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dfs-connect  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), s:Traversal, i:V.
|  ∃s':Traversal.
|  (∀j:V. (inr(j) ∈ s') ⇒ i-the_graph->*j)
|  & dfs(the_obj;s;i) = (s' @ s)
| by Auto
|    THEN
|    (Inst Thm: dfs_induction2
|     [the_graph
|     ;the_obj
|     ;λ2i,s1,s2.
|      ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
|     ;s
|     ;i])
|
|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. traversal(the_graph)
| 4. Vertices(the_graph)
| 5. s1 : traversal(the_graph)
| 6. i1 : Vertices(the_graph)
| 7. (inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1)
| 8. j : Vertices(the_graph)
| 9. (inr(j) ∈ nil)
| ⊢  i1-the_graph->*j  by ObviousContradiction [-1]
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|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. traversal(the_graph)
| 4. Vertices(the_graph)
| 5. traversal(the_graph)
| 6. s2 : traversal(the_graph)
| 7. s3 : traversal(the_graph)
| 8. i1 : Vertices(the_graph)
| 9. j : Vertices(the_graph)
| 10. i1-the_graph->j
| 11. paren(Vertices(the_graph);s2)
| 12. paren(Vertices(the_graph);s3)
| 13. ∀j@0:Vertices(the_graph). (inr(j@0) ∈ s2) ⇒ j-the_graph->*j@0
| 14. ∀j:Vertices(the_graph). (inr(j) ∈ s3) ⇒ i1-the_graph->*j
| 15. j1 : Vertices(the_graph)
| 16. (inr(j1) ∈ s3 @ s2)
| ⊢  i1-the_graph->*j1
| | by (RWO
| |     Thm* ∀x:T, l1,l2:T List. (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| |     {member_append}
| |     -1)
| |    THEN
| |    (Analyze -1)
| |    THEN
| |    EasyHyp
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| |
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. traversal(the_graph)
| 4. Vertices(the_graph)
| 5. traversal(the_graph)
| 6. s2 : traversal(the_graph)
| 7. s3 : traversal(the_graph)
| 8. i1 : Vertices(the_graph)
| 9. j : Vertices(the_graph)
| 10. i1-the_graph->j
| 11. paren(Vertices(the_graph);s2)
| 12. paren(Vertices(the_graph);s3)
| 13. ∀j@0:Vertices(the_graph). (inr(j@0) ∈ s2) ⇒ j-the_graph->*j@0
| 14. ∀j:Vertices(the_graph). (inr(j) ∈ s3) ⇒ i1-the_graph->*j
| 15. j1 : Vertices(the_graph)
| 16. (inr(j1) ∈ s2)
| ⊢  i1-the_graph->*j1  by (InstHyp [j1] -4) THEN Connect
|
|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. traversal(the_graph)
| 4. Vertices(the_graph)
| 5. s1 : traversal(the_graph)
| 6. s2 : traversal(the_graph)
| 7. i1 : Vertices(the_graph)
| 8. ¬(inl(i1) ∈ s1)
| 9. ¬(inr(i1) ∈ s1)
| 10. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| 11. j : Vertices(the_graph)
| 12. (inr(j) ∈ [inl(i1) / (s2 @ [inr(i1)])])
| ⊢  i1-the_graph->*j
| | by (RWO
| |     Thm* ∀l:T List, a,x:T. (x ∈ [a / l]) ⟺ x = a ∨ (x ∈ l)
| |     {cons_member}
| |     -1)
| |    THEN
| |    (Analyze -1)
| |
| |\
| | 1. the_graph : Graph
| | 2. GraphObject(the_graph)
| | 3. traversal(the_graph)
| | 4. Vertices(the_graph)
| | 5. s1 : traversal(the_graph)
| | 6. s2 : traversal(the_graph)
| | 7. i1 : Vertices(the_graph)
| | 8. ¬(inl(i1) ∈ s1)
| | 9. ¬(inr(i1) ∈ s1)
| | 10. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | 11. j : Vertices(the_graph)
| | 12. inr(j) = inl(i1)
| | ⊢  i1-the_graph->*j  by ObviousContradiction [-1]
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| .\
| . 1. the_graph : Graph
| . 2. GraphObject(the_graph)
| . 3. traversal(the_graph)
| . 4. Vertices(the_graph)
| . 5. s1 : traversal(the_graph)
| . 6. s2 : traversal(the_graph)
| . 7. i1 : Vertices(the_graph)
| . 8. ¬(inl(i1) ∈ s1)
| . 9. ¬(inr(i1) ∈ s1)
| . 10. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| . 11. j : Vertices(the_graph)
| . 12. (inr(j) ∈ s2 @ [inr(i1)])
| . ⊢  i1-the_graph->*j
| . | by ((RWO
| . |      Thm* ∀x:T, l1,l2:T List.
| . |           (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| . |      {member_append}
| . |      -1)
| . |     THEN
| . |     (RWO Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a    {member_singleton}
| . |      -1))
| . |    THEN
| . |    (Analyze -1)
| . |    THEN
| . |    EasyHyp
| . |
| . 1. the_graph : Graph
| . 2. GraphObject(the_graph)
| . 3. traversal(the_graph)
| . 4. Vertices(the_graph)
| . 5. s1 : traversal(the_graph)
| . 6. s2 : traversal(the_graph)
| . 7. i1 : Vertices(the_graph)
| . 8. ¬(inl(i1) ∈ s1)
| . 9. ¬(inr(i1) ∈ s1)
| . 10. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| . 11. j : Vertices(the_graph)
| . 12. inr(j) = inr(i1)
| . ⊢  i1-the_graph->*j  by Subst (j = i1) 0
| . |
| . |\
| . | .....equality .....
| . | 1. the_graph : Graph
| . | 2. GraphObject(the_graph)
| . | 3. traversal(the_graph)
| . | 4. Vertices(the_graph)
| . | 5. s1 : traversal(the_graph)
| . | 6. s2 : traversal(the_graph)
| . | 7. i1 : Vertices(the_graph)
| . | 8. ¬(inl(i1) ∈ s1)
| . | 9. ¬(inr(i1) ∈ s1)
| . | 10. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| . | 11. j : Vertices(the_graph)
| . | 12. inr(j) = inr(i1)
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| . | ⊢  j = i1  by ObviousFrom [-1]
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| . .\
| . . 1. the_graph : Graph
| . . 2. GraphObject(the_graph)
| . . 3. traversal(the_graph)
| . . 4. Vertices(the_graph)
| . . 5. s1 : traversal(the_graph)
| . . 6. s2 : traversal(the_graph)
| . . 7. i1 : Vertices(the_graph)
| . . 8. ¬(inl(i1) ∈ s1)
| . . 9. ¬(inr(i1) ∈ s1)
| . . 10. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| . . 11. j : Vertices(the_graph)
| . . 12. inr(j) = inr(i1)
| . . ⊢  i1-the_graph->*i1  by Connect
|
.\
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. s : traversal(the_graph)
. 4. i : Vertices(the_graph)
. 5. ∃s':traversal(the_graph).
.    (∀j:Vertices(the_graph). (inr(j) ∈ s') ⇒ i-the_graph->*j)
.    & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
.    & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
.    & paren(Vertices(the_graph);s')
.    & dfs(the_obj;s;i) = (s' @ s)
. ⊢  ∃s':traversal(the_graph).
.    (∀j:Vertices(the_graph). (inr(j) ∈ s') ⇒ i-the_graph->*j)
.    & dfs(the_obj;s;i) = (s' @ s)  by (ParallelOp -1) THEN EasyHyp
END
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