. A : ℕ List
| | | | | | 6. m : ℕ
| | | | | | 7. m≤||A||
| | | | | | 8. ∀x:ℕ||A||. A[x]<n & G([A[x]]) = c
| | | | | | 9. ∀x,y:ℕ||A||. x<y ⇒ A[x]>A[y]
| | | | | | ⊢  increasing(λx.A[(||A||-(x+1))];m)
| | | | | |   by (Analyze 0) THEN (Reduce 0)
| | | | | |      THEN
| | | | | |      (InstHyp [||A||-(i+1+1);||A||-(i+1)] -2)
| | | | | |
| | | | | .\
| | | | | . 1. L : ℕ List
| | | | | . 2. n : ℕ
| | | | | . 3. G :
| | | | | .    {s:(ℕn List)
| | | | | .     | ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
| | | | | .    →ℕ||L||
| | | | | . 4. c : ℕ||L||
| | | | | . 5. A : ℕ List
| | | | | . 6. m : ℕ
| | | | | . 7. m≤||A||
| | | | | . 8. ∀x:ℕ||A||. A[x]<n & G([A[x]]) = c
| | | | | . 9. ∀x,y:ℕ||A||. x<y ⇒ A[x]>A[y]
| | | | | . 10. increasing(λx.A[(||A||-(x+1))];m)
| | | | | . ⊢  ∀s:ℕm. G([A[(||A||-(s+1))]]) = c
| | | | | .   by Auto THEN (InstHyp [||A||-(s+1)] -4)
| | | | | .      THEN
| | | | | .      (Analyze -1)
| | | | |
| | | | .\
| | | | . .....wf .....
| | | | . 1. L : ℕ List
| | | | . 2. n : ℕ
| | | | . 3. G :
| | | | .    {s:(ℕn List)
| | | | .     | ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
| | | | .    →ℕ||L||
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| | | | . 4. c : ℕ||L||
| | | | . 5. A : ℕ List
| | | | . 6. m : ℕ
| | | | . 7. m≤||A||
| | | | . 8. ∀x:ℕ||A||. A[x]<n & G([A[x]]) = c
| | | | . 9. ∀x,y:ℕ||A||. x<y ⇒ A[x]>A[y]
| | | | . 10. f : ℕm→ℕn
| | | | . ⊢  (increasing(f;m) & (∀s:ℕm. G([(f(s))]) = c)) ∈ Prop
| | | | .   by Auto THEN Analyze THEN (Reduce 0)
| | | |
| | | .\
| | | . .....subterm .....
| | | . 1. L : ℕ List
| | | . 2. n : ℕ
| | | . 3. {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
| | | . |  →ℕ||L||
| | | . 4. ℕ||L||
| | | . 5. A : ℕ List
| | | . 6. m : ℕ
| | | . 7. m≤||A||
| | | . 8. x : ℕ||A||
| | | . 9. A[x]<n
| | | . ⊢  [A[x]]
| | | . |  ∈ {s:(ℕn List)
| | | . |      | ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
| | | . | by Analyze THEN (Reduce 0) THEN (Try (Complete Auto))
| | | . |
| | | . 1. L : ℕ List
| | | . 2. n : ℕ
| | | . 3. {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
| | | .    →ℕ||L||
| | | . 4. ℕ||L||
| | | . 5. A : ℕ List
| | | . 6. m : ℕ
| | | . 7. m≤||A||
| | | . 8. x : ℕ||A||
| | | . 9. A[x]<n
| | | . ⊢  [A[x]] ∈ ℕn List  by Analyze THEN (Try Analyze)
| | |
| | .\
| | . .....subterm .....
| | . 1. L : ℕ List
| | . 2. n : ℕ
| | . 3. sum(L[i]-1 | i < ||L||)+1≤n
| | . 4. G :
| | .    {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
| | .    ℕ||L||
| | . 5. p : ℕ||L||→(ℕ List)
| | . 6. sum(||p(j)|| | j < ||L||) = n
| | . 7. ∀j:ℕ||L||, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| | . 8. ∀j:ℕ||L||, x:ℕ||p(j)||.
| | .    (p(j))[x]<n & (λx.G([x]))((p(j))[x]) = j
| | . 9. c : ℕ||L||
| | . 10. L[c]≤||p(c)||
| | . 11. c1 : ℕ||L||
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| | . 12. f : ℕL[c1]→ℕn
| | . 13. increasing(f;L[c1])
| | . 14. s : ℕL[c1]
| | . ⊢  [(f(s))]
| | .    ∈ {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
| | .   by Analyze THEN (Reduce 0)
| |
| .\
| . 1. L : ℕ List
| . 2. n : ℕ
| . 3. sum(L[i]-1 | i < ||L||)+1≤n
| . 4. G :
| . |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
| . |  ℕ||L||
| . 5. p : ℕ||L||→(ℕ List)
| . 6. sum(||p(j)|| | j < ||L||) = n
| . 7. ∀j:ℕ||L||, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| . 8. ∀j:ℕ||L||, x:ℕ||p(j)||.
| . |  (p(j))[x]<n & (λx.G([x]))((p(j))[x]) = j
| . 9. ¬(∃c:ℕ||L||. L[c]≤||p(c)||)
| . ⊢  ∃c:ℕ||L||, f:(ℕL[c]→ℕn).
| . |  increasing(f;L[c]) & (∀s:ℕL[c]. G([(f(s))]) = c)
| . | by Assert (sum(||p(j)|| | j < ||L||)≤sum(L[i]-1 | i < ||L||))
| . |
| . |\
| . | .....assertion .....
| . | 1. L : ℕ List
| . | 2. n : ℕ
| . | 3. sum(L[i]-1 | i < ||L||)+1≤n
| . | 4. G :
| . | |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
| . | |  ℕ||L||
| . | 5. p : ℕ||L||→(ℕ List)
| . | 6. sum(||p(j)|| | j < ||L||) = n
| . | 7. ∀j:ℕ||L||, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| . | 8. ∀j:ℕ||L||, x:ℕ||p(j)||.
| . | |  (p(j))[x]<n & (λx.G([x]))((p(j))[x]) = j
| . | 9. ¬(∃c:ℕ||L||. L[c]≤||p(c)||)
| . | ⊢  sum(||p(j)|| | j < ||L||)≤sum(L[i]-1 | i < ||L||)
| . | | by BackThru
| . | |    Thm* ∀k:ℕ, f,g:(ℕk→ℤ).
| . | |         (∀x:ℕk. f(x)≤g(x))
| . | |         ⇒
| . | |         sum(f(x) | x < k)≤sum(g(x) | x < k)
| . | |    {sum_le}
| . | |
| . | 1. L : ℕ List
| . | 2. n : ℕ
| . | 3. sum(L[i]-1 | i < ||L||)+1≤n
| . | 4. G :
| . | |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
| . | |  ℕ||L||
| . | 5. p : ℕ||L||→(ℕ List)
| . | 6. sum(||p(j)|| | j < ||L||) = n
| . | 7. ∀j:ℕ||L||, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
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| . | 8. ∀j:ℕ||L||, x:ℕ||p(j)||.
| . | |  (p(j))[x]<n & (λx.G([x]))((p(j))[x]) = j
| . | 9. ¬(∃c:ℕ||L||. L[c]≤||p(c)||)
| . | 10. i : ℕ||L||
| . | ⊢  ||p(i)||≤L[i]-1  by SupposeNot
| . | |
| . | 1. L : ℕ List
| . | 2. n : ℕ
| . | 3. sum(L[i]-1 | i < ||L||)+1≤n
| . | 4. G :
| . | |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
| . | |  ℕ||L||
| . | 5. p : ℕ||L||→(ℕ List)
| . | 6. sum(||p(j)|| | j < ||L||) = n
| . | 7. ∀j:ℕ||L||, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| . | 8. ∀j:ℕ||L||, x:ℕ||p(j)||.
| . | |  (p(j))[x]<n & (λx.G([x]))((p(j))[x]) = j
| . | 9. ¬(∃c:ℕ||L||. L[c]≤||p(c)||)
| . | 10. i : ℕ||L||
| . | 11. ¬||p(i)||≤L[i]-1
| . | ⊢  ||p(i)||≤L[i]-1  by Analyze -3
| . | |
| . | 1. L : ℕ List
| . | 2. n : ℕ
| . | 3. sum(L[i]-1 | i < ||L||)+1≤n
| . | 4. G :
| . |    {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
| . |    ℕ||L||
| . | 5. p : ℕ||L||→(ℕ List)
| . | 6. sum(||p(j)|| | j < ||L||) = n
| . | 7. ∀j:ℕ||L||, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| . | 8. ∀j:ℕ||L||, x:ℕ||p(j)||.
| . |    (p(j))[x]<n & (λx.G([x]))((p(j))[x]) = j
| . | 9. i : ℕ||L||
| . | 10. ¬||p(i)||≤L[i]-1
| . | ⊢  ∃c:ℕ||L||. L[c]≤||p(c)||  by InstConcl [i]
| . |
| . .\
| . . 1. L : ℕ List
| . . 2. n : ℕ
| . . 3. sum(L[i]-1 | i < ||L||)+1≤n
| . . 4. G :
| . .    {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
| . .    ℕ||L||
| . . 5. p : ℕ||L||→(ℕ List)
| . . 6. sum(||p(j)|| | j < ||L||) = n
| . . 7. ∀j:ℕ||L||, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| . . 8. ∀j:ℕ||L||, x:ℕ||p(j)||.
| . .    (p(j))[x]<n & (λx.G([x]))((p(j))[x]) = j
| . . 9. ¬(∃c:ℕ||L||. L[c]≤||p(c)||)
| . . 10. sum(||p(j)|| | j < ||L||)≤sum(L[i]-1 | i < ||L||)
| . . ⊢  ∃c:ℕ||L||, f:(ℕL[c]→ℕn).
| . .    increasing(f;L[c]) & (∀s:ℕL[c]. G([(f(s))]) = c)  by Auto
|
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. 1. L : ℕ List
. 2. n : ℕ
. 3. sum(L[i]-1 | i < ||L||)+1≤n
. 4. G :
. |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
. |  ℕ||L||
. 5. ∃c:ℕ||L||, f:(ℕL[c]→ℕn).
. |  increasing(f;L[c]) & (∀s:ℕL[c]. G([(f(s))]) = c)
. ⊢  ∃c:ℕ||L||, f:(ℕL[c]→ℕn).
. |  increasing(f;L[c])
. |  & (∀s:ℕL[c] List.
. |     ||s|| = 1
. |     ⇒
. |     (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) ⇒ G(map(f;s)) = c)
. | by ParallelOp -1
. |
. |\
. | 1. L : ℕ List
. | 2. n : ℕ
. | 3. sum(L[i]-1 | i < ||L||)+1≤n
. | 4. G :
. | |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
. | |  ℕ||L||
. | 5. c : ℕ||L||
. | 6. ∃f:(ℕL[c]→ℕn).
. | |  increasing(f;L[c]) & (∀s:ℕL[c]. G([(f(s))]) = c)
. | ⊢  ∃f:(ℕL[c]→ℕn).
. | |  increasing(f;L[c])
. | |  & (∀s:ℕL[c] List.
. | |     ||s|| = 1
. | |     ⇒
. | |     (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) ⇒ G(map(f;s)) = c)
. | | by ParallelOp -1
. | |
. | |\
. | | 1. L : ℕ List
. | | 2. n : ℕ
. | | 3. sum(L[i]-1 | i < ||L||)+1≤n
. | | 4. G :
. | | |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
. | | |  ℕ||L||
. | | 5. c : ℕ||L||
. | | 6. f : ℕL[c]→ℕn
. | | 7. increasing(f;L[c]) & (∀s:ℕL[c]. G([(f(s))]) = c)
. | | ⊢  increasing(f;L[c])
. | | |  & (∀s:ℕL[c] List.
. | | |     ||s|| = 1
. | | |     ⇒
. | | |     (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) ⇒ G(map(f;s)) = c)
. | | | by (Analyze -1) THEN (Analyze 0) THEN (Try Trivial)
. | | |
. | | 1. L : ℕ List
. | | 2. n : ℕ
. | | 3. sum(L[i]-1 | i < ||L||)+1≤n
. | | 4. G :
```

```
. | | |  {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
. | | |  ℕ||L||
. | | 5. c : ℕ||L||
. | | 6. f : ℕL[c]→ℕn
. | | 7. increasing(f;L[c])
. | | 8. ∀s:ℕL[c]. G([(f(s))]) = c
. | | 9. increasing(f;L[c])
. | | ⊢  ∀s:ℕL[c] List.
. | | |  ||s|| = 1
. | | |  ⇒
. | | |  (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) ⇒ G(map(f;s)) = c
. | | | by UnivCD
. | | |    THEN
. | | |    (RepeatFor 2
. | | |     ((Analyze -3) THEN (All Reduce)
. | | |      THEN
. | | |      (Try (Complete Auto))))
. | | |
. | | 1. L : ℕ List
. | | 2. n : ℕ
. | | 3. sum(L[i]-1 | i < ||L||)+1≤n
. | | 4. G :
. | |    {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
. | |    ℕ||L||
. | | 5. c : ℕ||L||
. | | 6. f : ℕL[c]→ℕn
. | | 7. increasing(f;L[c])
. | | 8. ∀s:ℕL[c]. G([(f(s))]) = c
. | | 9. increasing(f;L[c])
. | | 10. u : ℕL[c]
. | | 11. 1 = 1
. | | 12. ∀x,y:ℕ1. x<y ⇒ [u][x]<[u][y]
. | | ⊢  G([(f(u))]) = c  by EasyHyp
. | |
. | .\
. | . .....subterm .....
. | . 1. L : ℕ List
. | . 2. n : ℕ
. | . 3. sum(L[i]-1 | i < ||L||)+1≤n
. | . 4. G :
. | .    {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
. | .    ℕ||L||
. | . 5. c : ℕ||L||
. | . 6. f : ℕL[c]→ℕn
. | . 7. increasing(f;L[c]) & (∀s:ℕL[c]. G([(f(s))]) = c)
. | . 8. f1 : ℕL[c]→ℕn
. | . 9. increasing(f1;L[c])
. | . 10. s : ℕL[c] List
. | . 11. ||s|| = 1
. | . 12. ∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]
. | . ⊢  map(f1;s)
. | .    ∈ {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
. | .   by Analyze THEN (Try (Complete Auto)) THEN (Analyze -3)
. | .      THEN
. | .      (All Reduce)
```

```
. | .      THEN
. | .      (Try (Complete Auto))
. |
. .\
. . .....subterm .....
. . 1. L : ℕ List
. . 2. n : ℕ

. . 3. sum(L[i]-1 | i < ||L||)+1≤n
. . 4. G :
. .    {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }→
. .    ℕ||L||
. . 5. c : ℕ||L||
. . 6. ∃f:(ℕL[c]→ℕn).
. .    increasing(f;L[c]) & (∀s:ℕL[c]. G([(f(s))]) = c)
. . 7. c1 : ℕ||L||
. . 8. f : ℕL[c1]→ℕn
. . 9. increasing(f;L[c1])
. . 10. s : ℕL[c1] List
. . 11. ||s|| = 1
. . 12. ∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]
. . ⊢  map(f;s)
. .    ∈ {s:(ℕn List)| ||s|| = 1 & (∀x,y:ℕ||s||. x<y ⇒ s[x]<s[y]) }
. .   by Analyze THEN (Try (Complete Auto)) THEN (Analyze -3)
. .      THEN
. .      (All Reduce)
. .      THEN
. .      (Try (Complete Auto))
END
```