```
dfs-df-traversal  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), s:Traversal, i:V.
|  df-traversal(the_graph;s)
|  ⇒
|  (∀j:V. (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i)
|  ⇒
|  df-traversal(the_graph;dfs(the_obj;s;i))
| by Auto
|    THEN
|    (Inst Thm: dfs_induction4
|     [the_graph
|     ;the_obj
|     ;λ₂i,s1,s2.
|      df-traversal(the_graph;s1)
|      ⇒
|      (∀j:Vertices(the_graph).
|       (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i)
|      ⇒
|      df-traversal(the_graph;s2 @ s1)
|     ;s
|     ;i])
|    THEN
|    (All Reduce)
|    THEN
|    ExRepD
|    THEN
|    (Repeat ThinTrivial)
|    THEN
|    (Try
|     (RWO
|      Thm* ∀a,b,c:Top List. ((a @ b) @ c) ∼ (a @ b @ c)
|      {append_assoc_sq}
|      0))
|    THEN
|    (AllHyps (λh.(HypSubst h 0) THEN (Complete Auto)))
|
|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. s : traversal(the_graph)
| 4. i : Vertices(the_graph)
| 5. df-traversal(the_graph;s)
| 6. ∀j:Vertices(the_graph).
| |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
| 7. s1 : traversal(the_graph)
| 8. s2 : traversal(the_graph)
| 9. s3 : traversal(the_graph)
| 10. i1 : Vertices(the_graph)
| 11. j : Vertices(the_graph)
| 12. i1-the_graph->j
| 13. paren(Vertices(the_graph);s2)
| 14. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| 15. paren(Vertices(the_graph);s3)
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| 16. df-traversal(the_graph;s2 @ s1)
| |   ⇒
| |   (∀j:Vertices(the_graph).
| |    (inr(j) ∈ s2 @ s1) ⇒ ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i1)
| |   ⇒
| |   df-traversal(the_graph;s3 @ s2 @ s1)
| 17. df-traversal(the_graph;s1)
| 18. ∀j:Vertices(the_graph).
| |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
| 19. (∀j@0:Vertices(the_graph).
| |    (inr(j@0) ∈ s1) ⇒ ¬(inl(j@0) ∈ s1) ⇒ j@0-the_graph->*j)
| |   ⇒
| |   df-traversal(the_graph;s2 @ s1)
| ⊢  df-traversal(the_graph;s3 @ s2 @ s1)  by Analyze -1
| |
| |\
| | 1. the_graph : Graph
| | 2. GraphObject(the_graph)
| | 3. s : traversal(the_graph)
| | 4. i : Vertices(the_graph)
| | 5. df-traversal(the_graph;s)
| | 6. ∀j:Vertices(the_graph).
| |    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
| | 7. s1 : traversal(the_graph)
| | 8. s2 : traversal(the_graph)
| | 9. s3 : traversal(the_graph)
| | 10. i1 : Vertices(the_graph)
| | 11. j : Vertices(the_graph)
| | 12. i1-the_graph->j
| | 13. paren(Vertices(the_graph);s2)
| | 14. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| | 15. paren(Vertices(the_graph);s3)
| | 16. df-traversal(the_graph;s2 @ s1)
| |     ⇒
| |     (∀j:Vertices(the_graph).
| |      (inr(j) ∈ s2 @ s1)
| |      ⇒
| |      ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i1)
| |     ⇒
| |     df-traversal(the_graph;s3 @ s2 @ s1)
| | 17. df-traversal(the_graph;s1)
| | 18. ∀j:Vertices(the_graph).
| |     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
| | 19. j@0 : Vertices(the_graph)
| | 20. (inr(j@0) ∈ s1)
| | 21. ¬(inl(j@0) ∈ s1)
| | ⊢  j@0-the_graph->*j  by (InstHyp [j@0] -4) THEN Connect
| |
| .\
| . 1. the_graph : Graph
| . 2. GraphObject(the_graph)
| . 3. s : traversal(the_graph)
| . 4. i : Vertices(the_graph)
| . 5. df-traversal(the_graph;s)
| . 6. ∀j:Vertices(the_graph).
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| . |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
| . 7. s1 : traversal(the_graph)
| . 8. s2 : traversal(the_graph)
| . 9. s3 : traversal(the_graph)
| . 10. i1 : Vertices(the_graph)
| . 11. j : Vertices(the_graph)
| . 12. i1-the_graph->j
| . 13. paren(Vertices(the_graph);s2)
| . 14. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| . 15. paren(Vertices(the_graph);s3)
| . 16. df-traversal(the_graph;s2 @ s1)
| . |   ⇒
| . |   (∀j:Vertices(the_graph).
| . |    (inr(j) ∈ s2 @ s1)
| . |    ⇒
| . |    ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i1)
| . |   ⇒
| . |   df-traversal(the_graph;s3 @ s2 @ s1)
| . 17. df-traversal(the_graph;s1)
| . 18. ∀j:Vertices(the_graph).
| . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
| . 19. df-traversal(the_graph;s2 @ s1)
| . ⊢  df-traversal(the_graph;s3 @ s2 @ s1)  by BackThruSomeHyp
| . |
| . 1. the_graph : Graph
| . 2. GraphObject(the_graph)
| . 3. s : traversal(the_graph)
| . 4. i : Vertices(the_graph)
| . 5. df-traversal(the_graph;s)
| . 6. ∀j:Vertices(the_graph).
| . |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
| . 7. s1 : traversal(the_graph)
| . 8. s2 : traversal(the_graph)
| . 9. s3 : traversal(the_graph)
| . 10. i1 : Vertices(the_graph)
| . 11. j : Vertices(the_graph)
| . 12. i1-the_graph->j
| . 13. paren(Vertices(the_graph);s2)
| . 14. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| . 15. paren(Vertices(the_graph);s3)
| . 16. df-traversal(the_graph;s2 @ s1)
| . |   ⇒
| . |   (∀j:Vertices(the_graph).
| . |    (inr(j) ∈ s2 @ s1)
| . |    ⇒
| . |    ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i1)
| . |   ⇒
| . |   df-traversal(the_graph;s3 @ s2 @ s1)
| . 17. df-traversal(the_graph;s1)
| . 18. ∀j:Vertices(the_graph).
| . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
| . 19. df-traversal(the_graph;s2 @ s1)
| . 20. j1 : Vertices(the_graph)
| . 21. (inr(j1) ∈ s2 @ s1)
| . 22. ¬(inl(j1) ∈ s2 @ s1)
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| . ⊢  j1-the_graph->*i1
| . | by (RWO
| . |     Thm* ∀x:T, l1,l2:T List.
| . |          (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| . |     {member_append}
| . |     -2)
| . |    THEN
| . |    SplitOrHyps
| . |
| . |\
| . | 1. the_graph : Graph
| . | 2. GraphObject(the_graph)
| . | 3. s : traversal(the_graph)
| . | 4. i : Vertices(the_graph)
| . | 5. df-traversal(the_graph;s)
| . | 6. ∀j:Vertices(the_graph).
| . | |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
| . | 7. s1 : traversal(the_graph)
| . | 8. s2 : traversal(the_graph)
| . | 9. s3 : traversal(the_graph)
| . | 10. i1 : Vertices(the_graph)
| . | 11. j : Vertices(the_graph)
| . | 12. i1-the_graph->j
| . | 13. paren(Vertices(the_graph);s2)
| . | 14. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| . | 15. paren(Vertices(the_graph);s3)
| . | 16. df-traversal(the_graph;s2 @ s1)
| . | |   ⇒
| . | |   (∀j:Vertices(the_graph).
| . | |    (inr(j) ∈ s2 @ s1)
| . | |    ⇒
| . | |    ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i1)
| . | |   ⇒
| . | |   df-traversal(the_graph;s3 @ s2 @ s1)
| . | 17. df-traversal(the_graph;s1)
| . | 18. ∀j:Vertices(the_graph).
| . | |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
| . | 19. df-traversal(the_graph;s2 @ s1)
| . | 20. j1 : Vertices(the_graph)
| . | 21. (inr(j1) ∈ s2)
| . | 22. ¬(inl(j1) ∈ s2 @ s1)
| . | ⊢  j1-the_graph->*i1
| . | | by (Analyze -1)
| . | |    THEN
| . | |    (RWO
| . | |     Thm* ∀x:T, l1,l2:T List.
| . | |          (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| . | |     {member_append}
| . | |     0)
| . | |    THEN
| . | |    OrLeft
| . | |
| . | 1. the_graph : Graph
| . | 2. GraphObject(the_graph)
| . | 3. s : traversal(the_graph)
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| . | 4. i : Vertices(the_graph)
| . | 5. df-traversal(the_graph;s)
| . | 6. ∀j:Vertices(the_graph).
| . |    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
| . | 7. s1 : traversal(the_graph)
| . | 8. s2 : traversal(the_graph)
| . | 9. s3 : traversal(the_graph)
| . | 10. i1 : Vertices(the_graph)
| . | 11. j : Vertices(the_graph)
| . | 12. i1-the_graph->j
| . | 13. paren(Vertices(the_graph);s2)
| . | 14. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| . | 15. paren(Vertices(the_graph);s3)
| . | 16. df-traversal(the_graph;s2 @ s1)
| . |     ⇒
| . |     (∀j:Vertices(the_graph).
| . |      (inr(j) ∈ s2 @ s1)
| . |      ⇒
| . |      ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i1)
| . |     ⇒
| . |     df-traversal(the_graph;s3 @ s2 @ s1)
| . | 17. df-traversal(the_graph;s1)
| . | 18. ∀j:Vertices(the_graph).
| . |     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
| . | 19. df-traversal(the_graph;s2 @ s1)
| . | 20. j1 : Vertices(the_graph)
| . | 21. (inr(j1) ∈ s2)
| . | ⊢  (inl(j1) ∈ s2)
| . |   by BackThru
| . |      Thm* ∀s':(T+T) List.
| . |           paren(T;s') ⇒ (∀i:T. (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
| . |      {paren_balance2}
| . |
| . .\
| . . 1. the_graph : Graph
| . . 2. GraphObject(the_graph)
| . . 3. s : traversal(the_graph)
| . . 4. i : Vertices(the_graph)
| . . 5. df-traversal(the_graph;s)
| . . 6. ∀j:Vertices(the_graph).
| . . |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
| . . 7. s1 : traversal(the_graph)
| . . 8. s2 : traversal(the_graph)
| . . 9. s3 : traversal(the_graph)
| . . 10. i1 : Vertices(the_graph)
| . . 11. j : Vertices(the_graph)
| . . 12. i1-the_graph->j
| . . 13. paren(Vertices(the_graph);s2)
| . . 14. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| . . 15. paren(Vertices(the_graph);s3)
| . . 16. df-traversal(the_graph;s2 @ s1)
| . . |   ⇒
| . . |   (∀j:Vertices(the_graph).
| . . |    (inr(j) ∈ s2 @ s1)
| . . |    ⇒
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| . . |    ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i1)
| . . |   ⇒
| . . |   df-traversal(the_graph;s3 @ s2 @ s1)
| . . 17. df-traversal(the_graph;s1)
| . . 18. ∀j:Vertices(the_graph).
| . . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
| . . 19. df-traversal(the_graph;s2 @ s1)
| . . 20. j1 : Vertices(the_graph)
| . . 21. (inr(j1) ∈ s1)
| . . 22. ¬(inl(j1) ∈ s2 @ s1)
| . . ⊢  j1-the_graph->*i1  by BackThruSomeHyp
| . . |
| . . 1. the_graph : Graph
| . . 2. GraphObject(the_graph)
| . . 3. s : traversal(the_graph)
| . . 4. i : Vertices(the_graph)
| . . 5. df-traversal(the_graph;s)
| . . 6. ∀j:Vertices(the_graph).
| . .    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
| . . 7. s1 : traversal(the_graph)
| . . 8. s2 : traversal(the_graph)
| . . 9. s3 : traversal(the_graph)
| . . 10. i1 : Vertices(the_graph)
| . . 11. j : Vertices(the_graph)
| . . 12. i1-the_graph->j
| . . 13. paren(Vertices(the_graph);s2)
| . . 14. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| . . 15. paren(Vertices(the_graph);s3)
| . . 16. df-traversal(the_graph;s2 @ s1)
| . .     ⇒
| . .     (∀j:Vertices(the_graph).
| . .      (inr(j) ∈ s2 @ s1)
| . .      ⇒
| . .      ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i1)
| . .     ⇒
| . .     df-traversal(the_graph;s3 @ s2 @ s1)
| . . 17. df-traversal(the_graph;s1)
| . . 18. ∀j:Vertices(the_graph).
| . .     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
| . . 19. df-traversal(the_graph;s2 @ s1)
| . . 20. j1 : Vertices(the_graph)
| . . 21. (inr(j1) ∈ s1)
| . . 22. ¬(inl(j1) ∈ s2 @ s1)
| . . ⊢  ¬(inl(j1) ∈ s1)  by ObviousFrom [-1]
|
.\
. 1. the_graph : Graph
. 2. GraphObject(the_graph)
. 3. s : traversal(the_graph)
. 4. i : Vertices(the_graph)
. 5. df-traversal(the_graph;s)
. 6. ∀j:Vertices(the_graph).
. |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
. 7. s1 : traversal(the_graph)
. 8. s2 : traversal(the_graph)
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. 9. i1 : Vertices(the_graph)
. 10. ¬(inr(i1) ∈ s1)
. 11. ¬(inl(i1) ∈ s1)
. 12. paren(Vertices(the_graph);s2)
. 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. 15. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. 16. ∀j:Vertices(the_graph).
. |   i1-the_graph->j
. |   ⇒
. |   ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. 17. df-traversal(the_graph;[inr(i1) / s1])
. |   ⇒
. |   (∀j:Vertices(the_graph).
. |    (inr(j) ∈ [inr(i1) / s1])
. |    ⇒
. |    ¬(inl(j) ∈ [inr(i1) / s1]) ⇒ j-the_graph->*i1)
. |   ⇒
. |   df-traversal(the_graph;s2 @ [inr(i1) / s1])
. 18. df-traversal(the_graph;s1)
. 19. ∀j:Vertices(the_graph).
. |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. ⊢  df-traversal(the_graph;[inl(i1) / (s2 @ [inr(i1)] @ s1)])
. | by Analyze -3
. |
. |\
. | .....antecedent .....
. | 1. the_graph : Graph
. | 2. GraphObject(the_graph)
. | 3. s : traversal(the_graph)
. | 4. i : Vertices(the_graph)
. | 5. df-traversal(the_graph;s)
. | 6. ∀j:Vertices(the_graph).
. |    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
. | 7. s1 : traversal(the_graph)
. | 8. s2 : traversal(the_graph)
. | 9. i1 : Vertices(the_graph)
. | 10. ¬(inr(i1) ∈ s1)
. | 11. ¬(inl(i1) ∈ s1)
. | 12. paren(Vertices(the_graph);s2)
. | 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. | 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. | 15. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. | 16. ∀j:Vertices(the_graph).
. |     i1-the_graph->j
. |     ⇒
. |     ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. | 17. df-traversal(the_graph;s1)
. | 18. ∀j:Vertices(the_graph).
. |     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. | ⊢  df-traversal(the_graph;[inr(i1) / s1])
. |   by BackThru
. |      Thm* For any graph
. |           ∀i:V, s:Traversal.
. |           (∀j:V. (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i)
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. |           ⇒
. |           df-traversal(the_graph;s)
. |           ⇒
. |           df-traversal(the_graph;[inr(i) / s])
. |      {df_DASH_traversal_DASH_cons2}
. |
. .\
. . 1. the_graph : Graph
. . 2. GraphObject(the_graph)
. . 3. s : traversal(the_graph)
. . 4. i : Vertices(the_graph)
. . 5. df-traversal(the_graph;s)
. . 6. ∀j:Vertices(the_graph).
. . |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
. . 7. s1 : traversal(the_graph)
. . 8. s2 : traversal(the_graph)
. . 9. i1 : Vertices(the_graph)
. . 10. ¬(inr(i1) ∈ s1)
. . 11. ¬(inl(i1) ∈ s1)
. . 12. paren(Vertices(the_graph);s2)
. . 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . 15. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . 16. ∀j:Vertices(the_graph).
. . |   i1-the_graph->j
. . |   ⇒
. . |   ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . 17. df-traversal(the_graph;s1)
. . 18. ∀j:Vertices(the_graph).
. . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . 19. (∀j:Vertices(the_graph).
. . |    (inr(j) ∈ [inr(i1) / s1])
. . |    ⇒
. . |    ¬(inl(j) ∈ [inr(i1) / s1]) ⇒ j-the_graph->*i1)
. . |   ⇒
. . |   df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . ⊢  df-traversal(the_graph;[inl(i1) / (s2 @ [inr(i1)] @ s1)])
. . | by Analyze -1
. . |
. . |\
. . | 1. the_graph : Graph
. . | 2. GraphObject(the_graph)
. . | 3. s : traversal(the_graph)
. . | 4. i : Vertices(the_graph)
. . | 5. df-traversal(the_graph;s)
. . | 6. ∀j:Vertices(the_graph).
. . | |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
. . | 7. s1 : traversal(the_graph)
. . | 8. s2 : traversal(the_graph)
. . | 9. i1 : Vertices(the_graph)
. . | 10. ¬(inr(i1) ∈ s1)
. . | 11. ¬(inl(i1) ∈ s1)
. . | 12. paren(Vertices(the_graph);s2)
. . | 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . | 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
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. . | 15. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . | 16. ∀j:Vertices(the_graph).
. . | |   i1-the_graph->j
. . | |   ⇒
. . | |   ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . | 17. df-traversal(the_graph;s1)
. . | 18. ∀j:Vertices(the_graph).
. . | |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . | 19. j : Vertices(the_graph)
. . | 20. (inr(j) ∈ [inr(i1) / s1])
. . | 21. ¬(inl(j) ∈ [inr(i1) / s1])
. . | ⊢  j-the_graph->*i1
. . | | by (RWO
. . | |     Thm* ∀l:T List, a,x:T. (x ∈ [a / l]) ⟺ x = a ∨ (x ∈ l)
. . | |     {cons_member}
. . | |     -2)
. . | |    THEN
. . | |    SplitOrHyps
. . | |
. . | |\
. . | | 1. the_graph : Graph
. . | | 2. GraphObject(the_graph)
. . | | 3. s : traversal(the_graph)
. . | | 4. i : Vertices(the_graph)
. . | | 5. df-traversal(the_graph;s)
. . | | 6. ∀j:Vertices(the_graph).
. . | |    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
. . | | 7. s1 : traversal(the_graph)
. . | | 8. s2 : traversal(the_graph)
. . | | 9. i1 : Vertices(the_graph)
. . | | 10. ¬(inr(i1) ∈ s1)
. . | | 11. ¬(inl(i1) ∈ s1)
. . | | 12. paren(Vertices(the_graph);s2)
. . | | 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . | | 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . | | 15. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . | | 16. ∀j:Vertices(the_graph).
. . | |     i1-the_graph->j
. . | |     ⇒
. . | |     ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . | | 17. df-traversal(the_graph;s1)
. . | | 18. ∀j:Vertices(the_graph).
. . | |     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . | | 19. j : Vertices(the_graph)
. . | | 20. inr(j) = inr(i1)
. . | | 21. ¬(inl(j) ∈ [inr(i1) / s1])
. . | | ⊢  j-the_graph->*i1
. . | |   by (Subst (j = i1) 0) THEN (Try Connect)
. . | |      THEN
. . | |      (ObviousFrom [-2])
. . | |
. . | .\
. . | . 1. the_graph : Graph
. . | . 2. GraphObject(the_graph)
. . | . 3. s : traversal(the_graph)
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. . | . 4. i : Vertices(the_graph)
. . | . 5. df-traversal(the_graph;s)
. . | . 6. ∀j:Vertices(the_graph).
. . | . |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
. . | . 7. s1 : traversal(the_graph)
. . | . 8. s2 : traversal(the_graph)
. . | . 9. i1 : Vertices(the_graph)
. . | . 10. ¬(inr(i1) ∈ s1)
. . | . 11. ¬(inl(i1) ∈ s1)
. . | . 12. paren(Vertices(the_graph);s2)
. . | . 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . | . 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . | . 15. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . | . 16. ∀j:Vertices(the_graph).
. . | . |   i1-the_graph->j
. . | . |   ⇒
. . | . |   ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . | . 17. df-traversal(the_graph;s1)
. . | . 18. ∀j:Vertices(the_graph).
. . | . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . | . 19. j : Vertices(the_graph)
. . | . 20. (inr(j) ∈ s1)
. . | . 21. ¬(inl(j) ∈ [inr(i1) / s1])
. . | . ⊢  j-the_graph->*i1  by BackThruSomeHyp
. . | . |
. . | . 1. the_graph : Graph
. . | . 2. GraphObject(the_graph)
. . | . 3. s : traversal(the_graph)
. . | . 4. i : Vertices(the_graph)
. . | . 5. df-traversal(the_graph;s)
. . | . 6. ∀j:Vertices(the_graph).
. . | .    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
. . | . 7. s1 : traversal(the_graph)
. . | . 8. s2 : traversal(the_graph)
. . | . 9. i1 : Vertices(the_graph)
. . | . 10. ¬(inr(i1) ∈ s1)
. . | . 11. ¬(inl(i1) ∈ s1)
. . | . 12. paren(Vertices(the_graph);s2)
. . | . 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . | . 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . | . 15. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . | . 16. ∀j:Vertices(the_graph).
. . | .     i1-the_graph->j
. . | .     ⇒
. . | .     ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . | . 17. df-traversal(the_graph;s1)
. . | . 18. ∀j:Vertices(the_graph).
. . | .     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . | . 19. j : Vertices(the_graph)
. . | . 20. (inr(j) ∈ s1)
. . | . 21. ¬(inl(j) ∈ [inr(i1) / s1])
. . | . ⊢  ¬(inl(j) ∈ s1)  by ObviousFrom [-1]
. . |
. . .\
. . . 1. the_graph : Graph
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. . . 2. GraphObject(the_graph)
. . . 3. s : traversal(the_graph)
. . . 4. i : Vertices(the_graph)
. . . 5. df-traversal(the_graph;s)
. . . 6. ∀j:Vertices(the_graph).
. . . |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i
. . . 7. s1 : traversal(the_graph)
. . . 8. s2 : traversal(the_graph)
. . . 9. i1 : Vertices(the_graph)
. . . 10. ¬(inr(i1) ∈ s1)
. . . 11. ¬(inl(i1) ∈ s1)
. . . 12. paren(Vertices(the_graph);s2)
. . . 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . . 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . 15. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . 16. ∀j:Vertices(the_graph).
. . . |   i1-the_graph->j
. . . |   ⇒
. . . |   ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . 17. df-traversal(the_graph;s1)
. . . 18. ∀j:Vertices(the_graph).
. . . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . 19. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . ⊢  df-traversal(the_graph;[inl(i1) / (s2 @ [inr(i1)] @ s1)])
. . . | by (Thin 6) THEN (Thin 5) THEN (Thin 4) THEN (Thin 3)
. . . |
. . . 1. the_graph : Graph
. . . 2. GraphObject(the_graph)
. . . 3. s1 : traversal(the_graph)
. . . 4. s2 : traversal(the_graph)
. . . 5. i1 : Vertices(the_graph)
. . . 6. ¬(inr(i1) ∈ s1)
. . . 7. ¬(inl(i1) ∈ s1)
. . . 8. paren(Vertices(the_graph);s2)
. . . 9. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . . 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . 11. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . 12. ∀j:Vertices(the_graph).
. . . |   i1-the_graph->j
. . . |   ⇒
. . . |   ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . 13. df-traversal(the_graph;s1)
. . . 14. ∀j:Vertices(the_graph).
. . . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . ⊢  df-traversal(the_graph;[inl(i1) / (s2 @ [inr(i1)] @ s1)])
. . . | by Unfold ‘df-traversal‘ 0
. . . |
. . . |\
. . . | 1. the_graph : Graph
. . . | 2. GraphObject(the_graph)
. . . | 3. s1 : traversal(the_graph)
. . . | 4. s2 : traversal(the_graph)
. . . | 5. i1 : Vertices(the_graph)
. . . | 6. ¬(inr(i1) ∈ s1)
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. . . | 7. ¬(inl(i1) ∈ s1)
. . . | 8. paren(Vertices(the_graph);s2)
. . . | 9. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . . | 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . | 11. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . | 12. ∀j:Vertices(the_graph).
. . . | |   i1-the_graph->j
. . . | |   ⇒
. . . | |   ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . | 13. df-traversal(the_graph;s1)
. . . | 14. ∀j:Vertices(the_graph).
. . . | |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . | 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . | 16. i : Vertices(the_graph)
. . . | 17. s1@0 : traversal(the_graph)
. . . | 18. s2@0 : traversal(the_graph)
. . . | 19. [inl(i1) / (s2 @ [inr(i1)] @ s1)]
. . . | |   =
. . . | |   (s1@0 @ [inr(i)] @ s2@0)
. . . | 20. j : Vertices(the_graph)
. . . | 21. (inr(j) ∈ s2@0)
. . . | 22. ¬(inl(j) ∈ s2@0)
. . . | ⊢  j-the_graph->*i
. . . | | by (Analyze -6) THEN (All Reduce) THEN (SplitCons -4)
. . . | |
. . . | |\
. . . | | 1. the_graph : Graph
. . . | | 2. GraphObject(the_graph)
. . . | | 3. s1 : traversal(the_graph)
. . . | | 4. s2 : traversal(the_graph)
. . . | | 5. i1 : Vertices(the_graph)
. . . | | 6. ¬(inr(i1) ∈ s1)
. . . | | 7. ¬(inl(i1) ∈ s1)
. . . | | 8. paren(Vertices(the_graph);s2)
. . . | | 9. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . . | | 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . | | 11. ∀j:Vertices(the_graph).
. . . | |     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . | | 12. ∀j:Vertices(the_graph).
. . . | |     i1-the_graph->j
. . . | |     ⇒
. . . | |     ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . | | 13. df-traversal(the_graph;s1)
. . . | | 14. ∀j:Vertices(the_graph).
. . . | |     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . | | 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . | | 16. i : Vertices(the_graph)
. . . | | 17. s2@0 : traversal(the_graph)
. . . | | 18. j : Vertices(the_graph)
. . . | | 19. (inr(j) ∈ s2@0)
. . . | | 20. ¬(inl(j) ∈ s2@0)
. . . | | 21. inl(i1) = inr(i)
. . . | | 22. (s2 @ [inr(i1) / s1]) = s2@0
. . . | | ⊢  j-the_graph->*i  by ObviousContradiction [-2]
. . . | |
```

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. . . | .\
. . . | . 1. the_graph : Graph
. . . | . 2. GraphObject(the_graph)
. . . | . 3. s1 : traversal(the_graph)
. . . | . 4. s2 : traversal(the_graph)
. . . | . 5. i1 : Vertices(the_graph)
. . . | . 6. ¬(inr(i1) ∈ s1)
. . . | . 7. ¬(inl(i1) ∈ s1)
. . . | . 8. paren(Vertices(the_graph);s2)
. . . | . 9. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . . | . 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . | . 11. ∀j:Vertices(the_graph).
. . . | .     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . | . 12. ∀j:Vertices(the_graph).
. . . | .     i1-the_graph->j
. . . | .     ⇒
. . . | .     ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . | . 13. df-traversal(the_graph;s1)
. . . | . 14. ∀j:Vertices(the_graph).
. . . | .     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . | . 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . | . 16. i : Vertices(the_graph)
. . . | . 17. u : Vertices(the_graph)+Vertices(the_graph)
. . . | . 18. v : (Vertices(the_graph)+Vertices(the_graph)) List
. . . | . 19. s2@0 : traversal(the_graph)
. . . | . 20. j : Vertices(the_graph)
. . . | . 21. (inr(j) ∈ s2@0)
. . . | . 22. ¬(inl(j) ∈ s2@0)
. . . | . 23. inl(i1) = u
. . . | . 24. (s2 @ [inr(i1) / s1]) = (v @ [inr(i) / s2@0])
. . . | . ⊢  j-the_graph->*i
. . . | .   by AllHyps
. . . | .      (λh.
. . . | .       (Unfold `df-traversal` h) THEN (Analyze h)
. . . | .       THEN
. . . | .       (InstHyp [i;v;s2@0;j] h)
. . . | .       THEN
. . . | .       (Reduce 0))
. . . |
. . . .\
. . . . 1. the_graph : Graph
. . . . 2. GraphObject(the_graph)
. . . . 3. s1 : traversal(the_graph)
. . . . 4. s2 : traversal(the_graph)
. . . . 5. i1 : Vertices(the_graph)
. . . . 6. ¬(inr(i1) ∈ s1)
. . . . 7. ¬(inl(i1) ∈ s1)
. . . . 8. paren(Vertices(the_graph);s2)
. . . . 9. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . . . 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . . 11. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . 12. ∀j:Vertices(the_graph).
. . . . |   i1-the_graph->j
. . . . |   ⇒
. . . . |   ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
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. . . . 13. df-traversal(the_graph;s1)
. . . . 14. ∀j:Vertices(the_graph).
. . . . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . 16. i : Vertices(the_graph)
. . . . 17. s1@0 : traversal(the_graph)
. . . . 18. s2@0 : traversal(the_graph)
. . . . 19. ∀j:Vertices(the_graph).
. . . . |   i-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . 20. [inl(i1) / (s2 @ [inr(i1)] @ s1)]
. . . . |   =
. . . . |   (s1@0 @ [inl(i)] @ s2@0)
. . . . 21. j : Vertices(the_graph)
. . . . 22. i-the_graph->*j
. . . . ⊢  (inr(j) ∈ s2@0)
. . . . | by (Analyze -6) THEN (All Reduce) THEN (SplitCons -3)
. . . . |
. . . . |\
. . . . | 1. the_graph : Graph
. . . . | 2. GraphObject(the_graph)
. . . . | 3. s1 : traversal(the_graph)
. . . . | 4. s2 : traversal(the_graph)
. . . . | 5. i1 : Vertices(the_graph)
. . . . | 6. ¬(inr(i1) ∈ s1)
. . . . | 7. ¬(inl(i1) ∈ s1)
. . . . | 8. paren(Vertices(the_graph);s2)
. . . . | 9. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . . . | 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . . | 11. ∀j:Vertices(the_graph).
. . . . | |   (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | 12. ∀j:Vertices(the_graph).
. . . . | |   i1-the_graph->j
. . . . | |   ⇒
. . . . | |   ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | 13. df-traversal(the_graph;s1)
. . . . | 14. ∀j:Vertices(the_graph).
. . . . | |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . | 16. i : Vertices(the_graph)
. . . . | 17. s2@0 : traversal(the_graph)
. . . . | 18. ∀j:Vertices(the_graph).
. . . . | |   i-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | 19. j : Vertices(the_graph)
. . . . | 20. i-the_graph->*j
. . . . | 21. inl(i1) = inl(i)
. . . . | 22. (s2 @ [inr(i1) / s1]) = s2@0
. . . . | ⊢  (inr(j) ∈ s2@0)
. . . . | | by (RevHypSubst -1 0) THEN (Thin -1) THEN (Thin -5)
. . . . | |    THEN
. . . . | |    (AssertBY (i = i1) (ObviousFrom [-1]))
. . . . | |    THEN
. . . . | |    (Thin -2)
. . . . | |    THEN
. . . . | |    (HypSubst -1 -2)
. . . . | |    THEN
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. . . . | |    (HypSubst -1 -4)
. . . . | |    THEN
. . . . | |    (Thin -1)
. . . . | |    THEN
. . . . | |    (Thin -4)
. . . . | |
. . . . | 1. the_graph : Graph
. . . . | 2. GraphObject(the_graph)
. . . . | 3. s1 : traversal(the_graph)
. . . . | 4. s2 : traversal(the_graph)
. . . . | 5. i1 : Vertices(the_graph)
. . . . | 6. ¬(inr(i1) ∈ s1)
. . . . | 7. ¬(inl(i1) ∈ s1)
. . . . | 8. paren(Vertices(the_graph);s2)
. . . . | 9. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . . . | 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . . | 11. ∀j:Vertices(the_graph).
. . . . | |   (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | 12. ∀j:Vertices(the_graph).
. . . . | |   i1-the_graph->j
. . . . | |   ⇒
. . . . | |   ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | 13. df-traversal(the_graph;s1)
. . . . | 14. ∀j:Vertices(the_graph).
. . . . | |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . | 16. ∀j:Vertices(the_graph).
. . . . | |   i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | 17. j : Vertices(the_graph)
. . . . | 18. i1-the_graph->*j
. . . . | ⊢  (inr(j) ∈ s2 @ [inr(i1) / s1])
. . . . | | by (RWO

. . . . | |     Thm* For any graph
. . . . | |          (∀x,y:V. Dec(x = y))
. . . . | |          ⇒
. . . . | |          (∀x,y:V.
. . . . | |           x-the_graph->*y
. . . . | |           ⟺
. . . . | |           x = y
. . . . | |           ∨ (∃z:V.
. . . . | |              ¬z = x & x-the_graph->z & z-the_graph->*y))
. . . . | |     {connect_DASH_iff}
. . . . | |     -1)
. . . . | |    THEN
. . . . | |    SplitOrHyps
. . . . | |
. . . . | |\
. . . . | | .....decidable? .....
. . . . | | 1. the_graph : Graph
. . . . | | 2. GraphObject(the_graph)
. . . . | | 3. s1 : traversal(the_graph)
. . . . | | 4. s2 : traversal(the_graph)
. . . . | | 5. i1 : Vertices(the_graph)
. . . . | | 6. ¬(inr(i1) ∈ s1)
. . . . | | 7. ¬(inl(i1) ∈ s1)
```

```
. . . . | | 8. paren(Vertices(the_graph);s2)
. . . . | | 9. l_disjoint(Vertices(the_graph)+
. . . . | |    Vertices(the_graph);s2;s1)
. . . . | | 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . . | | 11. ∀j:Vertices(the_graph).
. . . . | |     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | | 12. ∀j:Vertices(the_graph).
. . . . | |     i1-the_graph->j
. . . . | |     ⇒
. . . . | |     ¬j = i1
. . . . | |     ⇒
. . . . | |     (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | | 13. df-traversal(the_graph;s1)
. . . . | | 14. ∀j:Vertices(the_graph).
. . . . | |     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | | 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . | | 16. ∀j:Vertices(the_graph).
. . . . | |     i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | | 17. j : Vertices(the_graph)
. . . . | | 18. i1-the_graph->*j
. . . . | | 19. x : Vertices(the_graph)
. . . . | | 20. y : Vertices(the_graph)
. . . . | | ⊢  Dec(x = y)
. . . . | |   by (Inst Thm: graphobj_DASH_properties
. . . . | |       [the_graph;the_obj])
. . . . | |      THEN
. . . . | |      (Analyze -1)
. . . . | |      THEN
. . . . | |      (RW (SweepUpC (RevHypC -2)) 0)
. . . . | |
. . . . | |\
. . . . | | 1. the_graph : Graph
. . . . | | 2. GraphObject(the_graph)
. . . . | | 3. s1 : traversal(the_graph)
. . . . | | 4. s2 : traversal(the_graph)
. . . . | | 5. i1 : Vertices(the_graph)
. . . . | | 6. ¬(inr(i1) ∈ s1)
. . . . | | 7. ¬(inl(i1) ∈ s1)
. . . . | | 8. paren(Vertices(the_graph);s2)
. . . . | | 9. l_disjoint(Vertices(the_graph)+
. . . . | |    Vertices(the_graph);s2;s1)
. . . . | | 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . . | | 11. ∀j:Vertices(the_graph).
. . . . | |     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | | 12. ∀j:Vertices(the_graph).
. . . . | |     i1-the_graph->j
. . . . | |     ⇒
. . . . | |     ¬j = i1
. . . . | |     ⇒
. . . . | |     (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | | 13. df-traversal(the_graph;s1)
. . . . | | 14. ∀j:Vertices(the_graph).
. . . . | |     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | | 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . | | 16. ∀j:Vertices(the_graph).
```

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. . . . | |     i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | | 17. j : Vertices(the_graph)
. . . . | | 18. i1 = j
. . . . | | ⊢  (inr(j) ∈ s2 @ [inr(i1) / s1])
. . . . | |   by (RevHypSubst -1 0) THEN ObviousConcl
. . . . | |
. . . . | .\
. . . . | . 1. the_graph : Graph
. . . . | . 2. GraphObject(the_graph)
. . . . | . 3. s1 : traversal(the_graph)
. . . . | . 4. s2 : traversal(the_graph)
. . . . | . 5. i1 : Vertices(the_graph)
. . . . | . 6. ¬(inr(i1) ∈ s1)
. . . . | . 7. ¬(inl(i1) ∈ s1)
. . . . | . 8. paren(Vertices(the_graph);s2)
. . . . | . 9. l_disjoint(Vertices(the_graph)+
. . . . | . |  Vertices(the_graph);s2;s1)
. . . . | . 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . . | . 11. ∀j:Vertices(the_graph).
. . . . | . |   (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . 12. ∀j:Vertices(the_graph).
. . . . | . |   i1-the_graph->j
. . . . | . |   ⇒
. . . . | . |   ¬j = i1
. . . . | . |   ⇒
. . . . | . |   (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . 13. df-traversal(the_graph;s1)
. . . . | . 14. ∀j:Vertices(the_graph).
. . . . | . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . | . 16. ∀j:Vertices(the_graph).
. . . . | . |   i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | . 17. j : Vertices(the_graph)
. . . . | . 18. ∃z:Vertices(the_graph).
. . . . | . |   ¬z = i1 & i1-the_graph->z & z-the_graph->*j
. . . . | . ⊢  (inr(j) ∈ s2 @ [inr(i1) / s1])
. . . . | . | by ExRepD THEN (InstHyp [z] 12)
. . . . | . |
. . . . | . 1. the_graph : Graph
. . . . | . 2. GraphObject(the_graph)
. . . . | . 3. s1 : traversal(the_graph)
. . . . | . 4. s2 : traversal(the_graph)
. . . . | . 5. i1 : Vertices(the_graph)
. . . . | . 6. ¬(inr(i1) ∈ s1)
. . . . | . 7. ¬(inl(i1) ∈ s1)
. . . . | . 8. paren(Vertices(the_graph);s2)
. . . . | . 9. l_disjoint(Vertices(the_graph)+
. . . . | . |  Vertices(the_graph);s2;s1)
. . . . | . 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . . | . 11. ∀j:Vertices(the_graph).
. . . . | . |   (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . 12. ∀j:Vertices(the_graph).
. . . . | . |   i1-the_graph->j
. . . . | . |   ⇒
. . . . | . |   ¬j = i1
```

```
. . . . | . |   ⇒
. . . . | . |   (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . 13. df-traversal(the_graph;s1)
. . . . | . 14. ∀j:Vertices(the_graph).
. . . . | . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . | . 16. ∀j:Vertices(the_graph).
. . . . | . |   i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | . 17. j : Vertices(the_graph)
. . . . | . 18. z : Vertices(the_graph)
. . . . | . 19. ¬z = i1
. . . . | . 20. i1-the_graph->z
. . . . | . 21. z-the_graph->*j
. . . . | . 22. (inl(z) ∈ s2) ∨ (inl(z) ∈ s1) ∨ (inr(z) ∈ s1)
. . . . | . ⊢  (inr(j) ∈ s2 @ [inr(i1) / s1])  by Analyze -1
. . . . | . |
. . . . | . |\
. . . . | . | 1. the_graph : Graph
. . . . | . | 2. GraphObject(the_graph)
. . . . | . | 3. s1 : traversal(the_graph)
. . . . | . | 4. s2 : traversal(the_graph)
. . . . | . | 5. i1 : Vertices(the_graph)
. . . . | . | 6. ¬(inr(i1) ∈ s1)
. . . . | . | 7. ¬(inl(i1) ∈ s1)
. . . . | . | 8. paren(Vertices(the_graph);s2)
. . . . | . | 9. l_disjoint(Vertices(the_graph)+
. . . . | . | |  Vertices(the_graph);s2;s1)
. . . . | . | 10. no_repeats(Vertices(the_graph)+
. . . . | . | |   Vertices(the_graph);s2)
. . . . | . | 11. ∀j:Vertices(the_graph).
. . . . | . | |   (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . | 12. ∀j:Vertices(the_graph).
. . . . | . | |   i1-the_graph->j
. . . . | . | |   ⇒
. . . . | . | |   ¬j = i1
. . . . | . | |   ⇒
. . . . | . | |   (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . | 13. df-traversal(the_graph;s1)
. . . . | . | 14. ∀j:Vertices(the_graph).
. . . . | . | |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . | 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . | . | 16. ∀j:Vertices(the_graph).
. . . . | . | |   i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | . | 17. j : Vertices(the_graph)
. . . . | . | 18. z : Vertices(the_graph)
. . . . | . | 19. ¬z = i1
. . . . | . | 20. i1-the_graph->z
. . . . | . | 21. z-the_graph->*j
. . . . | . | 22. (inl(z) ∈ s2)
. . . . | . | ⊢  (inr(j) ∈ s2 @ [inr(i1) / s1])
. . . . | . | | by (Inst
. . . . | . | |     Thm* ∀s:T List, z:T.
. . . . | . | |          (z ∈ s)
. . . . | . | |          ⇒
. . . . | . | |          (∃s1,s2:T List. s = (s1 @ [z / s2]))
```

```
. . . . | . | |     {l_member_decomp}
. . . . | . | |     [Vertices(the_graph)+Vertices(the_graph)
. . . . | . | |     ;s2
. . . . | . | |     ;inl(z)])
. . . . | . | |    THEN
. . . . | . | |    ExRepD
. . . . | . | |    THEN
. . . . | . | |    (let h ≡ 15 in
. . . . | . | |     (Unfold `df-traversal` h) THEN (Analyze h)
. . . . | . | |     THEN
. . . . | . | |     (InstHyp [z;s3;s2@0 @ [inr(i1) / s1];j] h+1))
. . . . | . | |
. . . . | . | |\
. . . . | . | | 1. the_graph : Graph
. . . . | . | | 2. GraphObject(the_graph)
. . . . | . | | 3. s1 : traversal(the_graph)
. . . . | . | | 4. s2 : traversal(the_graph)
. . . . | . | | 5. i1 : Vertices(the_graph)
. . . . | . | | 6. ¬(inr(i1) ∈ s1)
. . . . | . | | 7. ¬(inl(i1) ∈ s1)
. . . . | . | | 8. paren(Vertices(the_graph);s2)
. . . . | . | | 9. l_disjoint(Vertices(the_graph)+
. . . . | . | |    Vertices(the_graph);s2;s1)
. . . . | . | | 10. no_repeats(Vertices(the_graph)+
. . . . | . | |     Vertices(the_graph);s2)
. . . . | . | | 11. ∀j:Vertices(the_graph).
. . . . | . | |     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . | | 12. ∀j:Vertices(the_graph).
. . . . | . | |     i1-the_graph->j
. . . . | . | |     ⇒
. . . . | . | |     ¬j = i1
. . . . | . | |     ⇒
. . . . | . | |     (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . | | 13. df-traversal(the_graph;s1)
. . . . | . | | 14. ∀j:Vertices(the_graph).
. . . . | . | |     (inr(j) ∈ s1)
. . . . | . | |     ⇒
. . . . | . | |     ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . | | 15. ∀i:Vertices(the_graph)
. . . . | . | |     , s1@0,s2@0:traversal(the_graph).
. . . . | . | |     (s2 @ [inr(i1) / s1]) = (s1@0 @ [inr(i)] @ s2@0)
. . . . | . | |     ⇒
. . . . | . | |     (∀j:Vertices(the_graph).
. . . . | . | |      (inr(j) ∈ s2@0)
. . . . | . | |      ⇒
. . . . | . | |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i)
. . . . | . | | 16. ∀i:Vertices(the_graph)
. . . . | . | |     , s1@0,s2@0:traversal(the_graph).
. . . . | . | |     ...
. . . . | . | |     ⇒
. . . . | . | |     (s2 @ [inr(i1) / s1]) = (s1@0 @ [inl(i)] @ s2@0)
. . . . | . | |     ⇒
. . . . | . | |     (∀j:Vertices(the_graph).
. . . . | . | |      i-the_graph->*j ⇒ (inr(j) ∈ s2@0))
. . . . | . | | 17. ∀j:Vertices(the_graph).
```

```
. . . . | . | |     i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | . | | 18. j : Vertices(the_graph)
. . . . | . | | 19. z : Vertices(the_graph)
. . . . | . | | 20. ¬z = i1
. . . . | . | | 21. i1-the_graph->z
. . . . | . | | 22. z-the_graph->*j
. . . . | . | | 23. (inl(z) ∈ s2)
. . . . | . | | 24. s3 :
. . . . | . | |     (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . | | 25. s2@0 :
. . . . | . | |     (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . | | 26. s2 = (s3 @ [inl(z) / s2@0])
. . . . | . | | 27. j1 : Vertices(the_graph)
. . . . | . | | 28. z-the_graph->*j1
. . . . | . | | ⊢  ¬non-trivial-loop(the_graph;j1)
. . . . | . | |   by BackThruSomeHyp THEN Connect
. . . . | . | |
. . . . | . | |\
. . . . | . | | .....antecedent .....
. . . . | . | | 1. the_graph : Graph
. . . . | . | | 2. GraphObject(the_graph)
. . . . | . | | 3. s1 : traversal(the_graph)
. . . . | . | | 4. s2 : traversal(the_graph)
. . . . | . | | 5. i1 : Vertices(the_graph)
. . . . | . | | 6. ¬(inr(i1) ∈ s1)
. . . . | . | | 7. ¬(inl(i1) ∈ s1)
. . . . | . | | 8. paren(Vertices(the_graph);s2)
. . . . | . | | 9. l_disjoint(Vertices(the_graph)+
. . . . | . | |    Vertices(the_graph);s2;s1)
. . . . | . | | 10. no_repeats(Vertices(the_graph)+
. . . . | . | |     Vertices(the_graph);s2)
. . . . | . | | 11. ∀j:Vertices(the_graph).
. . . . | . | |     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . | | 12. ∀j:Vertices(the_graph).
. . . . | . | |     i1-the_graph->j
. . . . | . | |     ⇒
. . . . | . | |     ¬j = i1
. . . . | . | |     ⇒
. . . . | . | |     (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . | | 13. df-traversal(the_graph;s1)
. . . . | . | | 14. ∀j:Vertices(the_graph).
. . . . | . | |     (inr(j) ∈ s1)
. . . . | . | |     ⇒
. . . . | . | |     ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . | | 15. ∀i:Vertices(the_graph)
. . . . | . | |     , s1@0,s2@0:traversal(the_graph).
. . . . | . | |     (s2 @ [inr(i1) / s1]) = (s1@0 @ [inr(i)] @ s2@0)
. . . . | . | |     ⇒
. . . . | . | |     (∀j:Vertices(the_graph).
. . . . | . | |      (inr(j) ∈ s2@0)
. . . . | . | |      ⇒
. . . . | . | |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i)
. . . . | . | | 16. ∀i:Vertices(the_graph)
. . . . | . | |     , s1@0,s2@0:traversal(the_graph).
. . . . | . | |     ...
```

```
. . . . | . | |     ⇒
. . . . | . | |     (s2 @ [inr(i1) / s1]) = (s1@0 @ [inl(i)] @ s2@0)

. . . . | . | |     ⇒
. . . . | . | |     (∀j:Vertices(the_graph).
. . . . | . | |      i-the_graph->*j ⇒ (inr(j) ∈ s2@0))
. . . . | . | 17. ∀j:Vertices(the_graph).
. . . . | . | |     i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | . | 18. j : Vertices(the_graph)
. . . . | . | 19. z : Vertices(the_graph)
. . . . | . | 20. ¬z = i1
. . . . | . | 21. i1-the_graph->z
. . . . | . | 22. z-the_graph->*j
. . . . | . | 23. (inl(z) ∈ s2)
. . . . | . | 24. s3 :
. . . . | . | |     (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . | 25. s2@0 :
. . . . | . | |     (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . | 26. s2 = (s3 @ [inl(z) / s2@0])
. . . . | . | ⊢  (s2 @ [inr(i1) / s1])
. . . . | . | |    =
. . . . | . | |    (s3 @ [inl(z)] @ s2@0 @ [inr(i1) / s1])
. . . . | . | |   by (HypSubst -1 0) THEN (Reduce 0)
. . . . | . | |      THEN
. . . . | . | |      (RWW
. . . . | . | |       Thm* ∀as,bs,cs:T List.
. . . . | . | |            ((as @ bs) @ cs) = (as @ (bs @ cs))
. . . . | . | |       {append_assoc}
. . . . | . | |       0)
. . . . | . | |      THEN
. . . . | . | |      (Reduce 0)
. . . . | . | |
. . . . | . | .\
. . . . | . | . 1. the_graph : Graph
. . . . | . | . 2. GraphObject(the_graph)
. . . . | . | . 3. s1 : traversal(the_graph)
. . . . | . | . 4. s2 : traversal(the_graph)
. . . . | . | . 5. i1 : Vertices(the_graph)
. . . . | . | . 6. ¬(inr(i1) ∈ s1)
. . . . | . | . 7. ¬(inl(i1) ∈ s1)
. . . . | . | . 8. paren(Vertices(the_graph);s2)
. . . . | . | . 9. l_disjoint(Vertices(the_graph)+
. . . . | . | . |  Vertices(the_graph);s2;s1)
. . . . | . | . 10. no_repeats(Vertices(the_graph)+
. . . . | . | . |   Vertices(the_graph);s2)
. . . . | . | . 11. ∀j:Vertices(the_graph).
. . . . | . | . |   (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . | . 12. ∀j:Vertices(the_graph).
. . . . | . | . |   i1-the_graph->j
. . . . | . | . |   ⇒
. . . . | . | . |   ¬j = i1
. . . . | . | . |   ⇒
. . . . | . | . |   (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . | . 13. df-traversal(the_graph;s1)
. . . . | . | . 14. ∀j:Vertices(the_graph).
. . . . | . | . |   (inr(j) ∈ s1)
```

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. . . . | . | . |   ⇒
. . . . | . | . |   ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . | . 15. ∀i:Vertices(the_graph)
. . . . | . | . |   , s1@0,s2@0:traversal(the_graph).
. . . . | . | . |   (s2 @ [inr(i1) / s1]) = (s1@0 @ [inr(i)] @ s2@0)
. . . . | . | . |   ⇒
. . . . | . | . |   (∀j:Vertices(the_graph).
. . . . | . | . |    (inr(j) ∈ s2@0)
. . . . | . | . |    ⇒
. . . . | . | . |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i)
. . . . | . | . 16. ∀i:Vertices(the_graph)
. . . . | . | . |   , s1@0,s2@0:traversal(the_graph).
. . . . | . | . |   ...
. . . . | . | . |   ⇒
. . . . | . | . |   (s2 @ [inr(i1) / s1]) = (s1@0 @ [inl(i)] @ s2@0)
. . . . | . | . |   ⇒
. . . . | . | . |   (∀j:Vertices(the_graph).
. . . . | . | . |    i-the_graph->*j ⇒ (inr(j) ∈ s2@0))
. . . . | . | . 17. ∀j:Vertices(the_graph).
. . . . | . | . |   i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | . | . 18. j : Vertices(the_graph)
. . . . | . | . 19. z : Vertices(the_graph)
. . . . | . | . 20. ¬z = i1
. . . . | . | . 21. i1-the_graph->z
. . . . | . | . 22. z-the_graph->*j
. . . . | . | . 23. (inl(z) ∈ s2)
. . . . | . | . 24. s3 :
. . . . | . | . |   (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . | . 25. s2@0 :
. . . . | . | . |   (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . | . 26. s2 = (s3 @ [inl(z) / s2@0])
. . . . | . | . 27. (inr(j) ∈ s2@0 @ [inr(i1) / s1])
. . . . | . | . ⊢  (inr(j) ∈ s2 @ [inr(i1) / s1])  by HypSubst -2 0
. . . . | . | . |
. . . . | . | . 1. the_graph : Graph
. . . . | . | . 2. GraphObject(the_graph)
. . . . | . | . 3. s1 : traversal(the_graph)
. . . . | . | . 4. s2 : traversal(the_graph)
. . . . | . | . 5. i1 : Vertices(the_graph)
. . . . | . | . 6. ¬(inr(i1) ∈ s1)
. . . . | . | . 7. ¬(inl(i1) ∈ s1)
. . . . | . | . 8. paren(Vertices(the_graph);s2)
. . . . | . | . 9. l_disjoint(Vertices(the_graph)+
. . . . | . | . |  Vertices(the_graph);s2;s1)
. . . . | . | . 10. no_repeats(Vertices(the_graph)+
. . . . | . | . |   Vertices(the_graph);s2)
. . . . | . | . 11. ∀j:Vertices(the_graph).
. . . . | . | . |   (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . | . 12. ∀j:Vertices(the_graph).
. . . . | . | . |   i1-the_graph->j
. . . . | . | . |   ⇒
. . . . | . | . |   ¬j = i1
. . . . | . | . |   ⇒
. . . . | . | . |   (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . | . 13. df-traversal(the_graph;s1)
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. . . . | . | . 14. ∀j:Vertices(the_graph).
. . . . | . | . |   (inr(j) ∈ s1)
. . . . | . | . |   ⇒
. . . . | . | . |   ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . | . 15. ∀i:Vertices(the_graph)
. . . . | . | . |   , s1@0,s2@0:traversal(the_graph).
. . . . | . | . |   (s2 @ [inr(i1) / s1]) = (s1@0 @ [inr(i)] @ s2@0)
. . . . | . | . |   ⇒
. . . . | . | . |   (∀j:Vertices(the_graph).
. . . . | . | . |    (inr(j) ∈ s2@0)
. . . . | . | . |    ⇒
. . . . | . | . |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i)
. . . . | . | . 16. ∀i:Vertices(the_graph)
. . . . | . | . |   , s1@0,s2@0:traversal(the_graph).
. . . . | . | . |   ...
. . . . | . | . |   ⇒
. . . . | . | . |   (s2 @ [inr(i1) / s1]) = (s1@0 @ [inl(i)] @ s2@0)
. . . . | . | . |   ⇒
. . . . | . | . |   (∀j:Vertices(the_graph).
. . . . | . | . |    i-the_graph->*j ⇒ (inr(j) ∈ s2@0))
. . . . | . | . 17. ∀j:Vertices(the_graph).
. . . . | . | . |   i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | . | . 18. j : Vertices(the_graph)
. . . . | . | . 19. z : Vertices(the_graph)
. . . . | . | . 20. ¬z = i1
. . . . | . | . 21. i1-the_graph->z
. . . . | . | . 22. z-the_graph->*j
. . . . | . | . 23. (inl(z) ∈ s2)
. . . . | . | . 24. s3 :
. . . . | . | . |   (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . | . 25. s2@0 :
. . . . | . | . |   (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . | . 26. s2 = (s3 @ [inl(z) / s2@0])
. . . . | . | . 27. (inr(j) ∈ s2@0 @ [inr(i1) / s1])
. . . . | . | . ⊢  (inr(j) ∈ (s3 @ [inl(z) / s2@0]) @ [inr(i1) / s1])
. . . . | . | . | by (MoveToConcl -1)
. . . . | . | . |    THEN
. . . . | . | . |    (RWW
. . . . | . | . |     Thm* ∀x:T, l1,l2:T List.
. . . . | . | . |          (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
. . . . | . | . |     {member_append}
. . . . | . | . |     0)
. . . . | . | . |    THEN
. . . . | . | . |    (RWW
. . . . | . | . |     Thm* ∀l:T List, a,x:T.
. . . . | . | . |          (x ∈ [a / l]) ⟺ x = a ∨ (x ∈ l)
. . . . | . | . |     {cons_member}
. . . . | . | . |     0)
. . . . | . | . |
. . . . | . | . 1. the_graph : Graph
. . . . | . | . 2. GraphObject(the_graph)
. . . . | . | . 3. s1 : traversal(the_graph)
. . . . | . | . 4. s2 : traversal(the_graph)
. . . . | . | . 5. i1 : Vertices(the_graph)
. . . . | . | . 6. ¬(inr(i1) ∈ s1)
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. . . . | . | . 7. ¬(inl(i1) ∈ s1)
. . . . | . | . 8. paren(Vertices(the_graph);s2)
. . . . | . | . 9. l_disjoint(Vertices(the_graph)+
. . . . | . | .    Vertices(the_graph);s2;s1)
. . . . | . | . 10. no_repeats(Vertices(the_graph)+
. . . . | . | .     Vertices(the_graph);s2)
. . . . | . | . 11. ∀j:Vertices(the_graph).
. . . . | . | .     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . | . 12. ∀j:Vertices(the_graph).
. . . . | . | .     i1-the_graph->j
. . . . | . | .     ⇒
. . . . | . | .     ¬j = i1
. . . . | . | .     ⇒
. . . . | . | .     (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . | . 13. df-traversal(the_graph;s1)
. . . . | . | . 14. ∀j:Vertices(the_graph).
. . . . | . | .     (inr(j) ∈ s1)
. . . . | . | .     ⇒
. . . . | . | .     ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . | . 15. ∀i:Vertices(the_graph)
. . . . | . | .     , s1@0,s2@0:traversal(the_graph).
. . . . | . | .     (s2 @ [inr(i1) / s1]) = (s1@0 @ [inr(i)] @ s2@0)
. . . . | . | .     ⇒
. . . . | . | .     (∀j:Vertices(the_graph).
. . . . | . | .      (inr(j) ∈ s2@0)
. . . . | . | .      ⇒
. . . . | . | .      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i)
. . . . | . | . 16. ∀i:Vertices(the_graph)
. . . . | . | .     , s1@0,s2@0:traversal(the_graph).
. . . . | . | .     ...
. . . . | . | .     ⇒
. . . . | . | .     (s2 @ [inr(i1) / s1]) = (s1@0 @ [inl(i)] @ s2@0)
. . . . | . | .     ⇒
. . . . | . | .     (∀j:Vertices(the_graph).
. . . . | . | .      i-the_graph->*j ⇒ (inr(j) ∈ s2@0))
. . . . | . | . 17. ∀j:Vertices(the_graph).
. . . . | . | .     i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)

. . . . | . | . 18. j : Vertices(the_graph)
. . . . | . | . 19. z : Vertices(the_graph)
. . . . | . | . 20. ¬z = i1
. . . . | . | . 21. i1-the_graph->z
. . . . | . | . 22. z-the_graph->*j
. . . . | . | . 23. (inl(z) ∈ s2)
. . . . | . | . 24. s3 :
. . . . | . | .     (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . | . 25. s2@0 :
. . . . | . | .     (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . | . 26. s2 = (s3 @ [inl(z) / s2@0])
. . . . | . | . ⊢  (inr(j) ∈ s2@0) ∨ inr(j) = inr(i1) ∨ (inr(j) ∈ s1)
. . . . | . | .    ⇒
. . . . | . | .    (inr(j) ∈ s3) ∨ inr(j) = inl(z) ∨ (inr(j) ∈ s2@0)
. . . . | . | .    ∨ inr(j) = inr(i1)
. . . . | . | .    ∨ (inr(j) ∈ s1)  by ObviousConcl
. . . . | . |
. . . . | . .\
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. . . . | . . 1. the_graph : Graph
. . . . | . . 2. GraphObject(the_graph)
. . . . | . . 3. s1 : traversal(the_graph)
. . . . | . . 4. s2 : traversal(the_graph)
. . . . | . . 5. i1 : Vertices(the_graph)
. . . . | . . 6. ¬(inr(i1) ∈ s1)
. . . . | . . 7. ¬(inl(i1) ∈ s1)
. . . . | . . 8. paren(Vertices(the_graph);s2)
. . . . | . . 9. l_disjoint(Vertices(the_graph)+
. . . . | . . |  Vertices(the_graph);s2;s1)
. . . . | . . 10. no_repeats(Vertices(the_graph)+
. . . . | . . |   Vertices(the_graph);s2)
. . . . | . . 11. ∀j:Vertices(the_graph).
. . . . | . . |   (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . . 12. ∀j:Vertices(the_graph).
. . . . | . . |   i1-the_graph->j
. . . . | . . |   ⇒
. . . . | . . |   ¬j = i1
. . . . | . . |   ⇒
. . . . | . . |   (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . . 13. df-traversal(the_graph;s1)
. . . . | . . 14. ∀j:Vertices(the_graph).
. . . . | . . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . . 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . | . . 16. ∀j:Vertices(the_graph).
. . . . | . . |   i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | . . 17. j : Vertices(the_graph)
. . . . | . . 18. z : Vertices(the_graph)
. . . . | . . 19. ¬z = i1
. . . . | . . 20. i1-the_graph->z
. . . . | . . 21. z-the_graph->*j
. . . . | . . 22. (inl(z) ∈ s1) ∨ (inr(z) ∈ s1)
. . . . | . . ⊢  (inr(j) ∈ s2 @ [inr(i1) / s1])
. . . . | . . | by (Inst
. . . . | . . |     Thm* (∀x,y:T. Dec(x = y))
. . . . | . . |          ⇒
. . . . | . . |          (∀s:(T+T) List, i:T. Dec((inl(i) ∈ s)))
. . . . | . . |     {decidable__l_member_paren}
. . . . | . . |     [Vertices(the_graph);s1;z])
. . . . | . . |    THEN
. . . . | . . |    (Analyze -1)
. . . . | . . |
. . . . | . . |\
. . . . | . . | .....decidable? .....
. . . . | . . | 1. the_graph : Graph
. . . . | . . | 2. GraphObject(the_graph)
. . . . | . . | 3. s1 : traversal(the_graph)
. . . . | . . | 4. s2 : traversal(the_graph)
. . . . | . . | 5. i1 : Vertices(the_graph)
. . . . | . . | 6. ¬(inr(i1) ∈ s1)
. . . . | . . | 7. ¬(inl(i1) ∈ s1)
. . . . | . . | 8. paren(Vertices(the_graph);s2)
. . . . | . . | 9. l_disjoint(Vertices(the_graph)+
. . . . | . . |    Vertices(the_graph);s2;s1)
. . . . | . . | 10. no_repeats(Vertices(the_graph)+
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. . . . | . . |     Vertices(the_graph);s2)
. . . . | . . | 11. ∀j:Vertices(the_graph).
. . . . | . . |     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . . | 12. ∀j:Vertices(the_graph).
. . . . | . . |     i1-the_graph->j
. . . . | . . |     ⇒
. . . . | . . |     ¬j = i1
. . . . | . . |     ⇒
. . . . | . . |     (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . . | 13. df-traversal(the_graph;s1)
. . . . | . . | 14. ∀j:Vertices(the_graph).
. . . . | . . |     (inr(j) ∈ s1)
. . . . | . . |     ⇒
. . . . | . . |     ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . . | 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . | . . | 16. ∀j:Vertices(the_graph).
. . . . | . . |     i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | . . | 17. j : Vertices(the_graph)
. . . . | . . | 18. z : Vertices(the_graph)
. . . . | . . | 19. ¬z = i1
. . . . | . . | 20. i1-the_graph->z
. . . . | . . | 21. z-the_graph->*j
. . . . | . . | 22. (inl(z) ∈ s1) ∨ (inr(z) ∈ s1)
. . . . | . . | 23. x : Vertices(the_graph)
. . . . | . . | 24. y : Vertices(the_graph)
. . . . | . . | ⊢  Dec(x = y)
. . . . | . . |   by (Inst Thm: graphobj_DASH_properties
. . . . | . . |       [the_graph;the_obj])
. . . . | . . |      THEN
. . . . | . . |      (Analyze -1)
. . . . | . . |      THEN
. . . . | . . |      (RW (SweepUpC (RevHypC -2)) 0)
. . . . | . . |
. . . . | . . |\
. . . . | . . | 1. the_graph : Graph
. . . . | . . | 2. GraphObject(the_graph)
. . . . | . . | 3. s1 : traversal(the_graph)
. . . . | . . | 4. s2 : traversal(the_graph)
. . . . | . . | 5. i1 : Vertices(the_graph)
. . . . | . . | 6. ¬(inr(i1) ∈ s1)
. . . . | . . | 7. ¬(inl(i1) ∈ s1)
. . . . | . . | 8. paren(Vertices(the_graph);s2)
. . . . | . . | 9. l_disjoint(Vertices(the_graph)+
. . . . | . . | |  Vertices(the_graph);s2;s1)
. . . . | . . | 10. no_repeats(Vertices(the_graph)+
. . . . | . . | |   Vertices(the_graph);s2)
. . . . | . . | 11. ∀j:Vertices(the_graph).
. . . . | . . | |   (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . . | 12. ∀j:Vertices(the_graph).
. . . . | . . | |   i1-the_graph->j
. . . . | . . | |   ⇒
. . . . | . . | |   ¬j = i1
. . . . | . . | |   ⇒
. . . . | . . | |   (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . . | 13. df-traversal(the_graph;s1)
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. . . . | . . | 14. ∀j:Vertices(the_graph).
. . . . | . . | |   (inr(j) ∈ s1)
. . . . | . . | |   ⇒
. . . . | . . | |   ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . . | 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . | . . | 16. ∀j:Vertices(the_graph).
. . . . | . . | |   i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | . . | 17. j : Vertices(the_graph)
. . . . | . . | 18. z : Vertices(the_graph)
. . . . | . . | 19. ¬z = i1
. . . . | . . | 20. i1-the_graph->z
. . . . | . . | 21. z-the_graph->*j
. . . . | . . | 22. (inl(z) ∈ s1) ∨ (inr(z) ∈ s1)
. . . . | . . | 23. (inl(z) ∈ s1)
. . . . | . . | ⊢  (inr(j) ∈ s2 @ [inr(i1) / s1])  by Thin -2
. . . . | . . | |
. . . . | . . | 1. the_graph : Graph
. . . . | . . | 2. GraphObject(the_graph)
. . . . | . . | 3. s1 : traversal(the_graph)
. . . . | . . | 4. s2 : traversal(the_graph)
. . . . | . . | 5. i1 : Vertices(the_graph)
. . . . | . . | 6. ¬(inr(i1) ∈ s1)
. . . . | . . | 7. ¬(inl(i1) ∈ s1)
. . . . | . . | 8. paren(Vertices(the_graph);s2)
. . . . | . . | 9. l_disjoint(Vertices(the_graph)+
. . . . | . . | |  Vertices(the_graph);s2;s1)
. . . . | . . | 10. no_repeats(Vertices(the_graph)+
. . . . | . . | |   Vertices(the_graph);s2)
. . . . | . . | 11. ∀j:Vertices(the_graph).
. . . . | . . | |   (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . . | 12. ∀j:Vertices(the_graph).
. . . . | . . | |   i1-the_graph->j
. . . . | . . | |   ⇒
. . . . | . . | |   ¬j = i1
. . . . | . . | |   ⇒
. . . . | . . | |   (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . . | 13. df-traversal(the_graph;s1)
. . . . | . . | 14. ∀j:Vertices(the_graph).
. . . . | . . | |   (inr(j) ∈ s1)
. . . . | . . | |   ⇒
. . . . | . . | |   ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . . | 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . | . . | 16. ∀j:Vertices(the_graph).
. . . . | . . | |   i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | . . | 17. j : Vertices(the_graph)
. . . . | . . | 18. z : Vertices(the_graph)
. . . . | . . | 19. ¬z = i1
. . . . | . . | 20. i1-the_graph->z
. . . . | . . | 21. z-the_graph->*j
. . . . | . . | 22. (inl(z) ∈ s1)
. . . . | . . | ⊢  (inr(j) ∈ s2 @ [inr(i1) / s1])
. . . . | . . | | by (Inst
. . . . | . . | |     Thm* ∀s:T List, z:T.
. . . . | . . | |          (z ∈ s)
. . . . | . . | |          ⇒
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```
. . . . | . . | |          (∃s1,s2:T List. s = (s1 @ [z / s2]))
. . . . | . . | |     {l_member_decomp}
. . . . | . . | |     [Vertices(the_graph)+Vertices(the_graph)
. . . . | . . | |     ;s1
. . . . | . . | |     ;inl(z)])
. . . . | . . | |    THEN
. . . . | . . | |    ExRepD
. . . . | . . | |    THEN
. . . . | . . | |    (let h ≡ 15 in
. . . . | . . | |     (Unfold `df-traversal` h) THEN (Analyze h)
. . . . | . . | |     THEN
. . . . | . . | |     (InstHyp [z;(s2 @ [inr(i1)]) @ s1@0;s3;j] h+1))
. . . . | . . | |
. . . . | . . | |\
. . . . | . . | | 1. the_graph : Graph
. . . . | . . | | 2. GraphObject(the_graph)
. . . . | . . | | 3. s1 : traversal(the_graph)
. . . . | . . | | 4. s2 : traversal(the_graph)
. . . . | . . | | 5. i1 : Vertices(the_graph)
. . . . | . . | | 6. ¬(inr(i1) ∈ s1)
. . . . | . . | | 7. ¬(inl(i1) ∈ s1)
. . . . | . . | | 8. paren(Vertices(the_graph);s2)
. . . . | . . | | 9. l_disjoint(Vertices(the_graph)+
. . . . | . . | |    Vertices(the_graph);s2;s1)
. . . . | . . | | 10. no_repeats(Vertices(the_graph)+
. . . . | . . | |     Vertices(the_graph);s2)
. . . . | . . | | 11. ∀j:Vertices(the_graph).
. . . . | . . | |     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . . | | 12. ∀j:Vertices(the_graph).
. . . . | . . | |     i1-the_graph->j
. . . . | . . | |     ⇒
. . . . | . . | |     ¬j = i1
. . . . | . . | |     ⇒
. . . . | . . | |     (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . . | | 13. df-traversal(the_graph;s1)
. . . . | . . | | 14. ∀j:Vertices(the_graph).
. . . . | . . | |     (inr(j) ∈ s1)
. . . . | . . | |     ⇒
. . . . | . . | |     ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . . | | 15. ∀i:Vertices(the_graph)
. . . . | . . | |     , s1@0,s2@0:traversal(the_graph).
. . . . | . . | |     (s2 @ [inr(i1) / s1]) = (s1@0 @ [inr(i)] @ s2@0)
. . . . | . . | |     ⇒
. . . . | . . | |     (∀j:Vertices(the_graph).
. . . . | . . | |      (inr(j) ∈ s2@0)
. . . . | . . | |      ⇒
. . . . | . . | |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i)
. . . . | . . | | 16. ∀i:Vertices(the_graph)
. . . . | . . | |     , s1@0,s2@0:traversal(the_graph).
. . . . | . . | |     (∀j:Vertices(the_graph).
. . . . | . . | |      i-the_graph->*j
. . . . | . . | |      ⇒
. . . . | . . | |      ¬non-trivial-loop(the_graph;j))
. . . . | . . | |     ⇒
. . . . | . . | |     (s2 @ [inr(i1) / s1]) = (s1@0 @ [inl(i)] @ s2@0)
```

```
. . . . | . . | |     ⇒
. . . . | . . | |     (∀j:Vertices(the_graph).
. . . . | . . | |      i-the_graph->*j ⇒ (inr(j) ∈ s2@0))
. . . . | . . | | 17. ∀j:Vertices(the_graph).
. . . . | . . | |     i1-the_graph->*j
. . . . | . . | |     ⇒
. . . . | . . | |     ¬non-trivial-loop(the_graph;j)
. . . . | . . | | 18. j : Vertices(the_graph)
. . . . | . . | | 19. z : Vertices(the_graph)
. . . . | . . | | 20. ¬z = i1
. . . . | . . | | 21. i1-the_graph->z
. . . . | . . | | 22. z-the_graph->*j
. . . . | . . | | 23. (inl(z) ∈ s1)
. . . . | . . | | 24. s1@0 :
. . . . | . . | |     (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . . | | 25. s3 :
. . . . | . . | |     (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . . | | 26. s1 = (s1@0 @ [inl(z) / s3])
. . . . | . . | | 27. j1 : Vertices(the_graph)
. . . . | . . | | 28. z-the_graph->*j1
. . . . | . . | | ⊢  ¬non-trivial-loop(the_graph;j1)
. . . . | . . | |   by BackThruSomeHyp THEN Connect
. . . . | . . | |
. . . . | . . | |\
. . . . | . . | | .....antecedent .....
. . . . | . . | | 1. the_graph : Graph
. . . . | . . | | 2. GraphObject(the_graph)
. . . . | . . | | 3. s1 : traversal(the_graph)
. . . . | . . | | 4. s2 : traversal(the_graph)
. . . . | . . | | 5. i1 : Vertices(the_graph)
. . . . | . . | | 6. ¬(inr(i1) ∈ s1)
. . . . | . . | | 7. ¬(inl(i1) ∈ s1)
. . . . | . . | | 8. paren(Vertices(the_graph);s2)
. . . . | . . | | 9. l_disjoint(Vertices(the_graph)+
. . . . | . . | |    Vertices(the_graph);s2;s1)
. . . . | . . | | 10. no_repeats(Vertices(the_graph)+
. . . . | . . | |     Vertices(the_graph);s2)
. . . . | . . | | 11. ∀j:Vertices(the_graph).
. . . . | . . | |     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . . | | 12. ∀j:Vertices(the_graph).
. . . . | . . | |     i1-the_graph->j
. . . . | . . | |     ⇒
. . . . | . . | |     ¬j = i1
. . . . | . . | |     ⇒
. . . . | . . | |     (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . . | | 13. df-traversal(the_graph;s1)
. . . . | . . | | 14. ∀j:Vertices(the_graph).
. . . . | . . | |     (inr(j) ∈ s1)
. . . . | . . | |     ⇒
. . . . | . . | |     ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . . | | 15. ∀i:Vertices(the_graph)
. . . . | . . | |     , s1@0,s2@0:traversal(the_graph).
. . . . | . . | |     (s2 @ [inr(i1) / s1]) = (s1@0 @ [inr(i)] @ s2@0)
. . . . | . . | |     ⇒
. . . . | . . | |     (∀j:Vertices(the_graph).
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. . . . | . . | |      (inr(j) ∈ s2@0)
. . . . | . . | |      ⇒
. . . . | . . | |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i)
. . . . | . . | | 16. ∀i:Vertices(the_graph)
. . . . | . . | |     , s1@0,s2@0:traversal(the_graph).
. . . . | . . | |     (∀j:Vertices(the_graph).
. . . . | . . | |      i-the_graph->*j
. . . . | . . | |      ⇒
. . . . | . . | |      ¬non-trivial-loop(the_graph;j))
. . . . | . . | |     ⇒
. . . . | . . | |     (s2 @ [inr(i1) / s1]) = (s1@0 @ [inl(i)] @ s2@0)
. . . . | . . | |     ⇒
. . . . | . . | |     (∀j:Vertices(the_graph).
. . . . | . . | |      i-the_graph->*j ⇒ (inr(j) ∈ s2@0))
. . . . | . . | | 17. ∀j:Vertices(the_graph).
. . . . | . . | |     i1-the_graph->*j
. . . . | . . | |     ⇒
. . . . | . . | |     ¬non-trivial-loop(the_graph;j)
. . . . | . . | | 18. j : Vertices(the_graph)
. . . . | . . | | 19. z : Vertices(the_graph)
. . . . | . . | | 20. ¬z = i1
. . . . | . . | | 21. i1-the_graph->z
. . . . | . . | | 22. z-the_graph->*j
. . . . | . . | | 23. (inl(z) ∈ s1)
. . . . | . . | | 24. s1@0 :
. . . . | . . | |     (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . . | | 25. s3 :
. . . . | . . | |     (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . . | | 26. s1 = (s1@0 @ [inl(z) / s3])
. . . . | . . | | ⊢  (s2 @ [inr(i1) / s1])
. . . . | . . | |    =
. . . . | . . | |    (((s2 @ [inr(i1)]) @ s1@0) @ [inl(z)] @ s3)
. . . . | . . | |   by (HypSubst -1 0) THEN (Reduce 0)
. . . . | . . | |      THEN
. . . . | . . | |      (RWW
. . . . | . . | |       Thm* ∀as,bs,cs:T List.
. . . . | . . | |            ((as @ bs) @ cs) = (as @ (bs @ cs))
. . . . | . . | |       {append_assoc}
. . . . | . . | |       0)
. . . . | . . | |      THEN
. . . . | . . | |      (Reduce 0)
. . . . | . . | |
. . . . | . . | .\
. . . . | . . | . 1. the_graph : Graph
. . . . | . . | . 2. GraphObject(the_graph)
. . . . | . . | . 3. s1 : traversal(the_graph)
. . . . | . . | . 4. s2 : traversal(the_graph)
. . . . | . . | . 5. i1 : Vertices(the_graph)
. . . . | . . | . 6. ¬(inr(i1) ∈ s1)
. . . . | . . | . 7. ¬(inl(i1) ∈ s1)
. . . . | . . | . 8. paren(Vertices(the_graph);s2)
. . . . | . . | . 9. l_disjoint(Vertices(the_graph)+
. . . . | . . | . |  Vertices(the_graph);s2;s1)
. . . . | . . | . 10. no_repeats(Vertices(the_graph)+
. . . . | . . | . |   Vertices(the_graph);s2)
```

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. . . . | . . | . 11. ∀j:Vertices(the_graph).
. . . . | . . | . |   (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . . | . 12. ∀j:Vertices(the_graph).
. . . . | . . | . |   i1-the_graph->j
. . . . | . . | . |   ⇒
. . . . | . . | . |   ¬j = i1
. . . . | . . | . |   ⇒
. . . . | . . | . |   (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . . | . 13. df-traversal(the_graph;s1)
. . . . | . . | . 14. ∀j:Vertices(the_graph).
. . . . | . . | . |   (inr(j) ∈ s1)
. . . . | . . | . |   ⇒
. . . . | . . | . |   ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . . | . 15. ∀i:Vertices(the_graph)
. . . . | . . | . |   , s1@0,s2@0:traversal(the_graph).
. . . . | . . | . |   (s2 @ [inr(i1) / s1]) = (s1@0 @ [inr(i)] @ s2@0)
. . . . | . . | . |   ⇒
. . . . | . . | . |   (∀j:Vertices(the_graph).
. . . . | . . | . |    (inr(j) ∈ s2@0)
. . . . | . . | . |    ⇒
. . . . | . . | . |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i)
. . . . | . . | . 16. ∀i:Vertices(the_graph)
. . . . | . . | . |   , s1@0,s2@0:traversal(the_graph).
. . . . | . . | . |   (∀j:Vertices(the_graph).
. . . . | . . | . |    i-the_graph->*j
. . . . | . . | . |    ⇒
. . . . | . . | . |    ¬non-trivial-loop(the_graph;j))
. . . . | . . | . |   ⇒
. . . . | . . | . |   (s2 @ [inr(i1) / s1]) = (s1@0 @ [inl(i)] @ s2@0)
. . . . | . . | . |   ⇒
. . . . | . . | . |   (∀j:Vertices(the_graph).
. . . . | . . | . |    i-the_graph->*j ⇒ (inr(j) ∈ s2@0))
. . . . | . . | . 17. ∀j:Vertices(the_graph).
. . . . | . . | . |   i1-the_graph->*j
. . . . | . . | . |   ⇒
. . . . | . . | . |   ¬non-trivial-loop(the_graph;j)
. . . . | . . | . 18. j : Vertices(the_graph)
. . . . | . . | . 19. z : Vertices(the_graph)
. . . . | . . | . 20. ¬z = i1
. . . . | . . | . 21. i1-the_graph->z
. . . . | . . | . 22. z-the_graph->*j
. . . . | . . | . 23. (inl(z) ∈ s1)
. . . . | . . | . 24. s1@0 :
. . . . | . . | . |   (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . . | . 25. s3 :
. . . . | . . | . |   (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . . | . 26. s1 = (s1@0 @ [inl(z) / s3])
. . . . | . . | . 27. (inr(j) ∈ s3)
. . . . | . . | . ⊢  (inr(j) ∈ s2 @ [inr(i1) / s1])
. . . . | . . | . | by ((HypSubst -2 0)
. . . . | . . | . |     THEN
. . . . | . . | . |     (RWW
. . . . | . . | . |      Thm* ∀x:T, l1,l2:T List.
. . . . | . . | . |           (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
. . . . | . . | . |      {member_append}
```

```
. . . . | . . | . |      0))
. . . . | . . | . |    THEN
. . . . | . . | . |    (RWW
. . . . | . . | . |     Thm* ∀l:T List, a,x:T.
. . . . | . . | . |          (x ∈ [a / l]) ⟺ x = a ∨ (x ∈ l)
. . . . | . . | . |     {cons_member}
. . . . | . . | . |     0)
. . . . | . . | . |    THEN
. . . . | . . | . |    (RWW
. . . . | . . | . |     Thm* ∀x:T, l1,l2:T List.
. . . . | . . | . |          (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
. . . . | . . | . |     {member_append}
. . . . | . . | . |     0)
. . . . | . . | . |    THEN
. . . . | . . | . |    (RWW
. . . . | . . | . |     Thm* ∀l:T List, a,x:T.
. . . . | . . | . |          (x ∈ [a / l]) ⟺ x = a ∨ (x ∈ l)
. . . . | . . | . |     {cons_member}
. . . . | . . | . |     0)
. . . . | . . | . |
. . . . | . . | . 1. the_graph : Graph
. . . . | . . | . 2. GraphObject(the_graph)
. . . . | . . | . 3. s1 : traversal(the_graph)
. . . . | . . | . 4. s2 : traversal(the_graph)
. . . . | . . | . 5. i1 : Vertices(the_graph)
. . . . | . . | . 6. ¬(inr(i1) ∈ s1)
. . . . | . . | . 7. ¬(inl(i1) ∈ s1)
. . . . | . . | . 8. paren(Vertices(the_graph);s2)
. . . . | . . | . 9. l_disjoint(Vertices(the_graph)+
. . . . | . . | .    Vertices(the_graph);s2;s1)
. . . . | . . | . 10. no_repeats(Vertices(the_graph)+
. . . . | . . | .     Vertices(the_graph);s2)
. . . . | . . | . 11. ∀j:Vertices(the_graph).
. . . . | . . | .     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . . | . 12. ∀j:Vertices(the_graph).
. . . . | . . | .     i1-the_graph->j
. . . . | . . | .     ⇒
. . . . | . . | .     ¬j = i1
. . . . | . . | .     ⇒
. . . . | . . | .     (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . . | . 13. df-traversal(the_graph;s1)
. . . . | . . | . 14. ∀j:Vertices(the_graph).
. . . . | . . | .     (inr(j) ∈ s1)
. . . . | . . | .     ⇒
. . . . | . . | .     ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . . | . 15. ∀i:Vertices(the_graph)
. . . . | . . | .     , s1@0,s2@0:traversal(the_graph).
. . . . | . . | .     (s2 @ [inr(i1) / s1]) = (s1@0 @ [inr(i)] @ s2@0)
. . . . | . . | .     ⇒
. . . . | . . | .     (∀j:Vertices(the_graph).
. . . . | . . | .      (inr(j) ∈ s2@0)
. . . . | . . | .      ⇒
. . . . | . . | .      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i)
. . . . | . . | . 16. ∀i:Vertices(the_graph)
. . . . | . . | .     , s1@0,s2@0:traversal(the_graph).
```

```
. . . . | . . | .     (∀j:Vertices(the_graph).
. . . . | . . | .      i-the_graph->*j
. . . . | . . | .      ⇒
. . . . | . . | .      ¬non-trivial-loop(the_graph;j))
. . . . | . . | .     ⇒
. . . . | . . | .     (s2 @ [inr(i1) / s1]) = (s1@0 @ [inl(i)] @ s2@0)
. . . . | . . | .     ⇒
. . . . | . . | .     (∀j:Vertices(the_graph).
. . . . | . . | .      i-the_graph->*j ⇒ (inr(j) ∈ s2@0))
. . . . | . . | . 17. ∀j:Vertices(the_graph).
. . . . | . . | .     i1-the_graph->*j
. . . . | . . | .     ⇒
. . . . | . . | .     ¬non-trivial-loop(the_graph;j)
. . . . | . . | . 18. j : Vertices(the_graph)
. . . . | . . | . 19. z : Vertices(the_graph)
. . . . | . . | . 20. ¬z = i1
. . . . | . . | . 21. i1-the_graph->z
. . . . | . . | . 22. z-the_graph->*j
. . . . | . . | . 23. (inl(z) ∈ s1)
. . . . | . . | . 24. s1@0 :
. . . . | . . | .     (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . . | . 25. s3 :
. . . . | . . | .     (Vertices(the_graph)+Vertices(the_graph)) List
. . . . | . . | . 26. s1 = (s1@0 @ [inl(z) / s3])
. . . . | . . | . 27. (inr(j) ∈ s3)
. . . . | . . | . ⊢  (inr(j) ∈ s2)
. . . . | . . | .    ∨ inr(j) = inr(i1)
. . . . | . . | .    ∨ (inr(j) ∈ s1@0)
. . . . | . . | .    ∨ inr(j) = inl(z)
. . . . | . . | .    ∨ (inr(j) ∈ s3)  by ObviousFrom [-1]
. . . . | . . |
. . . . | . . .\
. . . . | . . . 1. the_graph : Graph
. . . . | . . . 2. GraphObject(the_graph)
. . . . | . . . 3. s1 : traversal(the_graph)
. . . . | . . . 4. s2 : traversal(the_graph)
. . . . | . . . 5. i1 : Vertices(the_graph)
. . . . | . . . 6. ¬(inr(i1) ∈ s1)
. . . . | . . . 7. ¬(inl(i1) ∈ s1)
. . . . | . . . 8. paren(Vertices(the_graph);s2)
. . . . | . . . 9. l_disjoint(Vertices(the_graph)+
. . . . | . . .    Vertices(the_graph);s2;s1)
. . . . | . . . 10. no_repeats(Vertices(the_graph)+
. . . . | . . .     Vertices(the_graph);s2)
. . . . | . . . 11. ∀j:Vertices(the_graph).
. . . . | . . .     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . | . . . 12. ∀j:Vertices(the_graph).
. . . . | . . .     i1-the_graph->j
. . . . | . . .     ⇒
. . . . | . . .     ¬j = i1
. . . . | . . .     ⇒
. . . . | . . .     (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . | . . . 13. df-traversal(the_graph;s1)
. . . . | . . . 14. ∀j:Vertices(the_graph).
. . . . | . . .     (inr(j) ∈ s1)
```

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. . . . | . . .     ⇒
. . . . | . . .     ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . | . . . 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . | . . . 16. ∀j:Vertices(the_graph).
. . . . | . . .     i1-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . | . . . 17. j : Vertices(the_graph)
. . . . | . . . 18. z : Vertices(the_graph)
. . . . | . . . 19. ¬z = i1
. . . . | . . . 20. i1-the_graph->z
. . . . | . . . 21. z-the_graph->*j
. . . . | . . . 22. (inl(z) ∈ s1) ∨ (inr(z) ∈ s1)
. . . . | . . . 23. ¬(inl(z) ∈ s1)
. . . . | . . . ⊢  (inr(j) ∈ s2 @ [inr(i1) / s1])
. . . . | . . .   by (((Analyze -2) THEN (InstHyp [z] 14)
. . . . | . . .        THEN
. . . . | . . .        (InstHyp [i1] 16))
. . . . | . . .       THENA
. . . . | . . .       (Auto THEN Connect))
. . . . | . . .      THEN
. . . . | . . .      (Analyze -1)
. . . . | . . .      THEN
. . . . | . . .      (Unfold `non-trivial-loop` 0)
. . . . | . . .      THEN
. . . . | . . .      (InstConcl [z])
. . . . | . . .      THEN
. . . . | . . .      Connect
. . . . |
. . . . .\
. . . . . 1. the_graph : Graph
. . . . . 2. GraphObject(the_graph)
. . . . . 3. s1 : traversal(the_graph)
. . . . . 4. s2 : traversal(the_graph)
. . . . . 5. i1 : Vertices(the_graph)
. . . . . 6. ¬(inr(i1) ∈ s1)
. . . . . 7. ¬(inl(i1) ∈ s1)
. . . . . 8. paren(Vertices(the_graph);s2)
. . . . . 9. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . . . . 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . . . 11. ∀j:Vertices(the_graph).
. . . . .     (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . . . 12. ∀j:Vertices(the_graph).
. . . . .     i1-the_graph->j
. . . . .     ⇒
. . . . .     ¬j = i1 ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
. . . . . 13. df-traversal(the_graph;s1)
. . . . . 14. ∀j:Vertices(the_graph).
. . . . .     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i1
. . . . . 15. df-traversal(the_graph;s2 @ [inr(i1) / s1])
. . . . . 16. i : Vertices(the_graph)
. . . . . 17. u : Vertices(the_graph)+Vertices(the_graph)
. . . . . 18. v : (Vertices(the_graph)+Vertices(the_graph)) List
. . . . . 19. s2@0 : traversal(the_graph)
. . . . . 20. ∀j:Vertices(the_graph).
. . . . .     i-the_graph->*j ⇒ ¬non-trivial-loop(the_graph;j)
. . . . . 21. j : Vertices(the_graph)
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. . . . . 22. i-the_graph->*j
. . . . . 23. inl(i1) = u
. . . . . 24. (s2 @ [inr(i1) / s1]) = (v @ [inl(i) / s2@0])
. . . . . ⊢  (inr(j) ∈ s2@0)
. . . . .   by AllHyps
. . . . .      (λh.
. . . . .       (Unfold ‘df-traversal‘ h) THEN (Analyze h)
. . . . .       THEN
. . . . .       (InstHyp [i;v;s2@0;j] h+1)
. . . . .       THEN
. . . . .       (Reduce 0))
END
```