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pigeon-hole  THM

⊢  ∀n,m:ℕ, f:(ℕn→ℕm). Inj(ℕn; ℕm; f) ⇒ n≤m
| by Auto THEN (Inst Thm: finite_DASH_partition [n;m;f]) THEN ExRepD
|
1. n : ℕ
2. m : ℕ
3. f : ℕn→ℕm
4. Inj(ℕn; ℕm; f)
5. p : ℕm→(ℕ List)
6. sum(||p(j)|| | j < m) = n
7. ∀j:ℕm, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
8. ∀j:ℕm, x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
⊢  n≤m  by Assert (sum(||p(j)|| | j < m)≤1·m)
|
.....assertion .....
1. n : ℕ
2. m : ℕ
3. f : ℕn→ℕm
4. Inj(ℕn; ℕm; f)
5. p : ℕm→(ℕ List)
6. sum(||p(j)|| | j < m) = n
7. ∀j:ℕm, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
8. ∀j:ℕm, x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
⊢  sum(||p(j)|| | j < m)≤1·m
| by BackThru
|    Thm* ∀k,b:ℕ, f:(ℕk→ℤ). (∀x:ℕk. f(x)≤b) ⇒ sum(f(x) | x < k)≤b·k
|    {sum_bound}
|
1. n : ℕ
2. m : ℕ
3. f : ℕn→ℕm
4. Inj(ℕn; ℕm; f)
5. p : ℕm→(ℕ List)
6. sum(||p(j)|| | j < m) = n
7. ∀j:ℕm, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
8. ∀j:ℕm, x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
9. j : ℕm
⊢  ||p(j)||≤1  by (AllHyps (InstHyp [j])) THEN SupposeNot
|
1. n : ℕ
2. m : ℕ
3. f : ℕn→ℕm
4. Inj(ℕn; ℕm; f)
5. p : ℕm→(ℕ List)
6. sum(||p(j)|| | j < m) = n
7. ∀j:ℕm, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
8. ∀j:ℕm, x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
9. j : ℕm
10. ∀x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
11. ∀x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
12. ¬||p(j)||≤1
⊢  ||p(j)||≤1
| by (InstHyp [0] -3) THEN (InstHyp [1] -4) THEN (InstHyp [0;1] -4)
|
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1. n : ℕ
2. m : ℕ
3. f : ℕn→ℕm
4. Inj(ℕn; ℕm; f)
5. p : ℕm→(ℕ List)
6. sum(||p(j)|| | j < m) = n
7. ∀j:ℕm, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
8. ∀j:ℕm, x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
9. j : ℕm
10. ∀x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
11. ∀x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
12. ¬||p(j)||≤1
13. (p(j))[0]<n & f((p(j))[0]) = j
14. (p(j))[1]<n & f((p(j))[1]) = j
15. (p(j))[0]>(p(j))[1]
⊢ ||p(j)||≤1
| by ExRepD THEN (Unfold `inject` 4)
|    THEN
|    (InstHyp [(p(j))[0];(p(j))[1]] 4)
|    THEN
|    (Try (Complete Auto))
|
|\
| .....wf .....
| 1. n : ℕ
| 2. m : ℕ
| 3. f : ℕn→ℕm
| 4. ∀a1,a2:ℕn. f(a1) = f(a2) ⇒ a1 = a2
| 5. p : ℕm→(ℕ List)
| 6. sum(||p(j)|| | j < m) = n
| 7. ∀j:ℕm, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| 8. ∀j:ℕm, x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
| 9. j : ℕm
| 10. ∀x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
| 11. ∀x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| 12. ¬||p(j)||≤1
| 13. (p(j))[0]<n
| 14. f((p(j))[0]) = j
| 15. (p(j))[1]<n
| 16. f((p(j))[1]) = j
| 17. (p(j))[0]>(p(j))[1]
| ⊢ (p(j))[0] ∈ ℕn  by Analyze
| |
| 1. n : ℕ
| 2. m : ℕ
| 3. f : ℕn→ℕm
| 4. ∀a1,a2:ℕn. f(a1) = f(a2) ⇒ a1 = a2
| 5. p : ℕm→(ℕ List)
| 6. sum(||p(j)|| | j < m) = n
| 7. ∀j:ℕm, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| 8. ∀j:ℕm, x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
| 9. j : ℕm
| 10. ∀x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
| 11. ∀x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
| 12. ¬||p(j)||≤1
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| 13. (p(j))[0]<n
| 14. f((p(j))[0]) = j
| 15. (p(j))[1]<n
| 16. f((p(j))[1]) = j
| 17. (p(j))[0]>(p(j))[1]
| ⊢  0≤(p(j))[0]
|   by (MoveToConcl -6) THEN (GenConclAtAddr [2;2;2])
|      THEN
|      (Lemmaize [-1])
|
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. .....wf .....
. 1. n : ℕ
. 2. m : ℕ
. 3. f : ℕn→ℕm
. 4. ∀a1,a2:ℕn. f(a1) = f(a2) ⇒ a1 = a2
. 5. p : ℕm→(ℕ List)
. 6. sum(||p(j)|| | j < m) = n
. 7. ∀j:ℕm, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. 8. ∀j:ℕm, x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
. 9. j : ℕm
. 10. ∀x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
. 11. ∀x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. 12. ¬||p(j)||≤1
. 13. (p(j))[0]<n
. 14. f((p(j))[0]) = j
. 15. (p(j))[1]<n
. 16. f((p(j))[1]) = j
. 17. (p(j))[0]>(p(j))[1]
. ⊢  (p(j))[1] ∈ ℕn  by Analyze
. |
. 1. n : ℕ
. 2. m : ℕ
. 3. f : ℕn→ℕm
. 4. ∀a1,a2:ℕn. f(a1) = f(a2) ⇒ a1 = a2
. 5. p : ℕm→(ℕ List)
. 6. sum(||p(j)|| | j < m) = n
. 7. ∀j:ℕm, x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. 8. ∀j:ℕm, x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
. 9. j : ℕm
. 10. ∀x:ℕ||p(j)||. (p(j))[x]<n & f((p(j))[x]) = j
. 11. ∀x,y:ℕ||p(j)||. x<y ⇒ (p(j))[x]>(p(j))[y]
. 12. ¬||p(j)||≤1
. 13. (p(j))[0]<n
. 14. f((p(j))[0]) = j
. 15. (p(j))[1]<n
. 16. f((p(j))[1]) = j
. 17. (p(j))[0]>(p(j))[1]
. ⊢  0≤(p(j))[1]
.   by (MoveToConcl -6) THEN (GenConclAtAddr [2;2;2])
.      THEN
.      (Lemmaize [-1])
END
```