```
finite-partition  THM

⊢  ∀n,k:ℕ, c:(ℕn→ℕk).
|  ∃p:(ℕk→(ℕ List)).
|  sum(||p(j)|| | j < k) = n
|  & (∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y])
|  & (∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n & c((p(j))[x]) = j)
| by InductionOnNat
|
|\
| .....basecase .....
| ⊢  ∀k:ℕ, c:(ℕ0→ℕk).
| |  ∃p:(ℕk→(ℕ List)).
| |  sum(||p(j)|| | j < k) = 0
| |  & (∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y])
| |  & (∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<0 & c((p(j))[x]) = j)
| | by Auto THEN (InstConcl [λj.nil]) THEN (Reduce 0)
| |    THEN
| |    (Try
| |     (RWO Thm* ∀n:ℕ, a:ℤ. sum(a | x < n) = a·n    {sum_constant}
| |      0))
| |
| .....subterm .....
| 1. k : ℕ
| 2. ℕ0→ℕk
| 3. p : ℕk→(ℕ List)
| 4. j : ℕk
| 5. x : ℕ||p(j)||
| 6. (p(j))[x]<0
| ⊢  p(j) ∈ ℕ0 List  by Obvious
|
.\
. .....upcase .....
. 1. n : ℤ
. 2. 0<n
. 3. ∀k:ℕ, c:(ℕ(n-1)→ℕk).
. |  ∃p:(ℕk→(ℕ List)).
. |  sum(||p(j)|| | j < k) = n-1
. |  & (∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y])
. |  & (∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j)
. ⊢  ∀k:ℕ, c:(ℕn→ℕk).
. |  ∃p:(ℕk→(ℕ List)).
. |  sum(||p(j)|| | j < k) = n
. |  & (∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y])
. |  & (∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n & c((p(j))[x]) = j)
. | by (((RepeatFor 2 ((ParallelOp -1) THEN (Thin -3))) THEN ExRepD
. |      THEN
. |      (InstConcl [λj.if c(n-1)=₂j→ [(n-1) / (p(j))] else p(j) fi]))
. |     THEN
. |     (Reduce 0)
. |     THEN
. |     (Analyze 0))
. |    THENA
. |    ((RepeatFor 5 (Analyze THEN (Try (Complete Auto))))
. |     THEN
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. |      (GenConcl ((p1(j))[x] = a))
. |      THEN
. |      (Try (Complete Auto))
. |      THEN
. |      Analyze)
. |
. |\
. | 1. n : ℤ
. | 2. 0<n
. | 3. k : ℕ
. | 4. c : ℕn→ℕk
. | 5. p : ℕk→(ℕ List)
. | 6. sum(||p(j)|| | j < k) = n-1
. | 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. | 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. | ⊢  sum(||if c(n-1)=₂j→ [(n-1) / (p(j))] else p(j) fi|| | j < k)
. | |  =
. | |  n
. | | by Subst'
. | |    (sum(||if c(n-1)=₂j→ [(n-1) / (p(j))] else p(j) fi|| | j < k)
. | |     =
. | |     sum(if c(n-1)=₂j→ ||[(n-1) / (p(j))]||
. | |         else ||p(j)|| fi | j < k))
. | |    0
. | |
. | |\
. | | 1. n : ℤ
. | | 2. 0<n
. | | 3. k : ℕ
. | | 4. c : ℕn→ℕk
. | | 5. p : ℕk→(ℕ List)
. | | 6. sum(||p(j)|| | j < k) = n-1
. | | 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. | | 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. | | ⊢  sum(||if c(n-1)=₂j→ [(n-1) / (p(j))] else p(j) fi|| | j < k)
. | |    =
. | |    sum(if c(n-1)=₂j→ ||[(n-1) / (p(j))]||
. | |        else ||p(j)|| fi | j < k)
. | |   by (BackThru
. | |       Thm* ∀n:ℕ, f,g:(ℕn→ℤ).
. | |            (∀i:ℕn. f(i) = g(i))
. | |            ⇒
. | |            sum(f(x) | x < n) = sum(g(x) | x < n)
. | |       {sum_functionality})
. | |      THEN
. | |      SplitOnConclITE
. | |
. | .\
. | . 1. n : ℤ
. | . 2. 0<n
. | . 3. k : ℕ
. | . 4. c : ℕn→ℕk
. | . 5. p : ℕk→(ℕ List)
. | . 6. sum(||p(j)|| | j < k) = n-1
. | . 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
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. | . 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. | . ⊢  sum(if c(n-1)=₂j→ ||[(n-1) / (p(j))]||
. | . |      else ||p(j)|| fi | j < k)
. | . |  =
. | . |  n
. | . | by (Reduce 0)
. | . |    THEN
. | . |    (RWO
. | . |     Thm* ∀k:ℕ, f,g:(ℕk→ℤ), p:(ℕk→𝔹).
. | . |          sum(if p(i)→ f(i)+g(i) else f(i) fi | i < k)
. | . |          =
. | . |          sum(f(i) | i < k)
. | . |          +sum(if p(i)→ g(i) else 0 fi | i < k)
. | . |     {sum_DASH_ite}
. | . |     0)
. | . |    THEN
. | . |    (HypSubst' -3 0)
. | . |
. | . 1. n : ℤ
. | . 2. 0<n
. | . 3. k : ℕ
. | . 4. c : ℕn→ℕk
. | . 5. p : ℕk→(ℕ List)
. | . 6. sum(||p(j)|| | j < k) = n-1
. | . 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. | . 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. | . ⊢  n-1+sum(if c(n-1)=₂j→ 1 else 0 fi | j < k) = n
. | . | by Assert (sum(if c(n-1)=₂j→ 1 else 0 fi | j < k) = 1)
. | . |
. | . |\
. | . | .....assertion .....
. | . | 1. n : ℤ
. | . | 2. 0<n
. | . | 3. k : ℕ
. | . | 4. c : ℕn→ℕk
. | . | 5. p : ℕk→(ℕ List)
. | . | 6. sum(||p(j)|| | j < k) = n-1
. | . | 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. | . | 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. | . | ⊢  sum(if c(n-1)=₂j→ 1 else 0 fi | j < k) = 1
. | . | | by Inst
. | . | |    Thm* ∀n:ℕ, f:(ℕn→ℤ), m:ℕn.
. | . | |         (∀x:ℕn. ¬x = m ⇒ f(x) = 0)
. | . | |         ⇒
. | . | |         sum(f(x) | x < n) = f(m)
. | . | |    {singleton_support_sum}
. | . | |    [k;λ₂j. if c(n-1)=₂j→ 1 else 0 fi;c(n-1)]
. | . | |
. | . | |\
. | . | | 1. n : ℤ
. | . | | 2. 0<n
. | . | | 3. k : ℕ
. | . | | 4. c : ℕn→ℕk
. | . | | 5. p : ℕk→(ℕ List)
. | . | | 6. sum(||p(j)|| | j < k) = n-1
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. | . | | 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. | . | | 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. | . | | 9. x : ℕk
. | . | | 10. ¬x = c(n-1)
. | . | | ⊢  if c(n-1)=₂x→ 1 else 0 fi = 0  by SplitOnConclITE
. | . | |
. | . | .\
. | . | . 1. n : ℤ
. | . | . 2. 0<n
. | . | . 3. k : ℕ
. | . | . 4. c : ℕn→ℕk
. | . | . 5. p : ℕk→(ℕ List)
. | . | . 6. sum(||p(j)|| | j < k) = n-1
. | . | . 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. | . | . 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. | . | . 9. sum(if c(n-1)=₂x→ 1 else 0 fi | x < k)
. | . | .    =
. | . | .    if c(n-1)=₂c(n-1)→ 1 else 0 fi
. | . | . ⊢  sum(if c(n-1)=₂j→ 1 else 0 fi | j < k) = 1
. | . | .   by (MoveToConcl -1) THEN SplitOnConclITE
. | . |
. | . .\
. | . . 1. n : ℤ
. | . . 2. 0<n
. | . . 3. k : ℕ
. | . . 4. c : ℕn→ℕk
. | . . 5. p : ℕk→(ℕ List)
. | . . 6. sum(||p(j)|| | j < k) = n-1
. | . . 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. | . . 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. | . . 9. sum(if c(n-1)=₂j→ 1 else 0 fi | j < k) = 1
. | . . ⊢  n-1+sum(if c(n-1)=₂j→ 1 else 0 fi | j < k) = n  by Auto
. |
. .\
. . 1. n : ℤ
. . 2. 0<n
. . 3. k : ℕ
. . 4. c : ℕn→ℕk
. . 5. p : ℕk→(ℕ List)
. . 6. sum(||p(j)|| | j < k) = n-1
. . 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . ⊢  (∀j:ℕk, x,y:ℕ||if c(n-1)=₂j→ [(n-1) / (p(j))] else p(j) fi||.
. . |   x<y
. . |   ⇒
. . |   if c(n-1)=₂j→ [(n-1) / (p(j))]
. . |   else p(j) fi[x]>if c(n-1)=₂j→ [(n-1) / (p(j))]
. . |                   else p(j) fi[y])
. . |  & (∀j:ℕk, x:ℕ||if c(n-1)=₂j→ [(n-1) / (p(j))] else p(j) fi||.
. . |      if c(n-1)=₂j→ [(n-1) / (p(j))] else p(j) fi[x]<n
. . |      & c(if c(n-1)=₂j→ [(n-1) / (p(j))] else p(j) fi[x]) = j)
. . | by Analyze 0
. . |
. . |\
. . | 1. n : ℤ
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. . | 2. 0<n
. . | 3. k : ℕ
. . | 4. c : ℕn→ℕk
. . | 5. p : ℕk→(ℕ List)
. . | 6. sum(||p(j)|| | j < k) = n-1
. . | 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . | 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . | ⊢  ∀j:ℕk, x,y:ℕ||if c(n-1)=₂j→ [(n-1) / (p(j))] else p(j) fi||.
. . | |  x<y
. . | |  ⇒
. . | |  if c(n-1)=₂j→ [(n-1) / (p(j))]
. . | |  else p(j) fi[x]>if c(n-1)=₂j→ [(n-1) / (p(j))]
. . | |                  else p(j) fi[y]
. . | | by (ParallelOp -2) THEN SplitOnConclITE THEN (Try Trivial)
. . | |
. . | .....truecase .....
. . | 1. n : ℤ
. . | 2. 0<n
. . | 3. k : ℕ
. . | 4. c : ℕn→ℕk
. . | 5. p : ℕk→(ℕ List)
. . | 6. sum(||p(j)|| | j < k) = n-1
. . | 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . | 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . | 9. j : ℕk
. . | 10. ∀x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . | 11. c(n-1) = j
. . | ⊢  ∀x@0,y:ℕ||[(n-1) / (p(j))]||.
. . | |  x@0<y ⇒ [(n-1) / (p(j))][x@0]>[(n-1) / (p(j))][y]
. . | | by (((Reduce 0) THEN (Analyze 0)) THEN (Analyze 0)
. . | |     THEN
. . | |     (CaseNat 0 ‘x@0’)
. . | |     THEN
. . | |     (Reduce 0))
. . | |    THEN
. . | |    (RWW
. . | |     Thm* ∀a:T, as:T List, i:ℤ.
. . | |          0<i ⇒ i≤||as|| ⇒ [a / as][i] = as[(i-1)]
. . | |     {select_cons_tl}
. . | |     0)
. . | |
. . | |\
. . | | 1. n : ℤ
. . | | 2. 0<n
. . | | 3. k : ℕ
. . | | 4. c : ℕn→ℕk
. . | | 5. p : ℕk→(ℕ List)
. . | | 6. sum(||p(j)|| | j < k) = n-1
. . | | 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . | | 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . | | 9. j : ℕk
. . | | 10. ∀x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . | | 11. c(n-1) = j
. . | | 12. x@0 : ℕ(||p(j)||+1)
. . | | 13. y : ℕ(||p(j)||+1)
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. . | | 14. x@0 = 0
. . | | 15. 0<y
. . | | ⊢  n-1>(p(j))[(y-1)]
. . | |   by (InstHyp [j;y-1] 8) THEN (Analyze -1)
. . | |
. . | .\
. . | . 1. n : ℤ
. . | . 2. 0<n
. . | . 3. k : ℕ
. . | . 4. c : ℕn→ℕk
. . | . 5. p : ℕk→(ℕ List)
. . | . 6. sum(||p(j)|| | j < k) = n-1
. . | . 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . | . 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . | . 9. j : ℕk
. . | . 10. ∀x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . | . 11. c(n-1) = j
. . | . 12. x@0 : ℕ(||p(j)||+1)
. . | . 13. y : ℕ(||p(j)||+1)
. . | . 14. ¬x@0 = 0
. . | . 15. x@0<y
. . | . ⊢  (p(j))[(x@0-1)]>(p(j))[(y-1)]  by EasyHyp'
. . |
. . .\
. . . 1. n : ℤ
. . . 2. 0<n
. . . 3. k : ℕ
. . . 4. c : ℕn→ℕk
. . . 5. p : ℕk→(ℕ List)
. . . 6. sum(||p(j)|| | j < k) = n-1
. . . 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . . 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . ⊢  ∀j:ℕk, x:ℕ||if c(n-1)=₂j→ [(n-1) / (p(j))] else p(j) fi||.
. . . |  if c(n-1)=₂j→ [(n-1) / (p(j))] else p(j) fi[x]<n
. . . |  & c(if c(n-1)=₂j→ [(n-1) / (p(j))] else p(j) fi[x]) = j
. . . | by (ParallelOp -1) THEN SplitOnConclITE
. . . |
. . . |\
. . . | .....truecase .....
. . . | 1. n : ℤ
. . . | 2. 0<n
. . . | 3. k : ℕ
. . . | 4. c : ℕn→ℕk
. . . | 5. p : ℕk→(ℕ List)
. . . | 6. sum(||p(j)|| | j < k) = n-1
. . . | 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . . | 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | 9. j : ℕk
. . . | 10. ∀x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | 11. c(n-1) = j
. . . | ⊢  ∀x@0:ℕ||[(n-1) / (p(j))]||.
. . . | |  [(n-1) / (p(j))][x@0]<n & c([(n-1) / (p(j))][x@0]) = j
. . . | | by (Reduce 0) THEN (Analyze 0) THEN (CaseNat 0 `x@0')
. . . | |    THEN
. . . | |    (Reduce 0)
```

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. . . | |
. . . | |\
. . . | | 1. n : ℤ
. . . | | 2. 0<n
. . . | | 3. k : ℕ
. . . | | 4. c : ℕn→ℕk
. . . | | 5. p : ℕk→(ℕ List)
. . . | | 6. sum(||p(j)|| | j < k) = n-1
. . . | | 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . . | | 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | | 9. j : ℕk
. . . | | 10. ∀x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | | 11. c(n-1) = j
. . . | | 12. x@0 : ℕ(||p(j)||+1)
. . . | | 13. x@0 = 0
. . . | | ⊢  n-1<n & c(n-1) = j  by Auto
. . . | |
. . . | .\
. . . | . 1. n : ℤ
. . . | . 2. 0<n
. . . | . 3. k : ℕ
. . . | . 4. c : ℕn→ℕk
. . . | . 5. p : ℕk→(ℕ List)
. . . | . 6. sum(||p(j)|| | j < k) = n-1
. . . | . 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . . | . 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | . 9. j : ℕk
. . . | . 10. ∀x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | . 11. c(n-1) = j
. . . | . 12. x@0 : ℕ(||p(j)||+1)
. . . | . 13. ¬x@0 = 0
. . . | . ⊢  [(n-1) / (p(j))][x@0]<n & c([(n-1) / (p(j))][x@0]) = j
. . . | . | by (InstHyp [x@0-1] -4) THEN (Analyze -1)
. . . | . |
. . . | . 1. n : ℤ
. . . | . 2. 0<n
. . . | . 3. k : ℕ
. . . | . 4. c : ℕn→ℕk
. . . | . 5. p : ℕk→(ℕ List)
. . . | . 6. sum(||p(j)|| | j < k) = n-1
. . . | . 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . . | . 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | . 9. j : ℕk
. . . | . 10. ∀x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | . 11. c(n-1) = j
. . . | . 12. x@0 : ℕ(||p(j)||+1)
. . . | . 13. ¬x@0 = 0
. . . | . 14. (p(j))[(x@0-1)]<n-1
. . . | . 15. c((p(j))[(x@0-1)]) = j
. . . | . ⊢  [(n-1) / (p(j))][x@0]<n & c([(n-1) / (p(j))][x@0]) = j
. . . | . | by Analyze 0
. . . | . |
. . . | . |\
. . . | . | 1. n : ℤ
. . . | . | 2. 0<n
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. . . | . | 3. k : ℕ
. . . | . | 4. c : ℕn→ℕk
. . . | . | 5. p : ℕk→(ℕ List)
. . . | . | 6. sum(||p(j)|| | j < k) = n-1
. . . | . | 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . . | . | 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | . | 9. j : ℕk
. . . | . | 10. ∀x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | . | 11. c(n-1) = j
. . . | . | 12. x@0 : ℕ(||p(j)||+1)
. . . | . | 13. ¬x@0 = 0
. . . | . | 14. (p(j))[(x@0-1)]<n-1
. . . | . | 15. c((p(j))[(x@0-1)]) = j
. . . | . | ⊢  [(n-1) / (p(j))][x@0]<n
. . . | . |   by RWW
. . . | . |      Thm* ∀a:T, as:T List, i:ℤ.
. . . | . |           0<i ⇒ i≤||as|| ⇒ [a / as][i] = as[(i-1)]
. . . | . |      {select_cons_tl}
. . . | . |      0
. . . | . |
. . . | . .\
. . . | . . 1. n : ℤ
. . . | . . 2. 0<n
. . . | . . 3. k : ℕ
. . . | . . 4. c : ℕn→ℕk
. . . | . . 5. p : ℕk→(ℕ List)
. . . | . . 6. sum(||p(j)|| | j < k) = n-1
. . . | . . 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . . | . . 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | . . 9. j : ℕk
. . . | . . 10. ∀x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | . . 11. c(n-1) = j
. . . | . . 12. x@0 : ℕ(||p(j)||+1)
. . . | . . 13. ¬x@0 = 0
. . . | . . 14. (p(j))[(x@0-1)]<n-1
. . . | . . 15. c((p(j))[(x@0-1)]) = j
. . . | . . 16. [(n-1) / (p(j))][x@0]<n
. . . | . . ⊢  c([(n-1) / (p(j))][x@0]) = j
. . . | . . | by (NthHypSq -2) THEN Analyze THEN Analyze
. . . | . . |
. . . | . . 1. n : ℤ
. . . | . . 2. 0<n
. . . | . . 3. k : ℕ
. . . | . . 4. c : ℕn→ℕk
. . . | . . 5. p : ℕk→(ℕ List)
. . . | . . 6. sum(||p(j)|| | j < k) = n-1
. . . | . . 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . . | . . 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | . . 9. j : ℕk
. . . | . . 10. ∀x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . | . . 11. c(n-1) = j
. . . | . . 12. x@0 : ℕ(||p(j)||+1)
. . . | . . 13. ¬x@0 = 0
. . . | . . 14. (p(j))[(x@0-1)]<n-1
. . . | . . 15. c((p(j))[(x@0-1)]) = j
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. . . | . . 16. [(n-1) / (p(j))][x@0]<n
. . . | . . ⊢  [(n-1) / (p(j))][x@0] = (p(j))[(x@0-1)]
. . . | . .   by RWW
. . . | . .      Thm* ∀a:T, as:T List, i:ℤ.
. . . | . .           0<i ⇒ i≤||as|| ⇒ [a / as][i] = as[(i-1)]
. . . | . .      {select_cons_tl}
. . . | . .      0
. . . |
. . . .\
. . . . .....falsecase .....
. . . . 1. n : ℤ
. . . . 2. 0<n
. . . . 3. k : ℕ
. . . . 4. c : ℕn→ℕk
. . . . 5. p : ℕk→(ℕ List)
. . . . 6. sum(||p(j)|| | j < k) = n-1
. . . . 7. ∀j:ℕk, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. . . . 8. ∀j:ℕk, x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . . 9. j : ℕk
. . . . 10. ∀x:ℕ||p(j)||. (p(j))[x]<n-1 & c((p(j))[x]) = j
. . . . 11. ¬c(n-1) = j
. . . . ⊢  ∀x:ℕ||p(j)||. (p(j))[x]<n & c((p(j))[x]) = j
. . . .   by (Thin -1) THEN (RepeatFor 2 (ParallelOp -1))
END
```