fun-graph-rel-graph THM

```
⊢  ∀A,B:Type, f:(A→B→A).
|  (∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2)
|  ⇒
|  (∀a,a':A. Dec(∃b:B. a' = f(a,b)))
|  ⇒
|  Graph(a:A --> f(a,b) | b:B) ≡ Graph(a,a':A. ∃b:B. a' = f(a,b))
| by Auto
|    THEN
|    (Unfolds [`graph-isomorphic`;`fun-graph`;`rel-graph`] 0)
|    THEN
|    (Reduce 0)
|    THEN
|    (InstConcl [Id;λp.<1of(p),f(1of(p),2of(p))>])
|
|\
| .....subterm .....
| 1. A : Type
| 2. B : Type
| 3. f : A→B→A
| 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
| 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
| 6. p : A×B
| ⊢  <1of(p),f(1of(p),2of(p))>
|    ∈ {p:(A×A)| ∃b:B. 2of(p) = f(1of(p),b) }
|   by (Analyze -1) THEN Analyze THEN (Reduce 0)
|      THEN
|      (InstConcl [p2])
|
|\
| 1. A : Type
| 2. B : Type
| 3. f : A→B→A
| 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
| 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
| ⊢  Bij(A; A; Id)  by Easy
|
|\
| 1. A : Type
| 2. B : Type
| 3. f : A→B→A
| 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
| 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
| ⊢  Bij(A×B; {p:(A×A)| ∃b:B. 2of(p) = f(1of(p),b) };
| |      λp.<1of(p),f(1of(p),2of(p))>)
| | by (Unfold `biject` 0) THEN (Unfolds [`inject`;`surject`] 0)
| |    THEN
| |    (Reduce 0)
| |    THEN
| |    (Try (Analyze THEN (Reduce 0) THEN Obvious))
| |
| |\
| | 1. A : Type
| | 2. B : Type
```

```
| | 3. f : A→B→A
| | 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
| | 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
| | 6. a1 : A×B
| | 7. a2 : A×B
| | 8. <1of(a1),f(1of(a1),2of(a1))> = <1of(a2),f(1of(a2),2of(a2))>
| | ⊢  a1 = a2
| | | by (Analyze -3) THEN (Analyze -2) THEN (All Reduce)
| | |    THEN
| | |    (Analyze -1)
| | |    THEN
| | |    (Reduce -1)
| | |
| | 1. A : Type
| | 2. B : Type
| | 3. f : A→B→A
| | 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
| | 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
| | 6. a3 : A
| | 7. a4 : B
| | 8. a5 : A
| | 9. a6 : B
| | 10. <a3,f(a3,a4)> = <a5,f(a5,a6)>
| | 11. ∃b:B. f(a3,a4) = f(a3,b)
| | ⊢  <a3,a4> = <a5,a6>  by (SplitPair -2) THEN ExRepD THEN Analyze
| | |
| | .....subterm .....
| | 1. A : Type
| | 2. B : Type
| | 3. f : A→B→A
| | 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
| | 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
| | 6. a3 : A
| | 7. a4 : B
| | 8. a5 : A
| | 9. a6 : B
| | 10. b : B
| | 11. f(a3,a4) = f(a3,b)
| | 12. a3 = a5
| | 13. f(a3,a4) = f(a5,a6)
| | ⊢  a4 = a6  by HypSubst -2 -1
| | |
| | 1. A : Type
| | 2. B : Type
| | 3. f : A→B→A
| | 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
| | 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
| | 6. a3 : A
| | 7. a4 : B
| | 8. a5 : A
| | 9. a6 : B
| | 10. b : B
| | 11. f(a3,a4) = f(a3,b)
| | 12. a3 = a5
| | 13. f(a5,a4) = f(a5,a6)
```

```
| | ⊢  a4 = a6  by FwdThru 4 [-1]
| |
| .\
| . 1. A : Type
| . 2. B : Type
| . 3. f : A→B→A
| . 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
| . 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
| . 6. b : {p:(A×A)| ∃b:B. 2of(p) = f(1of(p),b) }
| . ⊢  ∃a:(A×B). <1of(a),f(1of(a),2of(a))> = b  by DD -1
| . |
| . 1. A : Type
| . 2. B : Type
| . 3. f : A→B→A
| . 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
| . 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
| . 6. b : A×A
| . 7. ∃b1:B. 2of(b) = f(1of(b),b1)
| . ⊢  ∃a:(A×B). <1of(a),f(1of(a),2of(a))> = b
| . | by ((Analyze -2) THEN (All Reduce) THEN ExRepD
| . |     THEN
| . |     (InstConcl [<b1,b1@0>])
| . |     THEN
| . |     (Reduce 0))
| . |    THENA
| . |    (Auto THEN (Analyze -1) THEN (Reduce 0) THEN Analyze
| . |     THEN
| . |     (Reduce 0)
| . |     THEN
| . |     Obvious)
| . |
| . 1. A : Type
| . 2. B : Type
| . 3. f : A→B→A
| . 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
| . 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
| . 6. b1 : A
| . 7. b2 : A
| . 8. b1@0 : B
| . 9. b2 = f(b1,b1@0)
| . ⊢  <b1,f(b1,b1@0)> = <b1,b2>  by Analyze
| . |
| . |\
| . | 1. A : Type
| . | 2. B : Type
| . | 3. f : A→B→A
| . | 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
| . | 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
| . | 6. b1 : A
| . | 7. b2 : A
| . | 8. b1@0 : B
| . | 9. b2 = f(b1,b1@0)
| . | ⊢  <b1,f(b1,b1@0)> = <b1,b2>  by Analyze
| . |
| . .\
```

```
| . . .....set predicate .....
| . . 1. A : Type
| . . 2. B : Type
| . . 3. f : A→B→A
| . . 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
| . . 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
| . . 6. b1 : A
| . . 7. b2 : A
| . . 8. b1@0 : B
| . . 9. b2 = f(b1,b1@0)
| . . ⊢  ∃b:B. 2of(<b1,f(b1,b1@0)>) = f(1of(<b1,f(b1,b1@0)>),b)
| . .   by Obvious
|
.\
. 1. A : Type
. 2. B : Type
. 3. f : A→B→A
. 4. ∀a:A, b1,b2:B. f(a,b1) = f(a,b2) ⇒ b1 = b2
. 5. ∀a,a':A. Dec(∃b:B. a' = f(a,b))
. ⊢  (Id,Id) o λp.<1of(p),f(1of(p),2of(p))>
.    =
.    Id o (λp.<1of(p),f(1of(p),2of(p))>)
.   by ((GenConcl ((λp.<1of(p),f(1of(p),2of(p))>) = g))
.       THEN
.       (RWO Thm* ∀g:(A→B×C). (Id,Id) o g = g    {comp2_id_l} 0))
.      THEN
.      (RWO Thm* ∀f:(A→B). Id o f = f    {comp_id_l} 0)
END
```