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dfl-traversal-consl  THM

⊢  For any graph
|  ∀L:V List, i:V, s:Traversal.
|  (inr(i) ∈ s)
|  ⇒
|  dfl-traversal(the_graph;L;s)
|  ⇒
|  dfl-traversal(the_graph;L;[inl(i) / s])
| by Auto THEN (Unfold `dfl-traversal` 0)
|
|\
| 1. the_graph : Graph
| 2. L : Vertices(the_graph) List
| 3. i : Vertices(the_graph)
| 4. s : traversal(the_graph)
| 5. (inr(i) ∈ s)
| 6. dfl-traversal(the_graph;L;s)
| 7. i@0 : Vertices(the_graph)
| 8. s1 : traversal(the_graph)
| 9. s2 : traversal(the_graph)
| 10. [inl(i) / s] = (s1 @ [inr(i@0)] @ s2)
| 11. j : Vertices(the_graph)
| 12. (inr(j) ∈ s2)
| 13. ¬(inl(j) ∈ s2)
| ⊢  j-the_graph->*i@0
| | by (Analyze -6) THEN (All Reduce) THEN (SplitCons -4)
| |    THEN
| |    (Try ((Reduce 0) THEN (Unfold `label` 0) THEN (Reduce 0)))
| |
| |\
| | 1. the_graph : Graph
| | 2. L : Vertices(the_graph) List
| | 3. i : Vertices(the_graph)
| | 4. s : traversal(the_graph)
| | 5. (inr(i) ∈ s)
| | 6. dfl-traversal(the_graph;L;s)
| | 7. i@0 : Vertices(the_graph)
| | 8. s2 : traversal(the_graph)
| | 9. j : Vertices(the_graph)
| | 10. (inr(j) ∈ s2)
| | 11. ¬(inl(j) ∈ s2)
| | 12. inl(i) = inr(i@0)
| | 13. s = s2
| | ⊢  j-the_graph->*i@0  by ObviousContradiction [-2]
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| . 1. the_graph : Graph
| . 2. L : Vertices(the_graph) List
| . 3. i : Vertices(the_graph)
| . 4. s : traversal(the_graph)
| . 5. (inr(i) ∈ s)
| . 6. dfl-traversal(the_graph;L;s)
| . 7. i@0 : Vertices(the_graph)
| . 8. u : Vertices(the_graph)+Vertices(the_graph)
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| . 9. v : (Vertices(the_graph)+Vertices(the_graph)) List
| . 10. s2 : traversal(the_graph)
| . 11. j : Vertices(the_graph)
| . 12. (inr(j) ∈ s2)
| . 13. ¬(inl(j) ∈ s2)
| . 14. inl(i) = u
| . 15. s = (v @ [inr(i@0) / s2])
| . ⊢  j-the_graph->*i@0
| .   by AllHyps
| .      (λh.
| .       (Unfold `dfl-traversal` h) THEN (Analyze h) THEN (Analyze h)
| .       THEN
| .       (InstHyp [i@0;v;s2;j] h)
| .       THEN
| .       (Reduce 0))
|
|\
| 1. the_graph : Graph
| 2. L : Vertices(the_graph) List
| 3. i : Vertices(the_graph)
| 4. s : traversal(the_graph)
| 5. (inr(i) ∈ s)
| 6. dfl-traversal(the_graph;L;s)
| 7. j : Vertices(the_graph)
| 8. (inr(j) ∈ [inl(i) / s])
| ⊢  L-the_graph->*j
| | by (RWO
| |     Thm* ∀l:T List, a,x:T. (x ∈ [a / l]) ⟺ x = a ∨ (x ∈ l)
| |     {cons_member}
| |     -1)
| |    THEN
| |    (Analyze -1)
| |
| |\
| | 1. the_graph : Graph
| | 2. L : Vertices(the_graph) List
| | 3. i : Vertices(the_graph)
| | 4. s : traversal(the_graph)
| | 5. (inr(i) ∈ s)
| | 6. dfl-traversal(the_graph;L;s)
| | 7. j : Vertices(the_graph)
| | 8. inr(j) = inl(i)
| | ⊢  L-the_graph->*j  by ObviousContradiction [-1]
| |
| .\
| . 1. the_graph : Graph
| . 2. L : Vertices(the_graph) List
| . 3. i : Vertices(the_graph)
| . 4. s : traversal(the_graph)
| . 5. (inr(i) ∈ s)
| . 6. dfl-traversal(the_graph;L;s)
| . 7. j : Vertices(the_graph)
| . 8. (inr(j) ∈ s)
| . ⊢  L-the_graph->*j
| .   by AllHyps
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| .      (λh.(Analyze h) THEN (Analyze h) THEN (BackThru h+1))
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. 1. the_graph : Graph
. 2. L : Vertices(the_graph) List
. 3. i : Vertices(the_graph)
. 4. s : traversal(the_graph)
. 5. (inr(i) ∈ s)
. 6. dfl-traversal(the_graph;L;s)
. 7. i@0 : Vertices(the_graph)
. 8. s1 : traversal(the_graph)
. 9. s2 : traversal(the_graph)
. 10. ∀j:Vertices(the_graph).
. |   i@0-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. 11. [inl(i) / s] = (s1 @ [inl(i@0)] @ s2)
. ⊢  L-the_graph->*i@0
. | by (Analyze -4) THEN (All Reduce) THEN (SplitCons -1)
. |    THEN
. |    (Try ((Reduce 0) THEN (Reduce 0) THEN (Unfold `label` 0)))
. |
. |\
. | 1. the_graph : Graph
. | 2. L : Vertices(the_graph) List
. | 3. i : Vertices(the_graph)
. | 4. s : traversal(the_graph)
. | 5. (inr(i) ∈ s)
. | 6. dfl-traversal(the_graph;L;s)
. | 7. i@0 : Vertices(the_graph)
. | 8. s2 : traversal(the_graph)
. | 9. ∀j:Vertices(the_graph).
. |    i@0-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. | 10. inl(i) = inl(i@0)
. | 11. s = s2
. | ⊢  L-the_graph->*i@0
. |   by AllHyps
. |      (λh.
. |       (Analyze h) THEN (Analyze h) THEN (BackThru h+1)
. |       THEN
. |       (ObviousFrom [5;-2]))
. |
. .\
. . 1. the_graph : Graph
. . 2. L : Vertices(the_graph) List
. . 3. i : Vertices(the_graph)
. . 4. s : traversal(the_graph)
. . 5. (inr(i) ∈ s)
. . 6. dfl-traversal(the_graph;L;s)
. . 7. i@0 : Vertices(the_graph)
. . 8. u : Vertices(the_graph)+Vertices(the_graph)
. . 9. v : (Vertices(the_graph)+Vertices(the_graph)) List
. . 10. s2 : traversal(the_graph)
. . 11. ∀j:Vertices(the_graph).
. .     i@0-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . 12. inl(i) = u
. . 13. s = (v @ [inl(i@0) / s2])
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. . ⊢  L-the_graph->*i@0
. .   by AllHyps
. .      (λh.
. .       (Analyze h) THEN (InstHyp [i@0;v;s2] h+1) THEN (Reduce 0))
END
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