```
connect_transitivity  THM

⊢  For any graph
|  ∀x,y,z:V. x-the_graph->*y ⇒ y-the_graph->*z ⇒ x-the_graph->*z
| by Auto THEN (All (Unfold ‘connect‘)) THEN ExRepD
|    THEN
|    (All (Unfolds [‘last‘;‘path‘]))
|    THEN
|    ExRepD
|    THEN
|    (InstConcl [p1 @ tl(p)])
|    THEN
|    ExRepD
|    THEN
|    (Try
|     ((RWO
|       Thm* ∀as,bs:T List, i:ℕ||as||. (as @ bs)[i] = as[i]
|       {select_append_front}
|       0)
|      THEN
|      (Complete Auto)))
|
|\
| 1. the_graph : Graph
| 2. x : Vertices(the_graph)
| 3. y : Vertices(the_graph)
| 4. z : Vertices(the_graph)
| 5. p1 : Vertices(the_graph) List
| 6. 0<||p1||
| 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
| 8. p1[0] = x
| 9. p1[(||p1||-1)] = y
| 10. p : Vertices(the_graph) List
| 11. 0<||p||
| 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
| 13. p[0] = y
| 14. p[(||p||-1)] = z
| 15. i : ℕ(||p1 @ tl(p)||-1)
| ⊢  (p1 @ tl(p))[i]-the_graph->(p1 @ tl(p))[(i+1)]
| | by (Analyze -6) THEN (All Reduce) THEN (Decide (i+1<||p1||))
| |
| |\
| | 1. the_graph : Graph
| | 2. x : Vertices(the_graph)
| | 3. y : Vertices(the_graph)
| | 4. z : Vertices(the_graph)
| | 5. p1 : Vertices(the_graph) List
| | 6. 0<||p1||
| | 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
| | 8. p1[0] = x
| | 9. p1[(||p1||-1)] = y
| | 10. u : Vertices(the_graph)
| | 11. v : Vertices(the_graph) List
| | 12. 0<||v||+1
| | 13. ∀i:ℕ(||v||+1-1). [u / v][i]-the_graph->[u / v][(i+1)]
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| | 14. u = y
| | 15. [u / v][(||v||+1-1)] = z
| | 16. i : ℕ(||p1 @ v||-1)
| | 17. i+1<||p1||
| | ⊢  (p1 @ v)[i]-the_graph->(p1 @ v)[(i+1)]
| |   by (RWO
| |       Thm* ∀as,bs:T List, i:ℕ||as||. (as @ bs)[i] = as[i]
| |       {select_append_front}
| |       0)
| |      THEN
| |      EasyHyp
| |
| .\
| . 1. the_graph : Graph
| . 2. x : Vertices(the_graph)
| . 3. y : Vertices(the_graph)
| . 4. z : Vertices(the_graph)
| . 5. p1 : Vertices(the_graph) List
| . 6. 0<||p1||
| . 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
| . 8. p1[0] = x
| . 9. p1[(||p1||-1)] = y
| . 10. u : Vertices(the_graph)
| . 11. v : Vertices(the_graph) List
| . 12. 0<||v||+1
| . 13. ∀i:ℕ(||v||+1-1). [u / v][i]-the_graph->[u / v][(i+1)]
| . 14. u = y
| . 15. [u / v][(||v||+1-1)] = z
| . 16. i : ℕ(||p1 @ v||-1)
| . 17. ¬i+1<||p1||
| . ⊢  (p1 @ v)[i]-the_graph->(p1 @ v)[(i+1)]
| . | by Decide (i = ||p1||-1)
| . |
| . |\
| . | 1. the_graph : Graph
| . | 2. x : Vertices(the_graph)
| . | 3. y : Vertices(the_graph)
| . | 4. z : Vertices(the_graph)
| . | 5. p1 : Vertices(the_graph) List
| . | 6. 0<||p1||
| . | 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
| . | 8. p1[0] = x
| . | 9. p1[(||p1||-1)] = y
| . | 10. u : Vertices(the_graph)
| . | 11. v : Vertices(the_graph) List
| . | 12. 0<||v||+1
| . | 13. ∀i:ℕ(||v||+1-1). [u / v][i]-the_graph->[u / v][(i+1)]
| . | 14. u = y
| . | 15. [u / v][(||v||+1-1)] = z
| . | 16. i : ℕ(||p1 @ v||-1)
| . | 17. ¬i+1<||p1||
| . | 18. i = ||p1||-1
| . | ⊢  (p1 @ v)[i]-the_graph->(p1 @ v)[(i+1)]
| . | | by (HypSubst' -1 0) THEN (Subst ((p1 @ v)[(||p1||-1)] = u) 0)
| . | |
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| . | |\
| . | | .....equality .....
| . | | 1. the_graph : Graph
| . | | 2. x : Vertices(the_graph)
| . | | 3. y : Vertices(the_graph)
| . | | 4. z : Vertices(the_graph)
| . | | 5. p1 : Vertices(the_graph) List
| . | | 6. 0<||p1||
| . | | 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
| . | | 8. p1[0] = x
| . | | 9. p1[(||p1||-1)] = y
| . | | 10. u : Vertices(the_graph)
| . | | 11. v : Vertices(the_graph) List
| . | | 12. 0<||v||+1
| . | | 13. ∀i:ℕ(||v||+1-1). [u / v][i]-the_graph->[u / v][(i+1)]
| . | | 14. u = y
| . | | 15. [u / v][(||v||+1-1)] = z
| . | | 16. i : ℕ(||p1 @ v||-1)
| . | | 17. ¬i+1<||p1||
| . | | 18. i = ||p1||-1
| . | | ⊢  (p1 @ v)[(||p1||-1)] = u
| . | |   by RWO
| . | |      Thm* ∀as,bs:T List, i:ℕ||as||. (as @ bs)[i] = as[i]
| . | |      {select_append_front}
| . | |      0
| . | |
| . | .\
| . | . 1. the_graph : Graph
| . | . 2. x : Vertices(the_graph)
| . | . 3. y : Vertices(the_graph)
| . | . 4. z : Vertices(the_graph)
| . | . 5. p1 : Vertices(the_graph) List
| . | . 6. 0<||p1||
| . | . 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
| . | . 8. p1[0] = x
| . | . 9. p1[(||p1||-1)] = y
| . | . 10. u : Vertices(the_graph)
| . | . 11. v : Vertices(the_graph) List
| . | . 12. 0<||v||+1
| . | . 13. ∀i:ℕ(||v||+1-1). [u / v][i]-the_graph->[u / v][(i+1)]
| . | . 14. u = y
| . | . 15. [u / v][(||v||+1-1)] = z
| . | . 16. i : ℕ(||p1 @ v||-1)
| . | . 17. ¬i+1<||p1||
| . | . 18. i = ||p1||-1
| . | . ⊢  u-the_graph->(p1 @ v)[(||p1||-1+1)]
| . | . | by RWO
| . | . |    Thm* ∀as,bs:T List, i:{||as||..(||as||+||bs||)⁻}.
| . | . |         (as @ bs)[i] = bs[(i-||as||)]
| . | . |    {select_append_back}
| . | . |    0
| . | . |
| . | . 1. the_graph : Graph
| . | . 2. x : Vertices(the_graph)
| . | . 3. y : Vertices(the_graph)
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| . | . 4. z : Vertices(the_graph)
| . | . 5. p1 : Vertices(the_graph) List
| . | . 6. 0<||p1||
| . | . 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
| . | . 8. p1[0] = x
| . | . 9. p1[(||p1||-1)] = y
| . | . 10. u : Vertices(the_graph)
| . | . 11. v : Vertices(the_graph) List
| . | . 12. 0<||v||+1
| . | . 13. ∀i:ℕ(||v||+1-1). [u / v][i]-the_graph->[u / v][(i+1)]
| . | . 14. u = y
| . | . 15. [u / v][(||v||+1-1)] = z
| . | . 16. i : ℕ(||p1 @ v||-1)
| . | . 17. ¬i+1<||p1||
| . | . 18. i = ||p1||-1
| . | . ⊢  u-the_graph->v[(||p1||-1+1-||p1||)]
| . | .   by (InstHyp [0] -6) THEN (Reduce -1) THEN (NthHypSq -1)
| . | .      THEN
| . | .      Analyze
| . | .      THEN
| . | .      Analyze
| . | .      THEN
| . | .      Analyze
| . |
| . .\
| . . 1. the_graph : Graph
| . . 2. x : Vertices(the_graph)
| . . 3. y : Vertices(the_graph)
| . . 4. z : Vertices(the_graph)
| . . 5. p1 : Vertices(the_graph) List
| . . 6. 0<||p1||
| . . 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
| . . 8. p1[0] = x
| . . 9. p1[(||p1||-1)] = y
| . . 10. u : Vertices(the_graph)
| . . 11. v : Vertices(the_graph) List
| . . 12. 0<||v||+1
| . . 13. ∀i:ℕ(||v||+1-1). [u / v][i]-the_graph->[u / v][(i+1)]
| . . 14. u = y
| . . 15. [u / v][(||v||+1-1)] = z
| . . 16. i : ℕ(||p1 @ v||-1)
| . . 17. ¬i+1<||p1||
| . . 18. ¬i = ||p1||-1
| . . ⊢  (p1 @ v)[i]-the_graph->(p1 @ v)[(i+1)]
| . .   by (RWO
| . .       Thm* ∀as,bs:T List, i:{||as||..(||as||+||bs||)⁻}.
| . .            (as @ bs)[i] = bs[(i-||as||)]
| . .       {select_append_back}
| . .       0)
| . .      THEN
| . .      (InstHyp [i-||p1||+1] -6)
| . .      THEN
| . .      (RWO
| . .       Thm* ∀a:T, as:T List, i:ℤ.
| . .            0<i ⇒ i≤||as|| ⇒ [a / as][i] = as[(i-1)]
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| . .       {select_cons_tl}
| . .       -1)
| . .      THEN
| . .      (NthHypSq -1)
| . .      THEN
| . .      Analyze
| . .      THEN
| . .      Analyze
|
.\
. 1. the_graph : Graph
. 2. x : Vertices(the_graph)
. 3. y : Vertices(the_graph)
. 4. z : Vertices(the_graph)
. 5. p1 : Vertices(the_graph) List
. 6. 0<||p1||
. 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
. 8. p1[0] = x
. 9. p1[(||p1||-1)] = y
. 10. p : Vertices(the_graph) List
. 11. 0<||p||
. 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
. 13. p[0] = y
. 14. p[(||p||-1)] = z
. ⊢  (p1 @ tl(p))[(||p1 @ tl(p)||-1)] = z  by Decide (1<||p||)
. |
. |\
. | 1. the_graph : Graph
. | 2. x : Vertices(the_graph)
. | 3. y : Vertices(the_graph)
. | 4. z : Vertices(the_graph)
. | 5. p1 : Vertices(the_graph) List
. | 6. 0<||p1||
. | 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
. | 8. p1[0] = x
. | 9. p1[(||p1||-1)] = y
. | 10. p : Vertices(the_graph) List
. | 11. 0<||p||
. | 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
. | 13. p[0] = y
. | 14. p[(||p||-1)] = z
. | 15. 1<||p||
. | ⊢  (p1 @ tl(p))[(||p1 @ tl(p)||-1)] = z
. | | by (RWO
. | |     Thm* ∀as,bs:T List. ||as @ bs|| = ||as||+||bs|| ∈ ℤ
. | |     {length_append}
. | |     0)
. | |    THEN
. | |    (RWO
. | |     Thm* ∀as,bs:T List, i:{||as||..(||as||+||bs||)⁻}.
. | |          (as @ bs)[i] = bs[(i-||as||)]
. | |     {select_append_back}
. | |     0)
. | |
. | |\
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. | | .....wf .....
. | | 1. the_graph : Graph
. | | 2. x : Vertices(the_graph)
. | | 3. y : Vertices(the_graph)
. | | 4. z : Vertices(the_graph)
. | | 5. p1 : Vertices(the_graph) List
. | | 6. 0<||p1||
. | | 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
. | | 8. p1[0] = x
. | | 9. p1[(||p1||-1)] = y
. | | 10. p : Vertices(the_graph) List
. | | 11. 0<||p||
. | | 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
. | | 13. p[0] = y
. | | 14. p[(||p||-1)] = z
. | | 15. 1<||p||
. | | ⊢  ||p1||+||tl(p)||-1 ∈ {||p1||..(||p1||+||tl(p)||)⁻}
. | |   by Analyze
. | |      THEN
. | |      (RWO
. | |       Thm* ∀l:A List. ||l||≥1 ⇒ ||tl(l)|| = ||l||-1 ∈ ℤ
. | |       {length_tl}
. | |       0)
. | |
. | .\
. | . 1. the_graph : Graph
. | . 2. x : Vertices(the_graph)
. | . 3. y : Vertices(the_graph)
. | . 4. z : Vertices(the_graph)
. | . 5. p1 : Vertices(the_graph) List
. | . 6. 0<||p1||
. | . 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
. | . 8. p1[0] = x
. | . 9. p1[(||p1||-1)] = y
. | . 10. p : Vertices(the_graph) List
. | . 11. 0<||p||
. | . 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
. | . 13. p[0] = y
. | . 14. p[(||p||-1)] = z
. | . 15. 1<||p||
. | . ⊢  tl(p)[(||p1||+||tl(p)||-1-||p1||)] = z
. | . | by RWO
. | . |    Thm* ∀as:A List, n:ℕ(||as||-1). tl(as)[n] = as[(n+1)]
. | . |    {select_tl}
. | . |    0
. | . |
. | . |\
. | . | .....wf .....
. | . | 1. the_graph : Graph
. | . | 2. x : Vertices(the_graph)
. | . | 3. y : Vertices(the_graph)
. | . | 4. z : Vertices(the_graph)
. | . | 5. p1 : Vertices(the_graph) List
. | . | 6. 0<||p1||
. | . | 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
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. | . | 8. p1[0] = x
. | . | 9. p1[(||p1||-1)] = y
. | . | 10. p : Vertices(the_graph) List
. | . | 11. 0<||p||
. | . | 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
. | . | 13. p[0] = y
. | . | 14. p[(||p||-1)] = z
. | . | 15. 1<||p||
. | . | ⊢  ||p1||+||tl(p)||-1-||p1|| ∈ ℕ(||p||-1)
. | . |   by Analyze
. | . |      THEN
. | . |      (RWO
. | . |       Thm* ∀l:A List. ||l||≥1 ⇒ ||tl(l)|| = ||l||-1 ∈ ℤ
. | . |       {length_tl}
. | . |       0)
. | . |
. | . .\
. | . . 1. the_graph : Graph
. | . . 2. x : Vertices(the_graph)
. | . . 3. y : Vertices(the_graph)
. | . . 4. z : Vertices(the_graph)
. | . . 5. p1 : Vertices(the_graph) List
. | . . 6. 0<||p1||
. | . . 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
. | . . 8. p1[0] = x
. | . . 9. p1[(||p1||-1)] = y
. | . . 10. p : Vertices(the_graph) List
. | . . 11. 0<||p||
. | . . 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
. | . . 13. p[0] = y
. | . . 14. p[(||p||-1)] = z
. | . . 15. 1<||p||
. | . . ⊢  p[(||p1||+||tl(p)||-1-||p1||+1)] = z
. | . . | by (NthHypSq -2) THEN Analyze THEN Analyze
. | . . |
. | . . 1. the_graph : Graph
. | . . 2. x : Vertices(the_graph)
. | . . 3. y : Vertices(the_graph)
. | . . 4. z : Vertices(the_graph)
. | . . 5. p1 : Vertices(the_graph) List
. | . . 6. 0<||p1||
. | . . 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
. | . . 8. p1[0] = x
. | . . 9. p1[(||p1||-1)] = y
. | . . 10. p : Vertices(the_graph) List
. | . . 11. 0<||p||
. | . . 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
. | . . 13. p[0] = y
. | . . 14. p[(||p||-1)] = z
. | . . 15. 1<||p||
. | . . ⊢  ||p1||+||tl(p)||-1-||p1||+1 = ||p||-1
. | . .   by RWO
. | . .      Thm* ∀l:A List. ||l||≥1 ⇒ ||tl(l)|| = ||l||-1 ∈ ℤ
. | . .      {length_tl}
. | . .      0
```

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. |
. .\
. . 1. the_graph : Graph
. . 2. x : Vertices(the_graph)
. . 3. y : Vertices(the_graph)
. . 4. z : Vertices(the_graph)
. . 5. p1 : Vertices(the_graph) List
. . 6. 0<||p1||
. . 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
. . 8. p1[0] = x
. . 9. p1[(||p1||-1)] = y
. . 10. p : Vertices(the_graph) List
. . 11. 0<||p||
. . 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
. . 13. p[0] = y
. . 14. p[(||p||-1)] = z
. . 15. ¬1<||p||
. . ⊢  (p1 @ tl(p))[(||p1 @ tl(p)||-1)] = z  by Subst (tl(p) ~ nil) 0
. . |
. . |\
. . | .....equality .....
. . | 1. the_graph : Graph
. . | 2. x : Vertices(the_graph)
. . | 3. y : Vertices(the_graph)
. . | 4. z : Vertices(the_graph)
. . | 5. p1 : Vertices(the_graph) List
. . | 6. 0<||p1||
. . | 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
. . | 8. p1[0] = x
. . | 9. p1[(||p1||-1)] = y
. . | 10. p : Vertices(the_graph) List
. . | 11. 0<||p||
. . | 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
. . | 13. p[0] = y
. . | 14. p[(||p||-1)] = z
. . | 15. ¬1<||p||
. . | ⊢  tl(p) ~ nil
. . |   by RepeatFor 2
. . |      ((Analyze -6) THEN (All Reduce) THEN (Try Trivial))
. . |
. . .\
. . . 1. the_graph : Graph
. . . 2. x : Vertices(the_graph)
. . . 3. y : Vertices(the_graph)
. . . 4. z : Vertices(the_graph)
. . . 5. p1 : Vertices(the_graph) List
. . . 6. 0<||p1||
. . . 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
. . . 8. p1[0] = x
. . . 9. p1[(||p1||-1)] = y
. . . 10. p : Vertices(the_graph) List
. . . 11. 0<||p||
. . . 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
. . . 13. p[0] = y
. . . 14. p[(||p||-1)] = z
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. . . 15. ¬1<||p||
. . . ⊢  (p1 @ nil)[(||p1 @ nil||-1)] = z
. . . | by Subst ((p1 @ nil) ~ p1) 0
. . . |
. . . |\
. . . | .....equality .....
. . . | 1. the_graph : Graph
. . . | 2. x : Vertices(the_graph)
. . . | 3. y : Vertices(the_graph)
. . . | 4. z : Vertices(the_graph)
. . . | 5. p1 : Vertices(the_graph) List
. . . | 6. 0<||p1||
. . . | 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
. . . | 8. p1[0] = x
. . . | 9. p1[(||p1||-1)] = y
. . . | 10. p : Vertices(the_graph) List
. . . | 11. 0<||p||
. . . | 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
. . . | 13. p[0] = y
. . . | 14. p[(||p||-1)] = z
. . . | 15. ¬1<||p||
. . . | ⊢  (p1 @ nil) ~ p1
. . . |   by BackThru
. . . |      Thm* ∀l:T List. (l @ nil) ~ l    {append_nil_sq}
. . . |
. . . .\
. . . . 1. the_graph : Graph
. . . . 2. x : Vertices(the_graph)
. . . . 3. y : Vertices(the_graph)
. . . . 4. z : Vertices(the_graph)
. . . . 5. p1 : Vertices(the_graph) List
. . . . 6. 0<||p1||
. . . . 7. ∀i:ℕ(||p1||-1). p1[i]-the_graph->p1[(i+1)]
. . . . 8. p1[0] = x
. . . . 9. p1[(||p1||-1)] = y
. . . . 10. p : Vertices(the_graph) List
. . . . 11. 0<||p||
. . . . 12. ∀i:ℕ(||p||-1). p[i]-the_graph->p[(i+1)]
. . . . 13. p[0] = y
. . . . 14. p[(||p||-1)] = z
. . . . 15. ¬1<||p||
. . . . ⊢  p1[(||p1||-1)] = z
. . . .   by (Subst' (||p|| = 1) -2) THEN (Reduce -2)
END
```