```
dfs-dfl-traversal  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), L:V List, s:Traversal, x:V.
|  dfl-traversal(the_graph;L;s)
|  ⇒
|  (∀j:V. (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x)
|  ⇒
|  dfl-traversal(the_graph;L @ [x];dfs(the_obj;s;x))
| by Auto
|    THEN
|    (Inst Thm: dfs_induction4
|     [the_graph
|     ;the_obj
|     ;λ2i,s1,s2.
|      x-the_graph->*i
|      ⇒
|      (∀j:Vertices(the_graph).
|       (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i)
|      ⇒
|      dfl-traversal(the_graph;L @ [x];s1)
|      ⇒
|      dfl-traversal(the_graph;L @ [x];s2 @ s1)
|     ;s
|     ;x])
|    THEN
|    (Reduce 0)
|
|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. L : Vertices(the_graph) List
| 4. s : traversal(the_graph)
| 5. x : Vertices(the_graph)
| 6. dfl-traversal(the_graph;L;s)
| 7. ∀j:Vertices(the_graph).
| |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| 8. s1 : traversal(the_graph)
| 9. s2 : traversal(the_graph)
| 10. s3 : traversal(the_graph)
| 11. i : Vertices(the_graph)
| 12. j : Vertices(the_graph)
| 13. i-the_graph->j
| 14. paren(Vertices(the_graph);s2)
| 15. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| 16. paren(Vertices(the_graph);s3)
| 17. x-the_graph->*j
| |   ⇒
| |   (∀j@0:Vertices(the_graph).
| |    (inr(j@0) ∈ s1) ⇒ ¬(inl(j@0) ∈ s1) ⇒ j@0-the_graph->*j)
| |   ⇒
| |   dfl-traversal(the_graph;L @ [x];s1)
| |   ⇒
| |   dfl-traversal(the_graph;L @ [x];s2 @ s1)
| 18. x-the_graph->*i
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| |   ⇒
| |   (∀j:Vertices(the_graph).
| |    (inr(j) ∈ s2 @ s1) ⇒ ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i)
| |   ⇒
| |   dfl-traversal(the_graph;L @ [x];s2 @ s1)
| |   ⇒
| |   dfl-traversal(the_graph;L @ [x];s3 @ s2 @ s1)
| 19. x-the_graph->*i
| 20. ∀j:Vertices(the_graph).
| |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| 21. dfl-traversal(the_graph;L @ [x];s1)
| ⊢  dfl-traversal(the_graph;L @ [x];(s3 @ s2) @ s1)
| | by (AssertBY x-the_graph->*j Connect) THEN (Repeat ThinTrivial)
| |    THEN
| |    (Analyze -1)
| |
| |\
| | .....antecedent .....
| | 1. the_graph : Graph
| | 2. GraphObject(the_graph)
| | 3. L : Vertices(the_graph) List
| | 4. s : traversal(the_graph)
| | 5. x : Vertices(the_graph)
| | 6. dfl-traversal(the_graph;L;s)
| | 7. ∀j:Vertices(the_graph).
| |    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| | 8. s1 : traversal(the_graph)
| | 9. s2 : traversal(the_graph)
| | 10. s3 : traversal(the_graph)
| | 11. i : Vertices(the_graph)
| | 12. j : Vertices(the_graph)
| | 13. i-the_graph->j
| | 14. paren(Vertices(the_graph);s2)
| | 15. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| | 16. paren(Vertices(the_graph);s3)
| | 17. x-the_graph->*i
| | 18. ∀j:Vertices(the_graph).
| |     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| | 19. dfl-traversal(the_graph;L @ [x];s1)
| | 20. x-the_graph->*j
| | 21. (∀j:Vertices(the_graph).
| |      (inr(j) ∈ s2 @ s1) ⇒ ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i)
| |     ⇒
| |     dfl-traversal(the_graph;L @ [x];s2 @ s1)
| |     ⇒
| |     dfl-traversal(the_graph;L @ [x];s3 @ s2 @ s1)
| | ⊢  ∀j@0:Vertices(the_graph).
| |    (inr(j@0) ∈ s1) ⇒ ¬(inl(j@0) ∈ s1) ⇒ j@0-the_graph->*j
| |   by Auto THEN (InstHyp [j@0] -7) THEN Connect
| |
| .\
| . 1. the_graph : Graph
| . 2. GraphObject(the_graph)
| . 3. L : Vertices(the_graph) List
| . 4. s : traversal(the_graph)
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| . 5. x : Vertices(the_graph)
| . 6. dfl-traversal(the_graph;L;s)
| . 7. ∀j:Vertices(the_graph).
| . |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| . 8. s1 : traversal(the_graph)
| . 9. s2 : traversal(the_graph)
| . 10. s3 : traversal(the_graph)
| . 11. i : Vertices(the_graph)
| . 12. j : Vertices(the_graph)
| . 13. i-the_graph->j
| . 14. paren(Vertices(the_graph);s2)
| . 15. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| . 16. paren(Vertices(the_graph);s3)
| . 17. x-the_graph->*i
| . 18. ∀j:Vertices(the_graph).
| . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| . 19. dfl-traversal(the_graph;L @ [x];s1)
| . 20. x-the_graph->*j
| . 21. (∀j:Vertices(the_graph).
| . |    (inr(j) ∈ s2 @ s1) ⇒ ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i)
| . |   ⇒
| . |   dfl-traversal(the_graph;L @ [x];s2 @ s1)
| . |   ⇒
| . |   dfl-traversal(the_graph;L @ [x];s3 @ s2 @ s1)
| . 22. dfl-traversal(the_graph;L @ [x];s1)
| . |   ⇒
| . |   dfl-traversal(the_graph;L @ [x];s2 @ s1)
| . ⊢  dfl-traversal(the_graph;L @ [x];(s3 @ s2) @ s1)
| . | by (RWO
| . |     Thm* ∀as,bs,cs:T List. ((as @ bs) @ cs) = (as @ (bs @ cs))
| . |     {append_assoc}
| . |     0)
| . |    THEN
| . |    BackThruSomeHyp
| . |
| . |\
| . | 1. the_graph : Graph
| . | 2. GraphObject(the_graph)
| . | 3. L : Vertices(the_graph) List
| . | 4. s : traversal(the_graph)
| . | 5. x : Vertices(the_graph)
| . | 6. dfl-traversal(the_graph;L;s)
| . | 7. ∀j:Vertices(the_graph).
| . | |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| . | 8. s1 : traversal(the_graph)
| . | 9. s2 : traversal(the_graph)
| . | 10. s3 : traversal(the_graph)
| . | 11. i : Vertices(the_graph)
| . | 12. j : Vertices(the_graph)
| . | 13. i-the_graph->j
| . | 14. paren(Vertices(the_graph);s2)
| . | 15. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| . | 16. paren(Vertices(the_graph);s3)
| . | 17. x-the_graph->*i
| . | 18. ∀j:Vertices(the_graph).
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| . | |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| . | 19. dfl-traversal(the_graph;L @ [x];s1)
| . | 20. x-the_graph->*j
| . | 21. (∀j:Vertices(the_graph).
| . | |    (inr(j) ∈ s2 @ s1)
| . | |    ⇒
| . | |    ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i)
| . | |   ⇒
| . | |   dfl-traversal(the_graph;L @ [x];s2 @ s1)
| . | |   ⇒
| . | |   dfl-traversal(the_graph;L @ [x];s3 @ s2 @ s1)
| . | 22. dfl-traversal(the_graph;L @ [x];s1)
| . | |   ⇒
| . | |   dfl-traversal(the_graph;L @ [x];s2 @ s1)
| . | ⊢  ∀j:Vertices(the_graph).
| . | |  (inr(j) ∈ s2 @ s1) ⇒ ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i
| . | | by Auto
| . | |    THEN
| . | |    (RWO
| . | |     Thm* ∀x:T, l1,l2:T List.
| . | |          (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| . | |     {member_append}
| . | |     -2)
| . | |    THEN
| . | |    SplitOrHyps
| . | |
| . | |\
| . | | 1. the_graph : Graph
| . | | 2. GraphObject(the_graph)
| . | | 3. L : Vertices(the_graph) List
| . | | 4. s : traversal(the_graph)
| . | | 5. x : Vertices(the_graph)
| . | | 6. dfl-traversal(the_graph;L;s)
| . | | 7. ∀j:Vertices(the_graph).
| . | |    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| . | | 8. s1 : traversal(the_graph)
| . | | 9. s2 : traversal(the_graph)
| . | | 10. s3 : traversal(the_graph)
| . | | 11. i : Vertices(the_graph)
| . | | 12. j : Vertices(the_graph)
| . | | 13. i-the_graph->j
| . | | 14. paren(Vertices(the_graph);s2)
| . | | 15. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| . | | 16. paren(Vertices(the_graph);s3)
| . | | 17. x-the_graph->*i
| . | | 18. ∀j:Vertices(the_graph).
| . | |     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| . | | 19. dfl-traversal(the_graph;L @ [x];s1)
| . | | 20. x-the_graph->*j
| . | | 21. (∀j:Vertices(the_graph).
| . | |      (inr(j) ∈ s2 @ s1)
| . | |      ⇒
| . | |      ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i)
| . | |     ⇒
| . | |     dfl-traversal(the_graph;L @ [x];s2 @ s1)
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| . | |     ⇒
| . | |     dfl-traversal(the_graph;L @ [x];s3 @ s2 @ s1)
| . | | 22. dfl-traversal(the_graph;L @ [x];s1)
| . | |     ⇒
| . | |     dfl-traversal(the_graph;L @ [x];s2 @ s1)
| . | | 23. j1 : Vertices(the_graph)
| . | | 24. (inr(j1) ∈ s2)
| . | | 25. ¬(inl(j1) ∈ s2 @ s1)
| . | | ⊢  j1-the_graph->*i
| . | |   by (Analyze -1)
| . | |      THEN
| . | |      (RWO
| . | |       Thm* ∀x:T, l1,l2:T List.
| . | |            (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| . | |       {member_append}
| . | |       0)
| . | |      THEN
| . | |      OrLeft
| . | |      THEN
| . | |      (BackThru
| . | |       Thm* ∀s':(T+T) List.
| . | |            paren(T;s')
| . | |            ⇒
| . | |            (∀i:T. (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
| . | |       {paren_balance2})
| . | |
| . | .\
| . | . 1. the_graph : Graph
| . | . 2. GraphObject(the_graph)
| . | . 3. L : Vertices(the_graph) List
| . | . 4. s : traversal(the_graph)
| . | . 5. x : Vertices(the_graph)
| . | . 6. dfl-traversal(the_graph;L;s)
| . | . 7. ∀j:Vertices(the_graph).
| . | .    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| . | . 8. s1 : traversal(the_graph)
| . | . 9. s2 : traversal(the_graph)
| . | . 10. s3 : traversal(the_graph)
| . | . 11. i : Vertices(the_graph)
| . | . 12. j : Vertices(the_graph)
| . | . 13. i-the_graph->j
| . | . 14. paren(Vertices(the_graph);s2)
| . | . 15. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| . | . 16. paren(Vertices(the_graph);s3)
| . | . 17. x-the_graph->*i
| . | . 18. ∀j:Vertices(the_graph).
| . | .     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| . | . 19. dfl-traversal(the_graph;L @ [x];s1)
| . | . 20. x-the_graph->*j
| . | . 21. (∀j:Vertices(the_graph).
| . | .      (inr(j) ∈ s2 @ s1)
| . | .      ⇒
| . | .      ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i)
| . | .     ⇒
| . | .     dfl-traversal(the_graph;L @ [x];s2 @ s1)
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| . | .     ⇒
| . | .     dfl-traversal(the_graph;L @ [x];s3 @ s2 @ s1)
| . | . 22. dfl-traversal(the_graph;L @ [x];s1)
| . | .     ⇒
| . | .     dfl-traversal(the_graph;L @ [x];s2 @ s1)
| . | . 23. j1 : Vertices(the_graph)
| . | . 24. (inr(j1) ∈ s1)
| . | . 25. ¬(inl(j1) ∈ s2 @ s1)
| . | . ⊢  j1-the_graph->*i
| . | .   by BackThruSomeHyp THEN (ObviousFrom [-1])
| . |
| . .\
| . . 1. the_graph : Graph
| . . 2. GraphObject(the_graph)
| . . 3. L : Vertices(the_graph) List
| . . 4. s : traversal(the_graph)
| . . 5. x : Vertices(the_graph)
| . . 6. dfl-traversal(the_graph;L;s)
| . . 7. ∀j:Vertices(the_graph).
| . .    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| . . 8. s1 : traversal(the_graph)
| . . 9. s2 : traversal(the_graph)
| . . 10. s3 : traversal(the_graph)
| . . 11. i : Vertices(the_graph)
| . . 12. j : Vertices(the_graph)
| . . 13. i-the_graph->j
| . . 14. paren(Vertices(the_graph);s2)
| . . 15. ∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k
| . . 16. paren(Vertices(the_graph);s3)
| . . 17. x-the_graph->*i
| . . 18. ∀j:Vertices(the_graph).
| . .     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| . . 19. dfl-traversal(the_graph;L @ [x];s1)
| . . 20. x-the_graph->*j
| . . 21. (∀j:Vertices(the_graph).
| . .      (inr(j) ∈ s2 @ s1)
| . .      ⇒
| . .      ¬(inl(j) ∈ s2 @ s1) ⇒ j-the_graph->*i)
| . .     ⇒
| . .     dfl-traversal(the_graph;L @ [x];s2 @ s1)
| . .     ⇒
| . .     dfl-traversal(the_graph;L @ [x];s3 @ s2 @ s1)
| . . 22. dfl-traversal(the_graph;L @ [x];s1)
| . .     ⇒
| . .     dfl-traversal(the_graph;L @ [x];s2 @ s1)
| . . ⊢  dfl-traversal(the_graph;L @ [x];s2 @ s1)  by ThinTrivial
|
|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. L : Vertices(the_graph) List
| 4. s : traversal(the_graph)
| 5. x : Vertices(the_graph)
| 6. dfl-traversal(the_graph;L;s)
| 7. ∀j:Vertices(the_graph).
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| |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| 8. s1 : traversal(the_graph)
| 9. s2 : traversal(the_graph)
| 10. i : Vertices(the_graph)
| 11. ¬(inr(i) ∈ s1)
| 12. ¬(inl(i) ∈ s1)
| 13. paren(Vertices(the_graph);s2)
| 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| 16. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| 17. ∀j:Vertices(the_graph).
| |   i-the_graph->j
| |   ⇒
| |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
| 18. x-the_graph->*i
| |   ⇒
| |   (∀j:Vertices(the_graph).
| |    (inr(j) ∈ [inr(i) / s1])
| |    ⇒
| |    ¬(inl(j) ∈ [inr(i) / s1]) ⇒ j-the_graph->*i)
| |   ⇒
| |   dfl-traversal(the_graph;L @ [x];[inr(i) / s1])
| |   ⇒
| |   dfl-traversal(the_graph;L @ [x];s2 @ [inr(i) / s1])
| 19. x-the_graph->*i
| 20. ∀j:Vertices(the_graph).
| |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| 21. dfl-traversal(the_graph;L @ [x];s1)
| ⊢  dfl-traversal(the_graph;L @ [x];[inl(i) /
| |                                  ((s2 @ [inr(i)]) @ s1)])
| | by ThinTrivial THEN (Analyze -1)
| |
| |\
| | .....antecedent .....
| | 1. the_graph : Graph
| | 2. GraphObject(the_graph)
| | 3. L : Vertices(the_graph) List
| | 4. s : traversal(the_graph)
| | 5. x : Vertices(the_graph)
| | 6. dfl-traversal(the_graph;L;s)
| | 7. ∀j:Vertices(the_graph).
| | |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| | 8. s1 : traversal(the_graph)
| | 9. s2 : traversal(the_graph)
| | 10. i : Vertices(the_graph)
| | 11. ¬(inr(i) ∈ s1)
| | 12. ¬(inl(i) ∈ s1)
| | 13. paren(Vertices(the_graph);s2)
| | 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 16. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| | 17. ∀j:Vertices(the_graph).
| | |   i-the_graph->j
| | |   ⇒
| | |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
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| | 18. x-the_graph->*i
| | 19. ∀j:Vertices(the_graph).
| | |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| | 20. dfl-traversal(the_graph;L @ [x];s1)
| | ⊢  ∀j:Vertices(the_graph).
| | |  (inr(j) ∈ [inr(i) / s1])
| | |  ⇒
| | |  ¬(inl(j) ∈ [inr(i) / s1]) ⇒ j-the_graph->*i
| | | by Auto
| | |    THEN
| | |    (RWO
| | |     Thm* ∀l:T List, a,x:T. (x ∈ [a / l]) ⟺ x = a ∨ (x ∈ l)
| | |     {cons_member}
| | |     -2)
| | |    THEN
| | |    SplitOrHyps
| | |

| | |\
| | | 1. the_graph : Graph
| | | 2. GraphObject(the_graph)
| | | 3. L : Vertices(the_graph) List
| | | 4. s : traversal(the_graph)
| | | 5. x : Vertices(the_graph)
| | | 6. dfl-traversal(the_graph;L;s)
| | | 7. ∀j:Vertices(the_graph).
| | |    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| | | 8. s1 : traversal(the_graph)
| | | 9. s2 : traversal(the_graph)
| | | 10. i : Vertices(the_graph)
| | | 11. ¬(inr(i) ∈ s1)
| | | 12. ¬(inl(i) ∈ s1)
| | | 13. paren(Vertices(the_graph);s2)
| | | 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | | 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | | 16. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| | | 17. ∀j:Vertices(the_graph).
| | |     i-the_graph->j
| | |     ⇒
| | |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
| | | 18. x-the_graph->*i
| | | 19. ∀j:Vertices(the_graph).
| | |     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| | | 20. dfl-traversal(the_graph;L @ [x];s1)
| | | 21. j : Vertices(the_graph)
| | | 22. inr(j) = inr(i)
| | | 23. ¬(inl(j) ∈ [inr(i) / s1])
| | | ⊢  j-the_graph->*i
| | |   by (Subst (j = i) 0) THEN (Try Connect)
| | |      THEN
| | |      (ObviousFrom [-2])
| | |

| | .\
| | . 1. the_graph : Graph
| | . 2. GraphObject(the_graph)
| | . 3. L : Vertices(the_graph) List
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| | . 4. s : traversal(the_graph)
| | . 5. x : Vertices(the_graph)
| | . 6. dfl-traversal(the_graph;L;s)
| | . 7. ∀j:Vertices(the_graph).
| | .    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| | . 8. s1 : traversal(the_graph)
| | . 9. s2 : traversal(the_graph)
| | . 10. i : Vertices(the_graph)
| | . 11. ¬(inr(i) ∈ s1)
| | . 12. ¬(inl(i) ∈ s1)
| | . 13. paren(Vertices(the_graph);s2)
| | . 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | . 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | . 16. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| | . 17. ∀j:Vertices(the_graph).
| | .     i-the_graph->j
| | .     ⇒
| | .     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
| | . 18. x-the_graph->*i
| | . 19. ∀j:Vertices(the_graph).
| | .     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| | . 20. dfl-traversal(the_graph;L @ [x];s1)
| | . 21. j : Vertices(the_graph)
| | . 22. (inr(j) ∈ s1)
| | . 23. ¬(inl(j) ∈ [inr(i) / s1])
| | . ⊢  j-the_graph->*i  by BackThruSomeHyp THEN (ObviousFrom [-1])
| |
| .\
| . 1. the_graph : Graph
| . 2. GraphObject(the_graph)
| . 3. L : Vertices(the_graph) List
| . 4. s : traversal(the_graph)
| . 5. x : Vertices(the_graph)
| . 6. dfl-traversal(the_graph;L;s)
| . 7. ∀j:Vertices(the_graph).
| . |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| . 8. s1 : traversal(the_graph)
| . 9. s2 : traversal(the_graph)
| . 10. i : Vertices(the_graph)
| . 11. ¬(inr(i) ∈ s1)
| . 12. ¬(inl(i) ∈ s1)
| . 13. paren(Vertices(the_graph);s2)
| . 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . 16. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . 17. ∀j:Vertices(the_graph).
| . |   i-the_graph->j
| . |   ⇒
| . |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
| . 18. x-the_graph->*i
| . 19. ∀j:Vertices(the_graph).
| . |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| . 20. dfl-traversal(the_graph;L @ [x];s1)
| . 21. dfl-traversal(the_graph;L @ [x];[inr(i) / s1])
| . |   ⇒
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| . |   dfl-traversal(the_graph;L @ [x];s2 @ [inr(i) / s1])
| . ⊢  dfl-traversal(the_graph;L @ [x];[inl(i) /
| . |                                  ((s2 @ [inr(i)]) @ s1)])
| . | by Analyze -1
| . |
| . |\
| . | .....antecedent .....
| . | 1. the_graph : Graph
| . | 2. GraphObject(the_graph)
| . | 3. L : Vertices(the_graph) List
| . | 4. s : traversal(the_graph)
| . | 5. x : Vertices(the_graph)
| . | 6. dfl-traversal(the_graph;L;s)
| . | 7. ∀j:Vertices(the_graph).
| . | |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| . | 8. s1 : traversal(the_graph)
| . | 9. s2 : traversal(the_graph)
| . | 10. i : Vertices(the_graph)
| . | 11. ¬(inr(i) ∈ s1)
| . | 12. ¬(inl(i) ∈ s1)
| . | 13. paren(Vertices(the_graph);s2)
| . | 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . | 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | 16. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . | 17. ∀j:Vertices(the_graph).
| . | |   i-the_graph->j
| . | |   ⇒
| . | |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
| . | 18. x-the_graph->*i
| . | 19. ∀j:Vertices(the_graph).
| . | |   (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| . | 20. dfl-traversal(the_graph;L @ [x];s1)
| . | ⊢  dfl-traversal(the_graph;L @ [x];[inr(i) / s1])
| . | | by BackThru Thm: dfl_DASH_traversal_DASH_consr
| . | |
| . | 1. the_graph : Graph
| . | 2. GraphObject(the_graph)
| . | 3. L : Vertices(the_graph) List
| . | 4. s : traversal(the_graph)
| . | 5. x : Vertices(the_graph)
| . | 6. dfl-traversal(the_graph;L;s)
| . | 7. ∀j:Vertices(the_graph).
| . |    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| . | 8. s1 : traversal(the_graph)
| . | 9. s2 : traversal(the_graph)
| . | 10. i : Vertices(the_graph)
| . | 11. ¬(inr(i) ∈ s1)
| . | 12. ¬(inl(i) ∈ s1)
| . | 13. paren(Vertices(the_graph);s2)
| . | 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . | 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | 16. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . | 17. ∀j:Vertices(the_graph).
| . |     i-the_graph->j
| . |     ⇒
```

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| . |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
| . | 18. x-the_graph->*i
| . | 19. ∀j:Vertices(the_graph).
| . |     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| . | 20. dfl-traversal(the_graph;L @ [x];s1)
| . | ⊢  L @ [x]-the_graph->*i
| . |   by (Unfold `list-connect` 0) THEN (Unfold `l_exists` 0)
| . |      THEN
| . |      (InstConcl [x])
| . |      THEN
| . |      ObviousConcl
| . |
| . .\
| . . 1. the_graph : Graph
| . . 2. GraphObject(the_graph)
| . . 3. L : Vertices(the_graph) List
| . . 4. s : traversal(the_graph)
| . . 5. x : Vertices(the_graph)
| . . 6. dfl-traversal(the_graph;L;s)
| . . 7. ∀j:Vertices(the_graph).
| . .    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
| . . 8. s1 : traversal(the_graph)
| . . 9. s2 : traversal(the_graph)
| . . 10. i : Vertices(the_graph)
| . . 11. ¬(inr(i) ∈ s1)
| . . 12. ¬(inl(i) ∈ s1)
| . . 13. paren(Vertices(the_graph);s2)
| . . 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . . 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . 16. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . 17. ∀j:Vertices(the_graph).
| . .     i-the_graph->j
| . .     ⇒
| . .     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
| . . 18. x-the_graph->*i
| . . 19. ∀j:Vertices(the_graph).
| . .     (inr(j) ∈ s1) ⇒ ¬(inl(j) ∈ s1) ⇒ j-the_graph->*i
| . . 20. dfl-traversal(the_graph;L @ [x];s1)
| . . 21. dfl-traversal(the_graph;L @ [x];s2 @ [inr(i) / s1])
| . . ⊢  dfl-traversal(the_graph;L @ [x];[inl(i) /
| . .                                    ((s2 @ [inr(i)]) @ s1)])
| . .   by (((BackThru
| . .         Thm* For any graph
| . .              ∀L:V List, i:V, s:Traversal.
| . .              (inr(i) ∈ s)
| . .              ⇒
| . .              dfl-traversal(the_graph;L;s)
| . .              ⇒
| . .              dfl-traversal(the_graph;L;[inl(i) / s])
| . .         {dfl_DASH_traversal_DASH_consl})
| . .        THEN
| . .        (RWO
| . .         Thm* ∀as,bs,cs:T List.
| . .              ((as @ bs) @ cs) = (as @ (bs @ cs))
| . .         {append_assoc}
```

```
| . .         0))
| . .       THEN
| . .       (Reduce 0))
| . .      THEN
| . .      ObviousConcl
|
.\
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. L : Vertices(the_graph) List
. 4. s : traversal(the_graph)
. 5. x : Vertices(the_graph)
. 6. dfl-traversal(the_graph;L;s)
. 7. ∀j:Vertices(the_graph).
. |  (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
. 8. ∃s':traversal(the_graph).
. |  (x-the_graph->*x
. |   ⇒
. |   (∀j:Vertices(the_graph).
. |    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x)
. |   ⇒
. |   dfl-traversal(the_graph;L @ [x];s)
. |   ⇒
. |   dfl-traversal(the_graph;L @ [x];s' @ s))
. |  & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. |  & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. |  & paren(Vertices(the_graph);s')
. |  & (∀j:Vertices(the_graph). (inr(j) ∈ s') ⇒ x-the_graph->*j)
. |  & dfs(the_obj;s;x) = (s' @ s)
. ⊢  dfl-traversal(the_graph;L @ [x];dfs(the_obj;s;x))
. | by ExRepD THEN (AssertBY x-the_graph->*x Connect)
. |    THEN
. |    (Repeat ThinTrivial)
. |    THEN
. |    (SubstFor dfs(the_obj;s;x) 0)
. |    THEN
. |    BackThruSomeHyp
. |
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. L : Vertices(the_graph) List
. 4. s : traversal(the_graph)
. 5. x : Vertices(the_graph)
. 6. dfl-traversal(the_graph;L;s)
. 7. ∀j:Vertices(the_graph).
.    (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*x
. 8. s' : traversal(the_graph)
. 9. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. 10. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. 11. paren(Vertices(the_graph);s')
. 12. ∀j:Vertices(the_graph). (inr(j) ∈ s') ⇒ x-the_graph->*j
. 13. dfs(the_obj;s;x) = (s' @ s)
. 14. x-the_graph->*x
. 15. dfl-traversal(the_graph;L @ [x];s)
.     ⇒
```

```
.     dfl-traversal(the_graph;L @ [x];s' @ s)
. ⊢  dfl-traversal(the_graph;L @ [x];s)
.   by (((Using ['L2',L]
.         (BackThru
.          Thm* For any graph
.               ∀L1,L2:V List, s:Traversal.
.               L1-->*L2
.               ⇒
.               dfl-traversal(the_graph;L2;s)
.               ⇒
.               dfl-traversal(the_graph;L1;s)
.          {dfl_DASH_traversal_DASH_connect}))
.        THEN
.        (BackThru
.         Thm* For any graph
.              ∀A,B,C:V List. A-->*C ∨ B-->*C ⇒ A @ B-->*C
.         {list_DASH_list_DASH_connect_DASH_append2})
.        THEN
.        OrLeft)
.       THEN
.       (BackThru
.        Thm* For any graph ∀A,B:V List. B ≤ A ⇒ A-->*B
.        {list_DASH_list_DASH_connect_DASH_iseg}))
.      THEN
.      ObviousConcl
END
```