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depthfirst-df-traversal  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph).
|  df-traversal(the_graph;depthfirst(the_obj))
| by Auto
|    THEN
|    (Inst
|     Thm* For any graph
|          ∀the_obj:GraphObject(the_graph), P:(Traversal→Prop).
|          P(nil)
|          ⇒
|          (∀s:Traversal, i:V. P(s) ⇒ P(dfs(the_obj;s;i)))
|          ⇒
|          P(depthfirst(the_obj))
|     {depthfirst_induction}
|     [the_graph
|     ;the_obj
|     ;λ₂s.
|      df-traversal(the_graph;s) & paren(Vertices(the_graph);s)])
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| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| ⊢  df-traversal(the_graph;nil)
|   by BackThru
|      Thm* For any graph df-traversal(the_graph;nil)
|      {df_DASH_traversal_DASH_nil}
|
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| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| ⊢  paren(Vertices(the_graph);nil)  by ObviousConcl
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| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. s : traversal(the_graph)
| 4. i : Vertices(the_graph)
| 5. df-traversal(the_graph;s)
| 6. paren(Vertices(the_graph);s)
| ⊢  df-traversal(the_graph;dfs(the_obj;s;i))
|   by (BackThru
|       Thm* For any graph
|            ∀the_obj:GraphObject(the_graph), s:Traversal, i:V.
|            df-traversal(the_graph;s)
|            ⇒
|            (∀j:V. (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i)
|            ⇒
|            df-traversal(the_graph;dfs(the_obj;s;i))
|       {dfs_DASH_df_DASH_traversal})
|      THEN
|      (FwdThru
|       Thm* ∀s':(T+T) List.
|            paren(T;s') ⇒ (∀i:T. (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
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|       {paren_balance2}
|       [-4;-2])
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. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. s : traversal(the_graph)
. 4. i : Vertices(the_graph)
. 5. df-traversal(the_graph;s)
. 6. paren(Vertices(the_graph);s)
. ⊢  paren(Vertices(the_graph);dfs(the_obj;s;i))
. | by (Inst Thm: dfs_member [the_graph;the_obj;s;i]) THEN ExRepD
. |    THEN
. |    (HypSubst -1 0)
. |
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. s : traversal(the_graph)
. 4. i : Vertices(the_graph)
. 5. df-traversal(the_graph;s)
. 6. paren(Vertices(the_graph);s)
. 7. s' : traversal(the_graph)
. 8. (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. 9. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. 11. paren(Vertices(the_graph);s')
. 12. dfs(the_obj;s;i) = (s' @ s)
. ⊢  paren(Vertices(the_graph);s' @ s)
.   by BackThru
.      Thm* ∀s',s3:(T+T) List.
.           paren(T;s') ⇒ paren(T;s3) ⇒ paren(T;s3 @ s')
.      {append_paren}
END
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