```
adjm_to_adjl_graph  THM

⊢  ∀m:AdjMatrix. adjl-graph(adjm_to_adjl(m)) ≡ adjm-graph(m)
| by ((Auto THEN (Analyze 1)
|       THEN
|       (Unfolds
|        [‘adjl-graph‘;‘adjm-graph‘;‘adjm_to_adjl‘;‘graph-isomorphic‘]
|        0)
|       THEN
|       (Reduce 0)
|       THEN
|       (InstConcl [Id]))
|      THENA
|      (Auto THEN (Analyze -1) THEN (Reduce 0)))
|     THEN
|     (Unfold ‘compose2‘ 0)
|     THEN
|     (Reduce 0)
|
1. size : ℕ
2. m1 : ℕsize→ℕsize→𝔹
⊢  ∃emap:((x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||)→
|          {p:(ℕsize×ℕsize)| m1(1of(p),2of(p)) }).
|
|  Bij(ℕsize; ℕsize; Id)
|  & Bij(x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||;
|        {p:(ℕsize×ℕsize)| m1(1of(p),2of(p)) };
|        emap)
|  & (λx.<1of(x),filter(λj.m1(1of(x),j);upto(0;size))[2of(x)]>)
|     =
|     (λe.e) o emap
| by InstConcl [λe.e/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>]
|
|\
| .....wf .....
| 1. size : ℕ
| 2. m1 : ℕsize→ℕsize→𝔹
| ⊢  (λe.e/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>)
| |  ∈ (x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||)→
| |     {p:(ℕsize×ℕsize)| m1(1of(p),2of(p)) }
| | by Auto THEN Analyze THEN (Reduce 0)
| |
| 1. size : ℕ
| 2. m1 : ℕsize→ℕsize→𝔹
| 3. e : x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||
| 4. x : ℕsize
| 5. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| 6. e = <x,i>
| ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | by Assert
| |    (∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| |     m1(x,filter(λj.m1(x,j);upto(0;size))[i]))
| |
| |\
| | .....assertion .....
```

```
| | 1. size : ℕ
| | 2. m1 : ℕsize→ℕsize→𝔹
| | 3. e : x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||
| | 4. x : ℕsize
| | 5. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | 6. e = <x,i>
| | ⊢  ∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | |  m1(x,filter(λj.m1(x,j);upto(0;size))[i])  by All Thin
| | |
| | 1. size : ℕ
| | 2. m1 : ℕsize→ℕsize→𝔹
| | 3. x : ℕsize
| | 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | | by Assert
| | |    (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | |    m1(x,j);upto(0;size)))
| | |
| | |\
| | | .....assertion .....
| | | 1. size : ℕ
| | | 2. m1 : ℕsize→ℕsize→𝔹
| | | 3. x : ℕsize
| | | 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | ⊢  (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | |    m1(x,j);upto(0;size)))
| | |   by (Unfold ‘l_member‘ 0) THEN (InstConcl [i])
| | |
| | .\
| | . 1. size : ℕ
| | . 2. m1 : ℕsize→ℕsize→𝔹
| | . 3. x : ℕsize
| | . 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . 5. (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | .    m1(x,j);upto(0;size)))
| | . ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | .   by (RWO
| | .       Thm* ∀P:(T→𝔹), L:T List, x:T.
| | .             (x ∈ filter(P;L)) ⟺ (x ∈ L) & P(x)
| | .       {member_filter}
| | .       -1)
| | .      THEN
| | .      (Reduce -1)
| |
| .\
| . 1. size : ℕ
| . 2. m1 : ℕsize→ℕsize→𝔹
| . 3. e : x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||
| . 4. x : ℕsize
| . 5. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| . 6. e = <x,i>
| . 7. ∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| .    m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| . ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i])  by BackThru -1
|
```

```
|\
| 1. size : ℕ
| 2. m1 : ℕsize→ℕsize→𝔹
| ⊢  Bij(ℕsize; ℕsize; Id)
| |  & Bij(x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||;
| |        {p:(ℕsize×ℕsize)| m1(1of(p),2of(p)) };
| |        λe.e/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>)
| |  & (λx.<1of(x),filter(λj.m1(1of(x),j);upto(0;size))[2of(x)]>)
| |    =
| |    (λe.e) o (λe.e/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>)
| | by Reduce 0
| |
| |\
| | 1. size : ℕ
| | 2. ℕsize→ℕsize→𝔹
| | ⊢  Bij(ℕsize; ℕsize; Id)
| |   by BackThru Thm* Bij(T; T; Id)    {identity_DASH_biject}
| |
| |\
| | 1. size : ℕ
| | 2. m1 : ℕsize→ℕsize→𝔹
| | ⊢  Bij(x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||;
| | |      {p:(ℕsize×ℕsize)| m1(1of(p),2of(p)) };
| | |      λe.e/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>)
| | | by (Unfold ‘biject‘ 0) THEN (Unfolds [‘surject‘;‘inject‘] 0)
| | |    THEN
| | |    (Reduce 0)
| | |    THEN
| | |    (Analyze 0)
| | |
| | |\
| | | 1. size : ℕ
| | | 2. m1 : ℕsize→ℕsize→𝔹
| | | ⊢  ∀a1,a2:(x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||).
| | | |  (a1/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>)
| | | |  =
| | | |  (a2/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>)
| | | |  ⇒
| | | |  a1 = a2
| | | | by RepeatFor 2
| | | |    ((Analyze 0) THEN (Analyze -1) THEN (Reduce 0))
| | | |
| | | 1. size : ℕ
| | | 2. m1 : ℕsize→ℕsize→𝔹
| | | 3. x : ℕsize
| | | 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | 5. x1 : ℕsize
| | | 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | ⊢  <x,filter(λj.m1(x,j);upto(0;size))[a2]>
| | | |  =
| | | |  <x1,filter(λj.m1(x1,j);upto(0;size))[a3]>
| | | |  ⇒
| | | |  <x,a2> = <x1,a3>
| | | | by (Analyze 0) THENA (Auto THEN Analyze THEN (Reduce 0))
| | | |
```

```
| | | | |\
| | | | | 1. size : ℕ
| | | | | 2. m1 : ℕsize→ℕsize→𝔹
| | | | | 3. x : ℕsize
| | | | | 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | | 5. x1 : ℕsize
| | | | | 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | | 7. <x,filter(λj.m1(x,j);upto(0;size))[a2]>
| | | | | |  =
| | | | | |  <x1,filter(λj.m1(x1,j);upto(0;size))[a3]>
| | | | | ⊢  <x,a2> = <x1,a3>  by (Analyze -1) THEN (Reduce -1)
| | | | | |
| | | | | 1. size : ℕ
| | | | | 2. m1 : ℕsize→ℕsize→𝔹
| | | | | 3. x : ℕsize
| | | | | 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | | 5. x1 : ℕsize
| | | | | 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | | 7. <x,filter(λj.m1(x,j);upto(0;size))[a2]>
| | | | | |  =
| | | | | |  <x1,filter(λj.m1(x1,j);upto(0;size))[a3]>
| | | | | 8. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | | ⊢  <x,a2> = <x1,a3>  by (SplitPair -2) THEN Analyze
| | | | | |
| | | | | .....subterm .....
| | | | | 1. size : ℕ
| | | | | 2. m1 : ℕsize→ℕsize→𝔹
| | | | | 3. x : ℕsize
| | | | | 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | | 5. x1 : ℕsize
| | | | | 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | | 7. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | | 8. x = x1
| | | | | 9. filter(λj.m1(x,j);upto(0;size))[a2]
| | | | | |  =
| | | | | |  filter(λj.m1(x1,j);upto(0;size))[a3]
| | | | | ⊢  a2 = a3

| | | | | | by Assert no_repeats(ℕsize;filter(λj.m1(x,j);upto(0;size)))
| | | | | |
| | | | | |\
| | | | | | .....assertion .....
| | | | | | 1. size : ℕ
| | | | | | 2. m1 : ℕsize→ℕsize→𝔹
| | | | | | 3. x : ℕsize
| | | | | | 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | | | 5. x1 : ℕsize
| | | | | | 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | | | 7. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | | | 8. x = x1
| | | | | | 9. filter(λj.m1(x,j);upto(0;size))[a2]
| | | | | | |  =
| | | | | | |  filter(λj.m1(x1,j);upto(0;size))[a3]
| | | | | | ⊢  no_repeats(ℕsize;filter(λj.m1(x,j);upto(0;size)))
| | | | | | | by BackThru
| | | | | | |    Thm* ∀P:(T→𝔹), l:T List.
```

```
| | | | | | |         no_repeats(T;l) ⇒ no_repeats(T;filter(P;l))
| | | | | | |    {no_repeats_filter}
| | | | | | |
| | | | | | 1. size : ℕ
| | | | | | 2. m1 : ℕsize→ℕsize→𝔹
| | | | | | 3. x : ℕsize
| | | | | | 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | | | 5. x1 : ℕsize
| | | | | | 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | | | 7. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | | | 8. x = x1
| | | | | | 9. filter(λj.m1(x,j);upto(0;size))[a2]
| | | | | | |  =
| | | | | | |  filter(λj.m1(x1,j);upto(0;size))[a3]
| | | | | | ⊢  no_repeats(ℕsize;upto(0;size))
| | | | | | | by Assert no_repeats(ℕ;upto(0;size))
| | | | | | |
| | | | | | |\
| | | | | | | .....assertion .....
| | | | | | | 1. size : ℕ
| | | | | | | 2. m1 : ℕsize→ℕsize→𝔹
| | | | | | | 3. x : ℕsize
| | | | | | | 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | | | | 5. x1 : ℕsize
| | | | | | | 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | | | | 7. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | | | | 8. x = x1
| | | | | | | 9. filter(λj.m1(x,j);upto(0;size))[a2]
| | | | | | |    =
| | | | | | |    filter(λj.m1(x1,j);upto(0;size))[a3]
| | | | | | | ⊢  no_repeats(ℕ;upto(0;size))
| | | | | | |   by BackThru
| | | | | | |      Thm* ∀i,j:ℕ. no_repeats(ℕ;upto(i;j))
| | | | | | |      {no_repeats_upto}
| | | | | | |
| | | | | | .\
| | | | | | . 1. size : ℕ
| | | | | | . 2. m1 : ℕsize→ℕsize→𝔹
| | | | | | . 3. x : ℕsize
| | | | | | . 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | | | . 5. x1 : ℕsize
| | | | | | . 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | | | . 7. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | | | . 8. x = x1
| | | | | | . 9. filter(λj.m1(x,j);upto(0;size))[a2]
| | | | | | .    =
| | | | | | .    filter(λj.m1(x1,j);upto(0;size))[a3]
| | | | | | . 10. no_repeats(ℕ;upto(0;size))
| | | | | | . ⊢  no_repeats(ℕsize;upto(0;size))
| | | | | | .   by Repeat (ParallelOp -1)
| | | | | |
| | | | | .\
| | | | | . 1. size : ℕ
| | | | | . 2. m1 : ℕsize→ℕsize→𝔹
| | | | | . 3. x : ℕsize
```

```
| | | | . 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | . 5. x1 : ℕsize
| | | | . 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | . 7. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | . 8. x = x1
| | | | . 9. filter(λj.m1(x,j);upto(0;size))[a2]
| | | | . |  =
| | | | . |  filter(λj.m1(x1,j);upto(0;size))[a3]
| | | | . 10. no_repeats(ℕsize;filter(λj.m1(x,j);upto(0;size)))
| | | | . ⊢  a2 = a3  by SupposeNot
| | | | . |
| | | | . |\
| | | | . | 1. size : ℕ
| | | | . | 2. m1 : ℕsize→ℕsize→𝔹
| | | | . | 3. x : ℕsize
| | | | . | 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | . | 5. x1 : ℕsize
| | | | . | 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | . | 7. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | . | 8. x = x1
| | | | . | 9. filter(λj.m1(x,j);upto(0;size))[a2]
| | | | . |    =
| | | | . |    filter(λj.m1(x1,j);upto(0;size))[a3]
| | | | . | 10. no_repeats(ℕsize;filter(λj.m1(x,j);upto(0;size)))
| | | | . | ⊢  a3<||filter(λj.m1(x,j);upto(0;size))||
| | | | . |   by HypSubst -3 0
| | | | . |

| | | | . .\
| | | | . . 1. size : ℕ
| | | | . . 2. m1 : ℕsize→ℕsize→𝔹
| | | | . . 3. x : ℕsize
| | | | . . 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | . . 5. x1 : ℕsize
| | | | . . 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | . . 7. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | . . 8. x = x1
| | | | . . 9. filter(λj.m1(x,j);upto(0;size))[a2]
| | | | . . |  =
| | | | . . |  filter(λj.m1(x1,j);upto(0;size))[a3]
| | | | . . 10. no_repeats(ℕsize;filter(λj.m1(x,j);upto(0;size)))
| | | | . . 11. ¬a2 = a3
| | | | . . ⊢  a2 = a3
| | | | . . | by (Unfold ‘no_repeats‘ -2) THEN (InstHyp [a2;a3] -2)
| | | | . . |
| | | | . . |\
| | | | . . | .....antecedent .....
| | | | . . | 1. size : ℕ
| | | | . . | 2. m1 : ℕsize→ℕsize→𝔹
| | | | . . | 3. x : ℕsize
| | | | . . | 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | . . | 5. x1 : ℕsize
| | | | . . | 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | . . | 7. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | . . | 8. x = x1
| | | | . . | 9. filter(λj.m1(x,j);upto(0;size))[a2]
```

```
| | | | . . |     =
| | | | . . |     filter(λj.m1(x1,j);upto(0;size))[a3]
| | | | . . | 10. ∀i,j:ℕ.
| | | | . . |      i<||filter(λj.m1(x,j);upto(0;size))||
| | | | . . |      ⇒
| | | | . . |      j<||filter(λj.m1(x,j);upto(0;size))||
| | | | . . |      ⇒
| | | | . . |      ¬i = j
| | | | . . |      ⇒
| | | | . . |      ¬filter(λj.m1(x,j);upto(0;size))[i]
| | | | . . |       =
| | | | . . |       filter(λj.m1(x,j);upto(0;size))[j]
| | | | . . | 11. ¬a2 = a3
| | | | . . | ⊢  a3<||filter(λj.m1(x,j);upto(0;size))||
| | | | . . |   by HypSubst 8 0
| | | | . . |
| | | | . . |\
| | | | . . | .....antecedent .....
| | | | . . | 1. size : ℕ
| | | | . . | 2. m1 : ℕsize→ℕsize→𝔹
| | | | . . | 3. x : ℕsize
| | | | . . | 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | . . | 5. x1 : ℕsize
| | | | . . | 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | . . | 7. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | . . | 8. x = x1
| | | | . . | 9. filter(λj.m1(x,j);upto(0;size))[a2]
| | | | . . |     =
| | | | . . |     filter(λj.m1(x1,j);upto(0;size))[a3]
| | | | . . | 10. ∀i,j:ℕ.
| | | | . . |      i<||filter(λj.m1(x,j);upto(0;size))||
| | | | . . |      ⇒
| | | | . . |      j<||filter(λj.m1(x,j);upto(0;size))||
| | | | . . |      ⇒
| | | | . . |      ¬i = j
| | | | . . |      ⇒
| | | | . . |      ¬filter(λj.m1(x,j);upto(0;size))[i]
| | | | . . |       =
| | | | . . |       filter(λj.m1(x,j);upto(0;size))[j]
| | | | . . | 11. ¬a2 = a3
| | | | . . | ⊢  ¬a2 = a3  by Repeat (ParallelOp -1)
| | | | . . |
| | | | . . .\
| | | | . . . 1. size : ℕ
| | | | . . . 2. m1 : ℕsize→ℕsize→𝔹
| | | | . . . 3. x : ℕsize
| | | | . . . 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | . . . 5. x1 : ℕsize
| | | | . . . 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | . . . 7. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | . . . 8. x = x1
| | | | . . . 9. filter(λj.m1(x,j);upto(0;size))[a2]
| | | | . . . |  =
| | | | . . . |  filter(λj.m1(x1,j);upto(0;size))[a3]
| | | | . . . 10. ∀i,j:ℕ.
```

```
| | | | . . . |   i<||filter(λj.m1(x,j);upto(0;size))||
| | | | . . . |   ⇒
| | | | . . . |   j<||filter(λj.m1(x,j);upto(0;size))||
| | | | . . . |   ⇒
| | | | . . . |   ¬i = j
| | | | . . . |   ⇒
| | | | . . . |   ¬filter(λj.m1(x,j);upto(0;size))[i]
| | | | . . . |    =
| | | | . . . |    filter(λj.m1(x,j);upto(0;size))[j]
| | | | . . . 11. ¬a2 = a3
| | | | . . . 12. ¬filter(λj.m1(x,j);upto(0;size))[a2]
| | | | . . . |    =
| | | | . . . |    filter(λj.m1(x,j);upto(0;size))[a3]
| | | | . . . ⊢  a2 = a3  by Analyze -1
| | | | . . . |
| | | | . . . 1. size : ℕ
| | | | . . . 2. m1 : ℕsize→ℕsize→𝔹
| | | | . . . 3. x : ℕsize
| | | | . . . 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | . . . 5. x1 : ℕsize
| | | | . . . 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | . . . 7. m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | . . . 8. x = x1
| | | | . . . 9. filter(λj.m1(x,j);upto(0;size))[a2]
| | | | . . .    =
| | | | . . .    filter(λj.m1(x1,j);upto(0;size))[a3]
| | | | . . . 10. ∀i,j:ℕ.
| | | | . . .     i<||filter(λj.m1(x,j);upto(0;size))||
| | | | . . .     ⇒
| | | | . . .     j<||filter(λj.m1(x,j);upto(0;size))||
| | | | . . .     ⇒
| | | | . . .     ¬i = j
| | | | . . .     ⇒
| | | | . . .     ¬filter(λj.m1(x,j);upto(0;size))[i]
| | | | . . .      =
| | | | . . .      filter(λj.m1(x,j);upto(0;size))[j]
| | | | . . . 11. ¬a2 = a3
| | | | . . . ⊢  filter(λj.m1(x,j);upto(0;size))[a2]
| | | | . . .    =
| | | | . . .    filter(λj.m1(x,j);upto(0;size))[a3]
| | | | . . .   by RevHypSubst' 8 9
| | | |
| | | |\
| | | | 1. size : ℕ
| | | | 2. m1 : ℕsize→ℕsize→𝔹
| | | | 3. x : ℕsize
| | | | 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | 5. x1 : ℕsize
| | | | 6. ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
| | | | | by Assert
| | | | |    (∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | | | |     m1(x,filter(λj.m1(x,j);upto(0;size))[i]))
| | | | |
| | | | |\
```

```
| | | | | | .....assertion .....
| | | | | | 1. size : ℕ
| | | | | | 2. m1 : ℕsize→ℕsize→𝔹
| | | | | | 3. x : ℕsize
| | | | | | 4. ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | | | 5. x1 : ℕsize
| | | | | | 6. ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | | | ⊢  ∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | | | | | |  m1(x,filter(λj.m1(x,j);upto(0;size))[i])  by All Thin
| | | | | | |
| | | | | | 1. size : ℕ
| | | | | | 2. m1 : ℕsize→ℕsize→𝔹
| | | | | | 3. x : ℕsize
| | | | | | 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | | | ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | | | | | | by Assert
| | | | | | |    (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | | | | | |    m1(x,j);upto(0;size)))
| | | | | | |
| | | | | | |\
| | | | | | | .....assertion .....
| | | | | | | 1. size : ℕ
| | | | | | | 2. m1 : ℕsize→ℕsize→𝔹
| | | | | | | 3. x : ℕsize
| | | | | | | 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | | | | ⊢  (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | | | | | |    m1(x,j);upto(0;size)))
| | | | | | |   by (Unfold ‘l_member‘ 0) THEN (InstConcl [i])
| | | | | | |
| | | | | | .\
| | | | | | . 1. size : ℕ
| | | | | | . 2. m1 : ℕsize→ℕsize→𝔹
| | | | | | . 3. x : ℕsize
| | | | | | . 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | | | . 5. (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | | | | | .    m1(x,j);upto(0;size)))
| | | | | | . ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | | | | | .   by (RWO
| | | | | | .       Thm* ∀P:(T→𝔹), L:T List, x:T.
| | | | | | .            (x ∈ filter(P;L)) ⟺ (x ∈ L) & P(x)
| | | | | | .       {member_filter}
| | | | | | .       -1)
| | | | | | .      THEN
| | | | | | .      (Reduce -1)
| | | | | |
| | | | | .\
| | | | | . 1. size : ℕ
| | | | | . 2. m1 : ℕsize→ℕsize→𝔹
| | | | | . 3. x : ℕsize
| | | | | . 4. a2 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | | | . 5. x1 : ℕsize
| | | | | . 6. ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | | | . 7. ∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | | | | .    m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | | | | . ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[a2])
```

```
| | | | .   by BackThru -1
| | | |
| | | .\
| | | . 1. size : ℕ
| | | . 2. m1 : ℕsize→ℕsize→𝔹
| | | . 3. x : ℕsize
| | | . 4. ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | . 5. x1 : ℕsize
| | | . 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | . ⊢  m1(x1,filter(λj.m1(x1,j);upto(0;size))[a3])
| | | . | by Assert
| | | . |    (∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | | . |     m1(x,filter(λj.m1(x,j);upto(0;size))[i]))
| | | . |
| | | . |\
| | | . | .....assertion .....
| | | . | 1. size : ℕ
| | | . | 2. m1 : ℕsize→ℕsize→𝔹
| | | . | 3. x : ℕsize
| | | . | 4. ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | . | 5. x1 : ℕsize
| | | . | 6. ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | . | ⊢  ∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | | . | |  m1(x,filter(λj.m1(x,j);upto(0;size))[i])  by All Thin
| | | . | |
| | | . | 1. size : ℕ
| | | . | 2. m1 : ℕsize→ℕsize→𝔹
| | | . | 3. x : ℕsize
| | | . | 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | . | ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | | . | | by Assert
| | | . | |    (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | | . | |    m1(x,j);upto(0;size)))
| | | . | |
| | | . | |\
| | | . | | .....assertion .....
| | | . | | 1. size : ℕ
| | | . | | 2. m1 : ℕsize→ℕsize→𝔹
| | | . | | 3. x : ℕsize
| | | . | | 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | . | | ⊢  (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | | . | |    m1(x,j);upto(0;size)))
| | | . | |   by (Unfold ‘l_member‘ 0) THEN (InstConcl [i])
| | | . | |
| | | . | .\
| | | . | . 1. size : ℕ
| | | . | . 2. m1 : ℕsize→ℕsize→𝔹
| | | . | . 3. x : ℕsize
| | | . | . 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | . | . 5. (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | | . | .    m1(x,j);upto(0;size)))
| | | . | . ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | | . | .   by (RWO
| | | . | .       Thm* ∀P:(T→𝔹), L:T List, x:T.
| | | . | .            (x ∈ filter(P;L)) ⟺ (x ∈ L) & P(x)
```

```
| | | . | .       {member_filter}
| | | . | .       -1)
| | | . | .      THEN
| | | . | .      (Reduce -1)
| | | . |
| | | . .\
| | | . . 1. size : ℕ
| | | . . 2. m1 : ℕsize→ℕsize→𝔹
| | | . . 3. x : ℕsize
| | | . . 4. ℕ||filter(λj.m1(x,j);upto(0;size))||
| | | . . 5. x1 : ℕsize
| | | . . 6. a3 : ℕ||filter(λj.m1(x1,j);upto(0;size))||
| | | . . 7. ∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | | . .    m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | | . . ⊢  m1(x1,filter(λj.m1(x1,j);upto(0;size))[a3])
| | | . .   by BackThru -1
| | |
| | .\
| | . 1. size : ℕ
| | . 2. m1 : ℕsize→ℕsize→𝔹
| | . ⊢  ∀b:{p:(ℕsize×ℕsize)| m1(1of(p),2of(p)) }.
| | . |  ∃a:(x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||).
| | . |  (a/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>) = b
| | . | by (Analyze 0) THEN (Analyze -1)
| | . |    THEN
| | . |    (Decide (m1(1of(b),2of(b))))
| | . |
| | . |\
| | . | 1. size : ℕ
| | . | 2. m1 : ℕsize→ℕsize→𝔹
| | . | 3. b : ℕsize×ℕsize
| | . | 4. m1(1of(b),2of(b))  [not for witness]
| | . | 5. m1(1of(b),2of(b))
| | . | ⊢  ∃a:(x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||).
| | . | |  (a/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>) = b
| | . | | by Thin -2
| | . | |
| | . | 1. size : ℕ
| | . | 2. m1 : ℕsize→ℕsize→𝔹
| | . | 3. b : ℕsize×ℕsize
| | . | 4. m1(1of(b),2of(b))
| | . | ⊢  ∃a:(x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||).
| | . | |  (a/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>) = b
| | . | | by Assert (2of(b) ∈ filter(λj.m1(1of(b),j);upto(0;size)))
| | . | |
| | . | |\
| | . | | .....assertion .....
| | . | | 1. size : ℕ
| | . | | 2. m1 : ℕsize→ℕsize→𝔹
| | . | | 3. b : ℕsize×ℕsize
| | . | | 4. m1(1of(b),2of(b))
| | . | | ⊢  (2of(b) ∈ filter(λj.m1(1of(b),j);upto(0;size)))
| | . | | | by (RWO
| | . | | |     Thm* ∀P:(T→𝔹), L:T List, x:T.
| | . | | |          (x ∈ filter(P;L)) ⟺ (x ∈ L) & P(x)
```

```
| | . | | |     {member_filter}
| | . | | |     0)
| | . | | |    THEN
| | . | | |    (Reduce 0)
| | . | | |
| | . | | 1. size : ℕ
| | . | | 2. m1 : ℕsize→ℕsize→𝔹
| | . | | 3. b : ℕsize×ℕsize
| | . | | 4. m1(1of(b),2of(b))
| | . | | ⊢  (2of(b) ∈ upto(0;size))
| | . | | | by AssertBY (2of(b) ∈ upto(0;size))
| | . | | |    (RWO
| | . | | |     Thm* ∀i,j,k:ℕ. (k ∈ upto(i;j)) ⟺ i≤k & k<j
| | . | | |     {member_upto}
| | . | | |     0)
| | . | | |
| | . | | 1. size : ℕ
| | . | | 2. m1 : ℕsize→ℕsize→𝔹
| | . | | 3. b : ℕsize×ℕsize
| | . | | 4. m1(1of(b),2of(b))
| | . | | 5. (2of(b) ∈ upto(0;size))
| | . | | ⊢  (2of(b) ∈ upto(0;size))
| | . | |   by (RepeatFor 2 (ParallelOp -1)) THEN (Analyze -1)
| | . | |
| | . | .\
| | . | . 1. size : ℕ
| | . | . 2. m1 : ℕsize→ℕsize→𝔹
| | . | . 3. b : ℕsize×ℕsize
| | . | . 4. m1(1of(b),2of(b))
| | . | . 5. (2of(b) ∈ filter(λj.m1(1of(b),j);upto(0;size)))
| | . | . ⊢  ∃a:(x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||).
| | . | . |  (a/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>) = b
| | . | . | by (Unfold ‘l_member‘ -1) THEN ExRepD
| | . | . |
| | . | . 1. size : ℕ
| | . | . 2. m1 : ℕsize→ℕsize→𝔹
| | . | . 3. b : ℕsize×ℕsize
| | . | . 4. m1(1of(b),2of(b))
| | . | . 5. i : ℕ
| | . | . 6. i<||filter(λj.m1(1of(b),j);upto(0;size))||
| | . | . 7. 2of(b) = filter(λj.m1(1of(b),j);upto(0;size))[i]
| | . | . ⊢  ∃a:(x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||).
| | . | . |  (a/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>) = b
| | . | . | by (InstConcl [<1of(b),i>]) THEN (Reduce 0)
| | . | . |
| | . | . |\
| | . | . | 1. size : ℕ
| | . | . | 2. m1 : ℕsize→ℕsize→𝔹
| | . | . | 3. b : ℕsize×ℕsize
| | . | . | 4. m1(1of(b),2of(b))
| | . | . | 5. i : ℕ
| | . | . | 6. i<||filter(λj.m1(1of(b),j);upto(0;size))||
| | . | . | 7. 2of(b) = filter(λj.m1(1of(b),j);upto(0;size))[i]
| | . | . | ⊢  <1of(b),filter(λj.m1(1of(b),j);upto(0;size))[i]> = b
| | . | . | | by (Analyze 3) THEN (All Reduce)
```

```
| | . | . | |
| | . | . | 1. size : ℕ
| | . | . | 2. m1 : ℕsize→ℕsize→𝔹
| | . | . | 3. b1 : ℕsize
| | . | . | 4. b2 : ℕsize
| | . | . | 5. m1(b1,b2)
| | . | . | 6. i : ℕ
| | . | . | 7. i<||filter(λj.m1(b1,j);upto(0;size))||
| | . | . | 8. b2 = filter(λj.m1(b1,j);upto(0;size))[i]
| | . | . | ⊢  <b1,filter(λj.m1(b1,j);upto(0;size))[i]> = <b1,b2>
| | . | . | | by Analyze
| | . | . | |
| | . | . | |\
| | . | . | | 1. size : ℕ
| | . | . | | 2. m1 : ℕsize→ℕsize→𝔹
| | . | . | | 3. b1 : ℕsize
| | . | . | | 4. b2 : ℕsize
| | . | . | | 5. m1(b1,b2)
| | . | . | | 6. i : ℕ
| | . | . | | 7. i<||filter(λj.m1(b1,j);upto(0;size))||
| | . | . | | 8. b2 = filter(λj.m1(b1,j);upto(0;size))[i]
| | . | . | | ⊢  <b1,filter(λj.m1(b1,j);upto(0;size))[i]> = <b1,b2>
| | . | . | |   by Analyze
| | . | . | |
| | . | . | .\
| | . | . | . .....set predicate .....
| | . | . | . 1. size : ℕ
| | . | . | . 2. m1 : ℕsize→ℕsize→𝔹
| | . | . | . 3. b1 : ℕsize
| | . | . | . 4. b2 : ℕsize
| | . | . | . 5. m1(b1,b2)
| | . | . | . 6. i : ℕ
| | . | . | . 7. i<||filter(λj.m1(b1,j);upto(0;size))||
| | . | . | . 8. b2 = filter(λj.m1(b1,j);upto(0;size))[i]
| | . | . | . ⊢  m1
| | . | . | . |  (1of(<b1,filter(λj.m1(b1,j);upto(0;size))[i]>)
| | . | . | . |  ,2of(<b1,filter(λj.m1(b1,j);upto(0;size))[i]>))
| | . | . | . | by Reduce 0
| | . | . | . |
| | . | . | . 1. size : ℕ
| | . | . | . 2. m1 : ℕsize→ℕsize→𝔹
| | . | . | . 3. b1 : ℕsize
| | . | . | . 4. b2 : ℕsize
| | . | . | . 5. m1(b1,b2)
| | . | . | . 6. i : ℕ
| | . | . | . 7. i<||filter(λj.m1(b1,j);upto(0;size))||
| | . | . | . 8. b2 = filter(λj.m1(b1,j);upto(0;size))[i]
| | . | . | . ⊢  m1(b1,filter(λj.m1(b1,j);upto(0;size))[i])
| | . | . | . | by Assert
| | . | . | . |    (∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | . | . | . |     m1(x,filter(λj.m1(x,j);upto(0;size))[i]))
| | . | . | . |
| | . | . | . |\
| | . | . | . | .....assertion .....
| | . | . | . | 1. size : ℕ
```

```
| | . | . | . | 2. m1 : ℕsize→ℕsize→𝔹
| | . | . | . | 3. b1 : ℕsize
| | . | . | . | 4. b2 : ℕsize
| | . | . | . | 5. m1(b1,b2)
| | . | . | . | 6. i : ℕ
| | . | . | . | 7. i<||filter(λj.m1(b1,j);upto(0;size))||
| | . | . | . | 8. b2 = filter(λj.m1(b1,j);upto(0;size))[i]
| | . | . | . | ⊢  ∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | . | . | . | |  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | . | . | . | | by All Thin
| | . | . | . | |
| | . | . | . | 1. size : ℕ
| | . | . | . | 2. m1 : ℕsize→ℕsize→𝔹
| | . | . | . | 3. x : ℕsize
| | . | . | . | 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . | . | ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | . | . | . | | by Assert
| | . | . | . | |    (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | . | . | . | |    m1(x,j);upto(0;size)))
| | . | . | . | |
| | . | . | . | |\
| | . | . | . | | .....assertion .....
| | . | . | . | | 1. size : ℕ
| | . | . | . | | 2. m1 : ℕsize→ℕsize→𝔹
| | . | . | . | | 3. x : ℕsize
| | . | . | . | | 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . | . | | ⊢  (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | . | . | . | |    m1(x,j);upto(0;size)))
| | . | . | . | |   by (Unfold `l_member` 0) THEN (InstConcl [i])
| | . | . | . | |
| | . | . | . | .\
| | . | . | . | . 1. size : ℕ
| | . | . | . | . 2. m1 : ℕsize→ℕsize→𝔹
| | . | . | . | . 3. x : ℕsize
| | . | . | . | . 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . | . | . 5. (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | . | . | . | .    m1(x,j);upto(0;size)))
| | . | . | . | . ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | . | . | . | .   by (RWO
| | . | . | . | .       Thm* ∀P:(T→𝔹), L:T List, x:T.
| | . | . | . | .            (x ∈ filter(P;L)) ⟺ (x ∈ L) & P(x)
| | . | . | . | .       {member_filter}
| | . | . | . | .       -1)
| | . | . | . | .      THEN
| | . | . | . | .      (Reduce -1)
| | . | . | . |
| | . | . | . .\
| | . | . | . . 1. size : ℕ
| | . | . | . . 2. m1 : ℕsize→ℕsize→𝔹
| | . | . | . . 3. b1 : ℕsize
| | . | . | . . 4. b2 : ℕsize
| | . | . | . . 5. m1(b1,b2)
| | . | . | . . 6. i : ℕ
| | . | . | . . 7. i<||filter(λj.m1(b1,j);upto(0;size))||
| | . | . | . . 8. b2 = filter(λj.m1(b1,j);upto(0;size))[i]
```

```
| | . | . | . . 9. ∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | . | . | . .    m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | . | . | . . ⊢  m1(b1,filter(λj.m1(b1,j);upto(0;size))[i])
| | . | . | . .   by BackThru -1
| | . | . |
| | . | . .\
| | . | . . .....subterm .....
| | . | . . 1. size : ℕ
| | . | . . 2. m1 : ℕsize→ℕsize→𝔹
| | . | . . 3. b : ℕsize×ℕsize
| | . | . . 4. m1(1of(b),2of(b))
| | . | . . 5. i : ℕ
| | . | . . 6. i<||filter(λj.m1(1of(b),j);upto(0;size))||
| | . | . . 7. 2of(b) = filter(λj.m1(1of(b),j);upto(0;size))[i]
| | . | . . 8. a : x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . . 9. x : ℕsize
| | . | . . 10. i1 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . . 11. a = <x,i1>
| | . | . . ⊢  <x,filter(λj.m1(x,j);upto(0;size))[i1]>
| | . | . . |  ∈ {p:(ℕsize×ℕsize)| m1(1of(p),2of(p)) }
| | . | . . | by Analyze THEN (Reduce 0)
| | . | . . |
| | . | . . 1. size : ℕ
| | . | . . 2. m1 : ℕsize→ℕsize→𝔹
| | . | . . 3. b : ℕsize×ℕsize
| | . | . . 4. m1(1of(b),2of(b))
| | . | . . 5. i : ℕ
| | . | . . 6. i<||filter(λj.m1(1of(b),j);upto(0;size))||
| | . | . . 7. 2of(b) = filter(λj.m1(1of(b),j);upto(0;size))[i]
| | . | . . 8. a : x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . . 9. x : ℕsize
| | . | . . 10. i1 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . . 11. a = <x,i1>
| | . | . . ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i1])
| | . | . . | by Assert
| | . | . . |    (∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | . | . . |     m1(x,filter(λj.m1(x,j);upto(0;size))[i]))
| | . | . . |
| | . | . . |\
| | . | . . | .....assertion .....
| | . | . . | 1. size : ℕ
| | . | . . | 2. m1 : ℕsize→ℕsize→𝔹
| | . | . . | 3. b : ℕsize×ℕsize
| | . | . . | 4. m1(1of(b),2of(b))
| | . | . . | 5. i : ℕ
| | . | . . | 6. i<||filter(λj.m1(1of(b),j);upto(0;size))||
| | . | . . | 7. 2of(b) = filter(λj.m1(1of(b),j);upto(0;size))[i]
| | . | . . | 8. a : x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . . | 9. x : ℕsize
| | . | . . | 10. i1 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . . | 11. a = <x,i1>
| | . | . . | ⊢  ∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | . | . . | |  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | . | . . | | by All Thin
| | . | . . | |
```

```
| | . | . . | 1. size : ℕ
| | . | . . | 2. m1 : ℕsize→ℕsize→𝔹
| | . | . . | 3. x : ℕsize
| | . | . . | 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . . | ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | . | . . | | by Assert
| | . | . . | |    (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | . | . . | |    m1(x,j);upto(0;size)))
| | . | . . | |
| | . | . . | |\
| | . | . . | | .....assertion .....
| | . | . . | | 1. size : ℕ
| | . | . . | | 2. m1 : ℕsize→ℕsize→𝔹
| | . | . . | | 3. x : ℕsize
| | . | . . | | 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . . | | ⊢  (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | . | . . | |    m1(x,j);upto(0;size)))
| | . | . . | |   by (Unfold ‘l_member‘ 0) THEN (InstConcl [i])
| | . | . . | |
| | . | . . | .\
| | . | . . | . 1. size : ℕ
| | . | . . | . 2. m1 : ℕsize→ℕsize→𝔹
| | . | . . | . 3. x : ℕsize
| | . | . . | . 4. i : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . . | . 5. (filter(λj.m1(x,j);upto(0;size))[i] ∈ filter(λj.
| | . | . . | .    m1(x,j);upto(0;size)))
| | . | . . | . ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | . | . . | .   by (RWO
| | . | . . | .       Thm* ∀P:(T→𝔹), L:T List, x:T.
| | . | . . | .            (x ∈ filter(P;L)) ⟺ (x ∈ L) & P(x)
| | . | . . | .       {member_filter}
| | . | . . | .       -1)
| | . | . . | .      THEN
| | . | . . | .      (Reduce -1)
| | . | . . |
| | . | . . .\
| | . | . . . 1. size : ℕ
| | . | . . . 2. m1 : ℕsize→ℕsize→𝔹
| | . | . . . 3. b : ℕsize×ℕsize
| | . | . . . 4. m1(1of(b),2of(b))
| | . | . . . 5. i : ℕ
| | . | . . . 6. i<||filter(λj.m1(1of(b),j);upto(0;size))||
| | . | . . . 7. 2of(b) = filter(λj.m1(1of(b),j);upto(0;size))[i]
| | . | . . . 8. a : x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . . . 9. x : ℕsize
| | . | . . . 10. i1 : ℕ||filter(λj.m1(x,j);upto(0;size))||
| | . | . . . 11. a = <x,i1>
| | . | . . . 12. ∀x:ℕsize, i:ℕ||filter(λj.m1(x,j);upto(0;size))||.
| | . | . .      m1(x,filter(λj.m1(x,j);upto(0;size))[i])
| | . | . . . ⊢  m1(x,filter(λj.m1(x,j);upto(0;size))[i1])
| | . | . . .   by BackThru -1
| | . |
| | . .\
| | . . 1. size : ℕ
| | . . 2. m1 : ℕsize→ℕsize→𝔹
```

```
| | . . 3. b : ℕsize×ℕsize
| | . . 4. m1(1of(b),2of(b))  [not for witness]
| | . . 5. ¬m1(1of(b),2of(b))
| | . . ⊢  ∃a:(x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||).
| | . .     (a/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>) = b
| | . .    by (Analyze -1) THEN Unhide
| |
| .\
| . 1. size : ℕ
| . 2. m1 : ℕsize→ℕsize→𝔹
| . ⊢  (λx.<1of(x),filter(λj.m1(1of(x),j);upto(0;size))[2of(x)]>)
| . |  =
| . |  (λe.e) o (λe.e/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>)
| . | by Ext THENA (Auto THEN (Analyze -1) THEN (Reduce 0))
| . |
| . 1. size : ℕ
| . 2. m1 : ℕsize→ℕsize→𝔹
| . 3. x : x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||
| . ⊢  (λx.<1of(x),filter(λj.m1(1of(x),j);upto(0;size))[2of(x)]>)(x)
| . |  =
| . |  ((λe.e) o (λe.e/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>))
| . |  (x)  by Reduce 0

| . |
| . 1. size : ℕ
| . 2. m1 : ℕsize→ℕsize→𝔹
| . 3. x : x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||
| . ⊢  <1of(x),filter(λj.m1(1of(x),j);upto(0;size))[2of(x)]>
| .    =
| .    (x/x,i. <x,filter(λj.m1(x,j);upto(0;size))[i]>)
| .   by (Analyze -1) THEN (Reduce 0)
|
.\
. .....wf .....
. 1. size : ℕ
. 2. m1 : ℕsize→ℕsize→𝔹
. 3. emap :
.    (x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||)→{p:(ℕsize×ℕsize)
.                                                          | m1
.                                                            (1of(p)
.                                                            ,2of(p)) }
. ⊢  (Bij(ℕsize; ℕsize; Id)
.     & Bij(x:ℕsize×ℕ||filter(λj.m1(x,j);upto(0;size))||;
.            {p:(ℕsize×ℕsize)| m1(1of(p),2of(p)) };
.            emap)
.     & (λx.<1of(x),filter(λj.m1(1of(x),j);upto(0;size))[2of(x)]>)
.       =
.       (λe.e) o emap)
.    ∈ Prop  by Auto THEN (Analyze -1) THEN (Reduce 0)
END
```