```
connect-iff  THM

⊢  For any graph
|  (∀x,y:V. Dec(x = y))
|  ⇒
|  (∀x,y:V.
|   x-the_graph->*y
|   ⟺
|   x = y ∨ (∃z:V. ¬z = x & x-the_graph->z & z-the_graph->*y))
| by Auto
|
|\
| 1. the_graph : Graph
| 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| 3. x : Vertices(the_graph)
| 4. y : Vertices(the_graph)
| 5. x-the_graph->*y
| ⊢  x = y
| |  ∨ (∃z:Vertices(the_graph).
| |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | by (Unfold `connect` -1) THEN ExRepD
| |
| 1. the_graph : Graph
| 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| 3. x : Vertices(the_graph)
| 4. y : Vertices(the_graph)
| 5. p : Vertices(the_graph) List
| 6. path(the_graph;p)
| 7. p[0] = x
| 8. last(p) = y
| ⊢  x = y
| |  ∨ (∃z:Vertices(the_graph).
| |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | by (RepeatFor 4 (MoveToConcl -1)) THEN InductionOnLength
| |
| |\
| | 1. the_graph : Graph
| | 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | 3. x : Vertices(the_graph)
| | 4. y : Vertices(the_graph)
| | 5. ℕ
| | 6. p : Vertices(the_graph) List
| | 7. ∀p1:Vertices(the_graph) List.
| | |  ||p1||<||p||
| | |  ⇒
| | |  path(the_graph;p1)
| | |  ⇒
| | |  p1[0] = x
| | |  ⇒
| | |  last(p1) = y
| | |  ⇒
| | |  x = y
| | |  ∨ (∃z:Vertices(the_graph).
| | |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | 8. path(the_graph;p)
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| | 9. p[0] = x
| | 10. last(p) = y
| | ⊢  x = y
| | |  ∨ (∃z:Vertices(the_graph).
| | |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | by Decide (1<||p||)
| | |
| | |\
| | | 1. the_graph : Graph
| | | 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | 3. x : Vertices(the_graph)
| | | 4. y : Vertices(the_graph)
| | | 5. ℕ
| | | 6. p : Vertices(the_graph) List
| | | 7. ∀p1:Vertices(the_graph) List.
| | | |  ||p1||<||p||
| | | |  ⇒
| | | |  path(the_graph;p1)
| | | |  ⇒
| | | |  p1[0] = x
| | | |  ⇒
| | | |  last(p1) = y
| | | |  ⇒
| | | |  x = y
| | | |  ∨ (∃z:Vertices(the_graph).
| | | |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | 8. path(the_graph;p)
| | | 9. p[0] = x
| | | 10. last(p) = y
| | | 11. 1<||p||
| | | ⊢  x = y
| | | |  ∨ (∃z:Vertices(the_graph).
| | | |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | | by Assert (p[1] = x ∨ ¬p[1] = x)
| | | |
| | | |\
| | | | .....assertion .....
| | | | 1. the_graph : Graph
| | | | 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | | 3. x : Vertices(the_graph)
| | | | 4. y : Vertices(the_graph)
| | | | 5. ℕ
| | | | 6. p : Vertices(the_graph) List
| | | | 7. ∀p1:Vertices(the_graph) List.
| | | |    ||p1||<||p||
| | | |    ⇒
| | | |    path(the_graph;p1)
| | | |    ⇒
| | | |    p1[0] = x
| | | |    ⇒
| | | |    last(p1) = y
| | | |    ⇒
| | | |    x = y
| | | |    ∨ (∃z:Vertices(the_graph).
| | | |       ¬z = x & x-the_graph->z & z-the_graph->*y)
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| | | | 8. path(the_graph;p)
| | | | 9. p[0] = x
| | | | 10. last(p) = y
| | | | 11. 1<||p||
| | | | ⊢  p[1] = x ∨ ¬p[1] = x  by (Fold ‘decidable‘ 0) THEN EasyHyp
| | | |
| | | .\
| | | . 1. the_graph : Graph
| | | . 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | . 3. x : Vertices(the_graph)
| | | . 4. y : Vertices(the_graph)
| | | . 5. ℕ
| | | . 6. p : Vertices(the_graph) List
| | | . 7. ∀p1:Vertices(the_graph) List.
| | | . |  ||p1||<||p||
| | | . |  ⇒
| | | . |  path(the_graph;p1)
| | | . |  ⇒
| | | . |  p1[0] = x
| | | . |  ⇒
| | | . |  last(p1) = y
| | | . |  ⇒
| | | . |  x = y
| | | . |  ∨ (∃z:Vertices(the_graph).
| | | . |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . 8. path(the_graph;p)
| | | . 9. p[0] = x
| | | . 10. last(p) = y
| | | . 11. 1<||p||
| | | . 12. p[1] = x ∨ ¬p[1] = x
| | | . ⊢  x = y
| | | . |  ∨ (∃z:Vertices(the_graph).
| | | . |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . | by AssertBY path(the_graph;tl(p))
| | | . |    (BackThru
| | | . |     Thm* For any graph
| | | . |          ∀p:V List.
| | | . |          1<||p||
| | | . |          ⇒
| | | . |          path(the_graph;p) ⇒ path(the_graph;tl(p))
| | | . |     {path_DASH_tl})
| | | . |
| | | . 1. the_graph : Graph
| | | . 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | . 3. x : Vertices(the_graph)
| | | . 4. y : Vertices(the_graph)
| | | . 5. ℕ
| | | . 6. p : Vertices(the_graph) List
| | | . 7. ∀p1:Vertices(the_graph) List.
| | | . |  ||p1||<||p||
| | | . |  ⇒
| | | . |  path(the_graph;p1)
| | | . |  ⇒
| | | . |  p1[0] = x
| | | . |  ⇒
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| | | . |  last(p1) = y
| | | . |  ⇒
| | | . |  x = y
| | | . |  ∨ (∃z:Vertices(the_graph).
| | | . |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . 8. path(the_graph;p)
| | | . 9. p[0] = x
| | | . 10. last(p) = y
| | | . 11. 1<||p||
| | | . 12. p[1] = x ∨ ¬p[1] = x
| | | . 13. path(the_graph;tl(p))
| | | . ⊢  x = y
| | | . |  ∨ (∃z:Vertices(the_graph).
| | | . |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . | by Analyze -2
| | | . |
| | | . |\
| | | . | 1. the_graph : Graph
| | | . | 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | . | 3. x : Vertices(the_graph)
| | | . | 4. y : Vertices(the_graph)
| | | . | 5. ℕ
| | | . | 6. p : Vertices(the_graph) List
| | | . | 7. ∀p1:Vertices(the_graph) List.
| | | . | |  ||p1||<||p||
| | | . | |  ⇒
| | | . | |  path(the_graph;p1)
| | | . | |  ⇒
| | | . | |  p1[0] = x
| | | . | |  ⇒
| | | . | |  last(p1) = y
| | | . | |  ⇒
| | | . | |  x = y
| | | . | |  ∨ (∃z:Vertices(the_graph).
| | | . | |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . | 8. path(the_graph;p)
| | | . | 9. p[0] = x
| | | . | 10. last(p) = y
| | | . | 11. 1<||p||
| | | . | 12. p[1] = x
| | | . | 13. path(the_graph;tl(p))
| | | . | ⊢  x = y
| | | . | |  ∨ (∃z:Vertices(the_graph).
| | | . | |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . | | by InstHyp [tl(p)] 7
| | | . | |
| | | . | |\
| | | . | | .....antecedent .....
| | | . | | 1. the_graph : Graph
| | | . | | 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | . | | 3. x : Vertices(the_graph)
| | | . | | 4. y : Vertices(the_graph)
| | | . | | 5. ℕ
| | | . | | 6. p : Vertices(the_graph) List
| | | . | | 7. ∀p1:Vertices(the_graph) List.
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| | | . | |    ||p1||<||p||
| | | . | |    ⇒
| | | . | |    path(the_graph;p1)
| | | . | |    ⇒
| | | . | |    p1[0] = x
| | | . | |    ⇒
| | | . | |    last(p1) = y
| | | . | |    ⇒
| | | . | |    x = y
| | | . | |    ∨ (∃z:Vertices(the_graph).
| | | . | |       ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . | | 8. path(the_graph;p)
| | | . | | 9. p[0] = x
| | | . | | 10. last(p) = y
| | | . | | 11. 1<||p||
| | | . | | 12. p[1] = x
| | | . | | 13. path(the_graph;tl(p))
| | | . | | ⊢  ||tl(p)||<||p||
| | | . | |   by RWO
| | | . | |      Thm* ∀l:A List. ||l||≥1 ⇒ ||tl(l)|| = ||l||-1 ∈ ℤ
| | | . | |      {length_tl}
| | | . | |      0
| | | . | |
| | | . | |\
| | | . | | .....antecedent .....
| | | . | | 1. the_graph : Graph
| | | . | | 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | . | | 3. x : Vertices(the_graph)
| | | . | | 4. y : Vertices(the_graph)
| | | . | | 5. ℕ
| | | . | | 6. p : Vertices(the_graph) List
| | | . | | 7. ∀p1:Vertices(the_graph) List.
| | | . | |    ||p1||<||p||
| | | . | |    ⇒
| | | . | |    path(the_graph;p1)
| | | . | |    ⇒
| | | . | |    p1[0] = x
| | | . | |    ⇒
| | | . | |    last(p1) = y
| | | . | |    ⇒
| | | . | |    x = y
| | | . | |    ∨ (∃z:Vertices(the_graph).
| | | . | |       ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . | | 8. path(the_graph;p)
| | | . | | 9. p[0] = x
| | | . | | 10. last(p) = y
| | | . | | 11. 1<||p||
| | | . | | 12. p[1] = x
| | | . | | 13. path(the_graph;tl(p))
| | | . | | ⊢  tl(p)[0] = x
| | | . | |   by (RWO
| | | . | |       Thm* ∀as:A List, n:ℕ(||as||-1).
| | | . | |            tl(as)[n] = as[(n+1)]
| | | . | |       {select_tl}
| | | . | |       0)
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| | | . | |      THEN
| | | . | |      (Reduce 0)
| | | . | |
| | | . | .\
| | | . | . .....antecedent .....
| | | . | . 1. the_graph : Graph
| | | . | . 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | . | . 3. x : Vertices(the_graph)
| | | . | . 4. y : Vertices(the_graph)
| | | . | . 5. ℕ
| | | . | . 6. p : Vertices(the_graph) List
| | | . | . 7. ∀p1:Vertices(the_graph) List.
| | | . | . |  ||p1||<||p||
| | | . | . |  ⇒
| | | . | . |  path(the_graph;p1)
| | | . | . |  ⇒
| | | . | . |  p1[0] = x
| | | . | . |  ⇒
| | | . | . |  last(p1) = y
| | | . | . |  ⇒
| | | . | . |  x = y
| | | . | . |  ∨ (∃z:Vertices(the_graph).
| | | . | . |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . | . 8. path(the_graph;p)
| | | . | . 9. p[0] = x
| | | . | . 10. last(p) = y
| | | . | . 11. 1<||p||

| | | . | . 12. p[1] = x
| | | . | . 13. path(the_graph;tl(p))
| | | . | . ⊢  last(tl(p)) = y  by (Analyze 6) THEN (All Reduce)
| | | . | . |
| | | . | . 1. the_graph : Graph
| | | . | . 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | . | . 3. x : Vertices(the_graph)
| | | . | . 4. y : Vertices(the_graph)
| | | . | . 5. ℕ
| | | . | . 6. u : Vertices(the_graph)
| | | . | . 7. v : Vertices(the_graph) List
| | | . | . 8. ∀p1:Vertices(the_graph) List.
| | | . | . |  ||p1||<||v||+1
| | | . | . |  ⇒
| | | . | . |  path(the_graph;p1)
| | | . | . |  ⇒
| | | . | . |  p1[0] = x
| | | . | . |  ⇒
| | | . | . |  last(p1) = y
| | | . | . |  ⇒
| | | . | . |  x = y
| | | . | . |  ∨ (∃z:Vertices(the_graph).
| | | . | . |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . | . 9. path(the_graph;[u / v])
| | | . | . 10. [u / v][0] = x
| | | . | . 11. last([u / v]) = y
| | | . | . 12. 1<||v||+1
| | | . | . 13. [u / v][1] = x
```

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| | | . | . 14. path(the_graph;v)
| | | . | . ⊢  last(v) = y
| | | . | . | by RWO
| | | . | . |    Thm* ∀L:T List, x:T.
| | | . | . |         ¬null(L) ⇒ last([x / L]) = last(L)
| | | . | . |    {last_cons}
| | | . | . |    -4
| | | . | . |
| | | . | . .....rewrite subgoal .....
| | | . | . 1. the_graph : Graph
| | | . | . 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | . | . 3. x : Vertices(the_graph)
| | | . | . 4. y : Vertices(the_graph)
| | | . | . 5. ℕ
| | | . | . 6. u : Vertices(the_graph)
| | | . | . 7. v : Vertices(the_graph) List
| | | . | . 8. ∀p1:Vertices(the_graph) List.
| | | . | .    ||p1||<||v||+1
| | | . | .    ⇒
| | | . | .    path(the_graph;p1)
| | | . | .    ⇒
| | | . | .    p1[0] = x
| | | . | .    ⇒
| | | . | .    last(p1) = y
| | | . | .    ⇒
| | | . | .    x = y
| | | . | .    ∨ (∃z:Vertices(the_graph).
| | | . | .       ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . | . 9. path(the_graph;[u / v])
| | | . | . 10. [u / v][0] = x
| | | . | . 11. last([u / v]) = y
| | | . | . 12. 1<||v||+1
| | | . | . 13. [u / v][1] = x
| | | . | . 14. path(the_graph;v)
| | | . | . ⊢  ¬null(v)
| | | . | .   by (RW assert_pushdownC 0) THEN (ObviousFrom [-3])
| | | . |
| | | . .\
| | | . . 1. the_graph : Graph
| | | . . 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | . . 3. x : Vertices(the_graph)
| | | . . 4. y : Vertices(the_graph)
| | | . . 5. ℕ
| | | . . 6. p : Vertices(the_graph) List
| | | . . 7. ∀p1:Vertices(the_graph) List.
| | | . . |  ||p1||<||p||
| | | . . |  ⇒
| | | . . |  path(the_graph;p1)
| | | . . |  ⇒
| | | . . |  p1[0] = x
| | | . . |  ⇒
| | | . . |  last(p1) = y
| | | . . |  ⇒
| | | . . |  x = y
| | | . . |  ∨ (∃z:Vertices(the_graph).
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| | | . . |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . . 8. path(the_graph;p)
| | | . . 9. p[0] = x
| | | . . 10. last(p) = y
| | | . . 11. 1<||p||
| | | . . 12. ¬p[1] = x
| | | . . 13. path(the_graph;tl(p))
| | | . . ⊢  x = y
| | | . . |  ∨ (∃z:Vertices(the_graph).
| | | . . |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . . | by (Unfold `path` -6) THEN OrRight
| | | . . |    THEN
| | | . . |    (InstConcl [p[1]])
| | | . . |
| | | . . |\
| | | . . | 1. the_graph : Graph
| | | . . | 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | . . | 3. x : Vertices(the_graph)
| | | . . | 4. y : Vertices(the_graph)
| | | . . | 5. ℕ
| | | . . | 6. p : Vertices(the_graph) List
| | | . . | 7. ∀p1:Vertices(the_graph) List.
| | | . . |    ||p1||<||p||
| | | . . |    ⇒
| | | . . |    path(the_graph;p1)
| | | . . |    ⇒
| | | . . |    p1[0] = x
| | | . . |    ⇒
| | | . . |    last(p1) = y
| | | . . |    ⇒
| | | . . |    x = y
| | | . . |    ∨ (∃z:Vertices(the_graph).
| | | . . |       ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . . | 8. 0<||p||
| | | . . | 9. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
| | | . . | 10. p[0] = x
| | | . . | 11. last(p) = y
| | | . . | 12. 1<||p||
| | | . . | 13. ¬p[1] = x
| | | . . | 14. path(the_graph;tl(p))
| | | . . | ⊢  x-the_graph->p[1]
| | | . . |   by (InstHyp [0] -6) THEN (NthHypEq -1) THEN Analyze
| | | . . |      THEN
| | | . . |      Analyze
| | | . . |
| | | . . .\
| | | . . . 1. the_graph : Graph
| | | . . . 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | . . . 3. x : Vertices(the_graph)
| | | . . . 4. y : Vertices(the_graph)
| | | . . . 5. ℕ
| | | . . . 6. p : Vertices(the_graph) List
| | | . . . 7. ∀p1:Vertices(the_graph) List.
| | | . . . |  ||p1||<||p||
| | | . . . |  ⇒
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| | | . . . |  path(the_graph;p1)
| | | . . . |  ⇒
| | | . . . |  p1[0] = x
| | | . . . |  ⇒
| | | . . . |  last(p1) = y
| | | . . . |  ⇒
| | | . . . |  x = y
| | | . . . |  ∨ (∃z:Vertices(the_graph).
| | | . . . |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . . . 8. 0<||p||
| | | . . . 9. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
| | | . . . 10. p[0] = x
| | | . . . 11. last(p) = y
| | | . . . 12. 1<||p||
| | | . . . 13. ¬p[1] = x
| | | . . . 14. path(the_graph;tl(p))
| | | . . . ⊢  p[1]-the_graph->*y
| | | . . . | by (Unfold `connect` 0) THEN (InstConcl [tl(p)])
| | | . . . |    THEN
| | | . . . |    (Try ((Unfold `path` -1) THEN ExRepD))
| | | . . . |    THEN
| | | . . . |    (Try (RW assert_pushdownC 0))
| | | . . . |    THEN
| | | . . . |    (Try (ObviousFrom [-2]))
| | | . . . |
| | | . . . 1. the_graph : Graph
| | | . . . 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | | . . . 3. x : Vertices(the_graph)
| | | . . . 4. y : Vertices(the_graph)
| | | . . . 5. ℕ
| | | . . . 6. p : Vertices(the_graph) List
| | | . . . 7. ∀p1:Vertices(the_graph) List.
| | | . . .    ||p1||<||p||
| | | . . .    ⇒
| | | . . .    path(the_graph;p1)
| | | . . .    ⇒
| | | . . .    p1[0] = x
| | | . . .    ⇒
| | | . . .    last(p1) = y
| | | . . .    ⇒
| | | . . .    x = y
| | | . . .    ∨ (∃z:Vertices(the_graph).
| | | . . .       ¬z = x & x-the_graph->z & z-the_graph->*y)
| | | . . . 8. 0<||p||
| | | . . . 9. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
| | | . . . 10. p[0] = x
| | | . . . 11. last(p) = y
| | | . . . 12. 1<||p||
| | | . . . 13. ¬p[1] = x
| | | . . . 14. 0<||tl(p)||
| | | . . . 15. ∀i:ℕ(||tl(p)||-1). tl(p)[i]-the_graph->tl(p)[(i+1)]
| | | . . . ⊢  last(tl(p)) = y
| | | . . .   by (Analyze 6) THEN (All Reduce)
| | | . . .      THEN
| | | . . .      (RWO
```

```
| | | . . .       Thm* ∀L:T List, x:T.
| | | . . .            ¬null(L) ⇒ last([x / L]) = last(L)
| | | . . .       {last_cons}
| | | . . .       -5)
| | | . . .      THEN
| | | . . .      (RW assert_pushdownC 0)
| | | . . .      THEN
| | | . . .      (ObviousFrom [-2])
| | |
| | .\
| | . 1. the_graph : Graph
| | . 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | . 3. x : Vertices(the_graph)
| | . 4. y : Vertices(the_graph)
| | . 5. ℕ
| | . 6. p : Vertices(the_graph) List
| | . 7. ∀p1:Vertices(the_graph) List.
| | . |  ||p1||<||p||
| | . |  ⇒
| | . |  path(the_graph;p1)
| | . |  ⇒
| | . |  p1[0] = x
| | . |  ⇒
| | . |  last(p1) = y
| | . |  ⇒
| | . |  x = y
| | . |  ∨ (∃z:Vertices(the_graph).
| | . |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | . 8. path(the_graph;p)
| | . 9. p[0] = x
| | . 10. last(p) = y
| | . 11. ¬1<||p||
| | . ⊢  x = y
| | . |  ∨ (∃z:Vertices(the_graph).
| | . |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | . | by OrLeft THEN (Unfold `last` -2) THEN (Subst (||p|| ∼ 1) -2)
| | . |
| | . |\
| | . | .....equality .....
| | . | 1. the_graph : Graph
| | . | 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | . | 3. x : Vertices(the_graph)
| | . | 4. y : Vertices(the_graph)
| | . | 5. ℕ
| | . | 6. p : Vertices(the_graph) List
| | . | 7. ∀p1:Vertices(the_graph) List.
| | . |    ||p1||<||p||
| | . |    ⇒
| | . |    path(the_graph;p1)
| | . |    ⇒
| | . |    p1[0] = x
| | . |    ⇒
| | . |    last(p1) = y
| | . |    ⇒
| | . |    x = y
```

```
| | . |    ∨ (∃z:Vertices(the_graph).
| | . |       ¬z = x & x-the_graph->z & z-the_graph->*y)
| | . | 8. path(the_graph;p)
| | . | 9. p[0] = x
| | . | 10. p[(||p||-1)] = y
| | . | 11. ¬1<||p||
| | . | ⊢  ||p|| ∼ 1  by (Unfold ‘path‘ -4) THEN ExRepD
| | . |
| | . .\
| | . . 1. the_graph : Graph
| | . . 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | . . 3. x : Vertices(the_graph)
| | . . 4. y : Vertices(the_graph)
| | . . 5. ℕ
| | . . 6. p : Vertices(the_graph) List
| | . . 7. ∀p1:Vertices(the_graph) List.
| | . .    ||p1||<||p||
| | . .    ⇒
| | . .    path(the_graph;p1)
| | . .    ⇒
| | . .    p1[0] = x
| | . .    ⇒
| | . .    last(p1) = y
| | . .    ⇒
| | . .    x = y
| | . .    ∨ (∃z:Vertices(the_graph).
| | . .       ¬z = x & x-the_graph->z & z-the_graph->*y)
| | . . 8. path(the_graph;p)
| | . . 9. p[0] = x
| | . . 10. p[(1-1)] = y
| | . . 11. ¬1<||p||
| | . . ⊢  x = y  by Reduce -2
| |
| |\
| | 1. the_graph : Graph
| | 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | 3. x : Vertices(the_graph)
| | 4. y : Vertices(the_graph)
| | 5. ℕ
| | 6. p : Vertices(the_graph) List
| | 7. ∀p1:Vertices(the_graph) List.
| | |  ||p1||<||p||
| | |  ⇒
| | |  path(the_graph;p1)
| | |  ⇒
| | |  p1[0] = x
| | |  ⇒
| | |  last(p1) = y
| | |  ⇒
| | |  x = y
| | |  ∨ (∃z:Vertices(the_graph).
| | |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | 8. path(the_graph;p)
| | 9. p[0] = x
| | ⊢  ¬null(p)  by (Unfold ‘path‘ -2) THEN ExRepD
```

```
| | |
| | 1. the_graph : Graph
| | 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | 3. x : Vertices(the_graph)
| | 4. y : Vertices(the_graph)
| | 5. ℕ
| | 6. p : Vertices(the_graph) List
| | 7. ∀p1:Vertices(the_graph) List.
| | |  ||p1||<||p||
| | |  ⇒
| | |  path(the_graph;p1)
| | |  ⇒
| | |  p1[0] = x
| | |  ⇒
| | |  last(p1) = y
| | |  ⇒
| | |  x = y
| | |  ∨ (∃z:Vertices(the_graph).
| | |     ¬z = x & x-the_graph->z & z-the_graph->*y)
| | 8. 0<||p||
| | 9. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
| | 10. p[0] = x
| | ⊢  ¬null(p)  by RW assert_pushdownC 0
| | |
| | 1. the_graph : Graph
| | 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| | 3. x : Vertices(the_graph)
| | 4. y : Vertices(the_graph)
| | 5. ℕ
| | 6. p : Vertices(the_graph) List
| | 7. ∀p1:Vertices(the_graph) List.
| |    ||p1||<||p||
| |    ⇒
| |    path(the_graph;p1)
| |    ⇒
| |    p1[0] = x
| |    ⇒
| |    last(p1) = y
| |    ⇒
| |    x = y
| |    ∨ (∃z:Vertices(the_graph).
| |       ¬z = x & x-the_graph->z & z-the_graph->*y)
| | 8. 0<||p||
| | 9. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
| | 10. p[0] = x
| | ⊢  ¬p = nil  by ObviousFrom [-3]
| |
| .\
| . 1. the_graph : Graph
| . 2. ∀x,y:Vertices(the_graph). Dec(x = y)
| . 3. x : Vertices(the_graph)
| . 4. y : Vertices(the_graph)
| . 5. ℕ
| . 6. p : Vertices(the_graph) List
| . 7. ∀p1:Vertices(the_graph) List.
```

```
| .    ||p1||<||p||
| .    ⇒
| .    path(the_graph;p1)
| .    ⇒
| .    p1[0] = x
| .    ⇒
| .    last(p1) = y
| .    ⇒
| .    x = y
| .    ∨ (∃z:Vertices(the_graph).
| .       ¬z = x & x-the_graph->z & z-the_graph->*y)
| . 8. path(the_graph;p)
| . ⊢  0<||p||  by (Unfold ‘path‘ -1) THEN ExRepD
|
.\
. 1. the_graph : Graph
. 2. ∀x,y:Vertices(the_graph). Dec(x = y)
. 3. x : Vertices(the_graph)
. 4. y : Vertices(the_graph)
. 5. x = y
. |  ∨ (∃z:Vertices(the_graph).
. |     ¬z = x & x-the_graph->z & z-the_graph->*y)
. ⊢  x-the_graph->*y  by SplitOrHyps
. |
. |\
. | 1. the_graph : Graph
. | 2. ∀x,y:Vertices(the_graph). Dec(x = y)
. | 3. x : Vertices(the_graph)
. | 4. y : Vertices(the_graph)
. | 5. x = y
. | ⊢  x-the_graph->*y  by (HypSubst -1 0) THEN Connect
. |
. .\
. . 1. the_graph : Graph
. . 2. ∀x,y:Vertices(the_graph). Dec(x = y)
. . 3. x : Vertices(the_graph)
. . 4. y : Vertices(the_graph)
. . 5. ∃z:Vertices(the_graph).
. .    ¬z = x & x-the_graph->z & z-the_graph->*y
. . ⊢  x-the_graph->*y  by ExRepD THEN Connect
END
```