```
dfs-properties  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), s:Traversal, i:V.
|  ∃s':Traversal.
|  ((inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil)
|  & (¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
|     ⇒
|     (∃s2:Traversal. s' = ([inl(i)] @ s2 @ [inr(i)])))
|  & l_disjoint(V+V;s';s) & no_repeats(V+V;s')
|  & paren(V;s')
|  & (∀j:V. (inr(j) ∈ s') ⇒ i-the_graph->*j)
|  & dfs(the_obj;s;i) = (s' @ s)
| by Auto THEN (Inst Thm: dfs_member [the_graph;the_obj;s;i])
|    THEN
|    (Inst
|     Thm* For any graph
|          ∀the_obj:GraphObject(the_graph), s:Traversal, i:V.
|          ∃s':Traversal.
|          (∀j:V. (inr(j) ∈ s') ⇒ i-the_graph->*j)
|          & dfs(the_obj;s;i) = (s' @ s)
|     {dfs_DASH_connect}
|     [the_graph;the_obj;s;i])
|    THEN
|    (Inst Thm: dfs_DASH_cases [the_graph;the_obj;s;i])
|    THEN
|    ExRepD
|    THEN
|    (InstConcl [s2])
|
|\
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. s : traversal(the_graph)
| 4. i : Vertices(the_graph)
| 5. s2 : traversal(the_graph)
| 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
| 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| 9. paren(Vertices(the_graph);s2)
| 10. dfs(the_obj;s;i) = (s2 @ s)
| 11. s1 : traversal(the_graph)
| 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
| 13. dfs(the_obj;s;i) = (s1 @ s)
| 14. s' : traversal(the_graph)
| 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
| 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
| |   ⇒
| |   (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
| 17. dfs(the_obj;s;i) = (s' @ s)
| 18. (inr(i) ∈ s) ∨ (inl(i) ∈ s)
| ⊢  s2 = nil  by Subst (s2 = s') 0
| |
| |\
| | .....equality .....
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| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. s : traversal(the_graph)
| | 4. i : Vertices(the_graph)
| | 5. s2 : traversal(the_graph)
| | 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
| | 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 9. paren(Vertices(the_graph);s2)
| | 10. dfs(the_obj;s;i) = (s2 @ s)
| | 11. s1 : traversal(the_graph)
| | 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
| | 13. dfs(the_obj;s;i) = (s1 @ s)
| | 14. s' : traversal(the_graph)
| | 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
| | 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
| | |   ⇒
| | |   (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
| | 17. dfs(the_obj;s;i) = (s' @ s)
| | 18. (inr(i) ∈ s) ∨ (inl(i) ∈ s)
| | ⊢  s2 = s'
| | | by (AssertBY ((s' @ s) = (s2 @ s)) Auto)
| | |    THEN
| | |    (RWO
| | |     Thm* ∀x,y,z:T List. (x @ z) = (y @ z) ⟺ x = y
| | |     {equal_append_front}
| | |     -1)
| | |
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. s : traversal(the_graph)
| | 4. i : Vertices(the_graph)
| | 5. s2 : traversal(the_graph)
| | 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
| | 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 9. paren(Vertices(the_graph);s2)
| | 10. dfs(the_obj;s;i) = (s2 @ s)
| | 11. s1 : traversal(the_graph)
| | 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
| | 13. dfs(the_obj;s;i) = (s1 @ s)
| | 14. s' : traversal(the_graph)
| | 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
| | 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
| |     ⇒
| |     (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
| | 17. dfs(the_obj;s;i) = (s' @ s)
| | 18. (inr(i) ∈ s) ∨ (inl(i) ∈ s)
| | 19. s' = s2
| | ⊢  s2 = s'  by HypSubst -1 0
| |
| .\
| . 1. the_graph : Graph
| . 2. the_obj : GraphObject(the_graph)
| . 3. s : traversal(the_graph)
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| . 4. i : Vertices(the_graph)
| . 5. s2 : traversal(the_graph)
| . 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
| . 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . 9. paren(Vertices(the_graph);s2)
| . 10. dfs(the_obj;s;i) = (s2 @ s)
| . 11. s1 : traversal(the_graph)
| . 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
| . 13. dfs(the_obj;s;i) = (s1 @ s)
| . 14. s' : traversal(the_graph)
| . 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
| . 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
| .     ⇒
| .     (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
| . 17. dfs(the_obj;s;i) = (s' @ s)
| . 18. (inr(i) ∈ s) ∨ (inl(i) ∈ s)
| . ⊢  s' = nil  by BackThruSomeHyp THEN (ObviousFrom [-1])
|
|\
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. s : traversal(the_graph)
| 4. i : Vertices(the_graph)
| 5. s2 : traversal(the_graph)
| 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
| 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| 9. paren(Vertices(the_graph);s2)
| 10. dfs(the_obj;s;i) = (s2 @ s)
| 11. s1 : traversal(the_graph)
| 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
| 13. dfs(the_obj;s;i) = (s1 @ s)
| 14. s' : traversal(the_graph)
| 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
| 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
| |   ⇒
| |   (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
| 17. dfs(the_obj;s;i) = (s' @ s)
| 18. ¬(inr(i) ∈ s)
| 19. ¬(inl(i) ∈ s)
| ⊢  ∃s2@0:traversal(the_graph). s2 = ([inl(i)] @ s2@0 @ [inr(i)])
| | by Subst (s2 = s') 0
| |
| |\
| | .....equality .....
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. s : traversal(the_graph)
| | 4. i : Vertices(the_graph)
| | 5. s2 : traversal(the_graph)
| | 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
| | 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 9. paren(Vertices(the_graph);s2)
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| | 10. dfs(the_obj;s;i) = (s2 @ s)
| | 11. s1 : traversal(the_graph)
| | 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
| | 13. dfs(the_obj;s;i) = (s1 @ s)
| | 14. s' : traversal(the_graph)
| | 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
| | 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
| | |   ⇒
| | |   (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
| | 17. dfs(the_obj;s;i) = (s' @ s)
| | 18. ¬(inr(i) ∈ s)
| | 19. ¬(inl(i) ∈ s)
| | ⊢  s2 = s'
| | | by (AssertBY ((s' @ s) = (s2 @ s)) Auto)
| | |    THEN
| | |    (RWO
| | |     Thm* ∀x,y,z:T List. (x @ z) = (y @ z) ⟺ x = y
| | |     {equal_append_front}
| | |     -1)
| | |
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. s : traversal(the_graph)
| | 4. i : Vertices(the_graph)
| | 5. s2 : traversal(the_graph)
| | 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
| | 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 9. paren(Vertices(the_graph);s2)
| | 10. dfs(the_obj;s;i) = (s2 @ s)
| | 11. s1 : traversal(the_graph)
| | 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
| | 13. dfs(the_obj;s;i) = (s1 @ s)
| | 14. s' : traversal(the_graph)
| | 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
| | 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
| |     ⇒
| |     (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
| | 17. dfs(the_obj;s;i) = (s' @ s)
| | 18. ¬(inr(i) ∈ s)
| | 19. ¬(inl(i) ∈ s)
| | 20. s' = s2
| | ⊢  s2 = s'  by HypSubst -1 0
| |
| .\
| . 1. the_graph : Graph
| . 2. the_obj : GraphObject(the_graph)
| . 3. s : traversal(the_graph)
| . 4. i : Vertices(the_graph)
| . 5. s2 : traversal(the_graph)
| . 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
| . 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . 9. paren(Vertices(the_graph);s2)
| . 10. dfs(the_obj;s;i) = (s2 @ s)
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| . 11. s1 : traversal(the_graph)
| . 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
| . 13. dfs(the_obj;s;i) = (s1 @ s)
| . 14. s' : traversal(the_graph)
| . 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
| . 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
| .     ⇒
| .     (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
| . 17. dfs(the_obj;s;i) = (s' @ s)
| . 18. ¬(inr(i) ∈ s)
| . 19. ¬(inl(i) ∈ s)
| . ⊢  ∃s2@0:traversal(the_graph). s' = ([inl(i)] @ s2@0 @ [inr(i)])
| .   by BackThruSomeHyp
|
.\
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. s : traversal(the_graph)
. 4. i : Vertices(the_graph)
. 5. s2 : traversal(the_graph)
. 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
. 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. 9. paren(Vertices(the_graph);s2)
. 10. dfs(the_obj;s;i) = (s2 @ s)
. 11. s1 : traversal(the_graph)
. 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
. 13. dfs(the_obj;s;i) = (s1 @ s)
. 14. s' : traversal(the_graph)
. 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
. 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
. |   ⇒
. |   (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
. 17. dfs(the_obj;s;i) = (s' @ s)
. 18. j : Vertices(the_graph)
. 19. (inr(j) ∈ s2)
. ⊢  i-the_graph->*j  by BackThruSomeHyp
. |
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. s : traversal(the_graph)
. 4. i : Vertices(the_graph)
. 5. s2 : traversal(the_graph)
. 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
. 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. 9. paren(Vertices(the_graph);s2)
. 10. dfs(the_obj;s;i) = (s2 @ s)
. 11. s1 : traversal(the_graph)
. 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
. 13. dfs(the_obj;s;i) = (s1 @ s)
. 14. s' : traversal(the_graph)
. 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
. 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
. |   ⇒
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. |   (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
. 17. dfs(the_obj;s;i) = (s' @ s)
. 18. j : Vertices(the_graph)
. 19. (inr(j) ∈ s2)
. ⊢  (inr(j) ∈ s1)  by Subst (s1 = s2) 0
. |
. .....equality .....
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. s : traversal(the_graph)
. 4. i : Vertices(the_graph)
. 5. s2 : traversal(the_graph)
. 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
. 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. 9. paren(Vertices(the_graph);s2)
. 10. dfs(the_obj;s;i) = (s2 @ s)
. 11. s1 : traversal(the_graph)
. 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
. 13. dfs(the_obj;s;i) = (s1 @ s)
. 14. s' : traversal(the_graph)
. 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
. 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
. |   ⇒
. |   (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
. 17. dfs(the_obj;s;i) = (s' @ s)
. 18. j : Vertices(the_graph)
. 19. (inr(j) ∈ s2)
. ⊢  s1 = s2
. | by Inst
. |    Thm* ∀x1,z,x2,x3:T List.
. |         ||x1|| = ||z|| ∈ ℤ
. |         ⇒
. |         (x1 @ x2) = (z @ x3) ⇒ x1 = z & x2 = x3
. |    {equal_appends_eq}
. |    [Vertices(the_graph)+Vertices(the_graph);s1;s2;s;s]
. |
. |\
. | .....antecedent .....
. | 1. the_graph : Graph
. | 2. the_obj : GraphObject(the_graph)
. | 3. s : traversal(the_graph)
. | 4. i : Vertices(the_graph)
. | 5. s2 : traversal(the_graph)
. | 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. | 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
. | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. | 9. paren(Vertices(the_graph);s2)
. | 10. dfs(the_obj;s;i) = (s2 @ s)
. | 11. s1 : traversal(the_graph)
. | 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
. | 13. dfs(the_obj;s;i) = (s1 @ s)
. | 14. s' : traversal(the_graph)
. | 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
. | 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
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. |     ⇒
. |     (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
. | 17. dfs(the_obj;s;i) = (s' @ s)
. | 18. j : Vertices(the_graph)
. | 19. (inr(j) ∈ s2)
. | ⊢  ||s1|| = ||s2||
. |   by (ApFunToHypEquands 'z' ||z|| ℤ 10)
. |      THEN
. |      (RWO
. |       Thm* ∀as,bs:T List. ||as @ bs|| = ||as||+||bs|| ∈ ℤ
. |       {length_append}
. |       -1)
. |      THEN
. |      (ApFunToHypEquands 'z' ||z|| ℤ 13)
. |      THEN
. |      (RWO
. |       Thm* ∀as,bs:T List. ||as @ bs|| = ||as||+||bs|| ∈ ℤ
. |       {length_append}
. |       -1)
. |
. .\
. . .....antecedent .....
. . 1. the_graph : Graph
. . 2. the_obj : GraphObject(the_graph)
. . 3. s : traversal(the_graph)
. . 4. i : Vertices(the_graph)
. . 5. s2 : traversal(the_graph)
. . 6. (inl(i) ∈ s2) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. . 7. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
. . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . 9. paren(Vertices(the_graph);s2)
. . 10. dfs(the_obj;s;i) = (s2 @ s)
. . 11. s1 : traversal(the_graph)
. . 12. ∀j:Vertices(the_graph). (inr(j) ∈ s1) ⇒ i-the_graph->*j
. . 13. dfs(the_obj;s;i) = (s1 @ s)
. . 14. s' : traversal(the_graph)
. . 15. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
. . 16. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
. .     ⇒
. .     (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
. . 17. dfs(the_obj;s;i) = (s' @ s)
. . 18. j : Vertices(the_graph)
. . 19. (inr(j) ∈ s2)
. . ⊢  (s1 @ s) = (s2 @ s)  by Fold 'traversal' 0
END
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