```
rel-graph_functionality  THM

⊢  ∀T:Type, R1,R2:(T→T→Prop).
|  (∀x,y:T. R1(x,y) ⟺ R2(x,y))
|  ⇒
|  Graph(x,y:T. R1(x,y)) ≡ Graph(x,y:T. R2(x,y))
| by Auto THEN (Unfolds [‘graph-isomorphic‘;‘rel-graph‘] 0)
|    THEN
|    (Reduce 0)
|    THEN
|    (InstConcl [Id;Id])
|    THEN
|    (Reduce 0)
|    THEN
|    (Try (BackThru Thm* Bij(T; T; Id)    {identity_DASH_biject}))
|
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| .....wf .....
| 1. T : Type
| 2. R1 : T→T→Prop
| 3. R2 : T→T→Prop
| 4. ∀x,y:T. R1(x,y) ⟺ R2(x,y)
| ⊢  Id ∈ {p:(T×T)| R1(1of(p),2of(p)) }→{p:(T×T)| R2(1of(p),2of(p)) }
|   by (Unfold ‘identity‘ 0) THEN (ParallelOp -1) THEN EasyHyp
|
|\
| 1. T : Type
| 2. R1 : T→T→Prop
| 3. R2 : T→T→Prop
| 4. ∀x,y:T. R1(x,y) ⟺ R2(x,y)
| ⊢  Bij({p:(T×T)| R1(1of(p),2of(p)) }; {p:(T×T)| R2(1of(p),2of(p)) };
| |      Id)
| | by (Unfold ‘biject‘ 0) THEN (Unfolds [‘inject‘;‘surject‘] 0)
| |    THEN
| |    (Reduce 0)
| |
| |\
| | 1. T : Type
| | 2. R1 : T→T→Prop
| | 3. R2 : T→T→Prop
| | 4. ∀x,y:T. R1(x,y) ⟺ R2(x,y)
| | 5. a1 : {p:(T×T)| R1(1of(p),2of(p)) }
| | 6. a2 : {p:(T×T)| R1(1of(p),2of(p)) }
| | 7. a1 = a2
| | ⊢  a1 = a2  by (ParallelOp -1) THEN (ParallelOp -1) THEN EasyHyp
| |
| |\
| | .....subterm .....
| | 1. T : Type
| | 2. R1 : T→T→Prop
| | 3. R2 : T→T→Prop
| | 4. ∀x,y:T. R1(x,y) ⟺ R2(x,y)
| | 5. a1 : {p:(T×T)| R1(1of(p),2of(p)) }
| | 6. {p:(T×T)| R1(1of(p),2of(p)) }
| | ⊢  a1 ∈ {p:(T×T)| R2(1of(p),2of(p)) }
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| |   by (ParallelOp -2) THEN EasyHyp
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| | .....subterm .....
| | 1. T : Type
| | 2. R1 : T→T→Prop
| | 3. R2 : T→T→Prop
| | 4. ∀x,y:T. R1(x,y) ⟺ R2(x,y)
| | 5. {p:(T×T)| R1(1of(p),2of(p)) }
| | 6. a2 : {p:(T×T)| R1(1of(p),2of(p)) }
| | ⊢  a2 ∈ {p:(T×T)| R2(1of(p),2of(p)) }
| |   by (ParallelOp -1) THEN EasyHyp
| |
| .\
| . 1. T : Type
| . 2. R1 : T→T→Prop
| . 3. R2 : T→T→Prop
| . 4. ∀x,y:T. R1(x,y) ⟺ R2(x,y)
| . 5. b : {p:(T×T)| R2(1of(p),2of(p)) }
| . ⊢  ∃a:{p:(T×T)| R1(1of(p),2of(p)) }. a = b
| .   by (InstConcl [b]) THEN (ParallelOp -1) THEN EasyHyp
|
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. 1. T : Type
. 2. R1 : T→T→Prop
. 3. R2 : T→T→Prop
. 4. ∀x,y:T. R1(x,y) ⟺ R2(x,y)
. ⊢  (Id,Id) o Id = Id o Id
.   by (ExtWith [‘z’] [T×T→T×T]) THEN (Reduce 0) THEN (Analyze -1)
.      THEN
.      (Analyze -2)
.      THEN
.      (Reduce 0)
END
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