```
dfs_accum_member  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), l:V List, s:Traversal.
|  ∃s':Traversal.
|  (∀i:V. (i ∈ l) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
|  & l_disjoint(V+V;s';s) & no_repeats(V+V;s')
|  & paren(V;s')
|  & list_accum(s',j.dfs(the_obj;s';j);s;l) = (s' @ s)
| by InductionOnList THEN (Reduce 0)
|
|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. Vertices(the_graph) List
| 4. s : traversal(the_graph)
| ⊢  ∃s':traversal(the_graph).
| |  (∀i:Vertices(the_graph).
| |   (i ∈ nil) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
| |  & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
| |  & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
| |  & paren(Vertices(the_graph);s')
| |  & s = (s' @ s)  by (InstConcl [nil]) THEN (Reduce 0)
| |
| |\
| | 1. the_graph : Graph
| | 2. GraphObject(the_graph)
| | 3. Vertices(the_graph) List
| | 4. s : traversal(the_graph)
| | 5. i : Vertices(the_graph)
| | 6. (i ∈ nil)
| | ⊢  (inl(i) ∈ nil) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)  by Obvious
| |
| |\
| | 1. the_graph : Graph
| | 2. GraphObject(the_graph)
| | 3. Vertices(the_graph) List
| | 4. s : traversal(the_graph)
| | ⊢  l_disjoint(Vertices(the_graph)+Vertices(the_graph);nil;s)
| |    by Obvious
| |
| |\
| | 1. the_graph : Graph
| | 2. GraphObject(the_graph)
| | 3. Vertices(the_graph) List
| | 4. traversal(the_graph)
| | ⊢  no_repeats(Vertices(the_graph)+Vertices(the_graph);nil)
| |    by Obvious
| |
| .\
| . 1. the_graph : Graph
| . 2. GraphObject(the_graph)
| . 3. Vertices(the_graph) List
| . 4. traversal(the_graph)
| . ⊢  paren(Vertices(the_graph);nil)  by Obvious
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|
.\
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. Vertices(the_graph) List
. 4. u : Vertices(the_graph)
. 5. v : Vertices(the_graph) List
. 6. ∀s:traversal(the_graph).
. |  ∃s':traversal(the_graph).
. |  (∀i:Vertices(the_graph).
. |   (i ∈ v) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
. |  & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. |  & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. |  & paren(Vertices(the_graph);s')
. |  & list_accum(s',j.dfs(the_obj;s';j);s;v) = (s' @ s)
. 7. s : traversal(the_graph)
. ⊢  ∃s':traversal(the_graph).
. |  (∀i:Vertices(the_graph).
. |   (i ∈ [u / v]) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
. |  & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. |  & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. |  & paren(Vertices(the_graph);s')
. |  & list_accum(s',j.dfs(the_obj;s';j);dfs(the_obj;s;u);v)
. |    =
. |    (s' @ s)
. | by (Inst Thm: dfs_member [the_graph;the_obj;s;u])
. |    THEN
. |    (InstHyp [dfs(the_obj;s;u)] -3)
. |    THEN
. |    ExRepD
. |    THEN
. |    (HypSubst -1 0)
. |    THEN
. |    (HypSubst -7 0)
. |    THEN
. |    (InstConcl [s' @ s1])
. |
. |\
. | 1. the_graph : Graph
. | 2. the_obj : GraphObject(the_graph)
. | 3. Vertices(the_graph) List
. | 4. u : Vertices(the_graph)
. | 5. v : Vertices(the_graph) List
. | 6. ∀s:traversal(the_graph).
. | |  ∃s':traversal(the_graph).
. | |  (∀i:Vertices(the_graph).
. | |   (i ∈ v) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
. | |  & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. | |  & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | |  & paren(Vertices(the_graph);s')
. | |  & list_accum(s',j.dfs(the_obj;s';j);s;v) = (s' @ s)
. | 7. s : traversal(the_graph)
. | 8. s1 : traversal(the_graph)
. | 9. (inl(u) ∈ s1) ∨ (inl(u) ∈ s) ∨ (inr(u) ∈ s)
. | 10. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s1;s)
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. | 11. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
. | 12. paren(Vertices(the_graph);s1)
. | 13. dfs(the_obj;s;u) = (s1 @ s)
. | 14. s' : traversal(the_graph)
. | 15. ∀i:Vertices(the_graph).
. | |    (i ∈ v)
. | |    ⇒
. | |    (inl(i) ∈ s')
. | |    ∨ (inl(i) ∈ dfs(the_obj;s;u))
. | |    ∨ (inr(i) ∈ dfs(the_obj;s;u))
. | 16. l_disjoint(Vertices(the_graph)+
. | |    Vertices(the_graph);s';dfs(the_obj;s;u))
. | 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | 18. paren(Vertices(the_graph);s')
. | 19. list_accum(s',j.dfs(the_obj;s';j);dfs(the_obj;s;u);v)
. | |    =
. | |    (s' @ dfs(the_obj;s;u))
. | 20. i : Vertices(the_graph)
. | 21. (i ∈ [u / v])
. | ⊢  (inl(i) ∈ s' @ s1) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. | | by ((RWO
. | |      Thm* ∀x:T, l1,l2:T List.
. | |           (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
. | |      {member_append}
. | |      0)
. | |     THEN
. | |     (RWO
. | |      Thm* ∀l:T List, a,x:T. (x ∈ [a / l]) ⟺ x = a ∨ (x ∈ l)
. | |      {cons_member}
. | |      -1))
. | |    THEN
. | |    (Analyze -1)
. | |
. | |\
. | | 1. the_graph : Graph
. | | 2. the_obj : GraphObject(the_graph)
. | | 3. Vertices(the_graph) List
. | | 4. u : Vertices(the_graph)
. | | 5. v : Vertices(the_graph) List
. | | 6. ∀s:traversal(the_graph).
. | |    ∃s':traversal(the_graph).
. | |    (∀i:Vertices(the_graph).
. | |     (i ∈ v) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
. | |    & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. | |    & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | |    & paren(Vertices(the_graph);s')
. | |    & list_accum(s',j.dfs(the_obj;s';j);s;v) = (s' @ s)
. | | 7. s : traversal(the_graph)
. | | 8. s1 : traversal(the_graph)
. | | 9. (inl(u) ∈ s1) ∨ (inl(u) ∈ s) ∨ (inr(u) ∈ s)
. | | 10. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s1;s)
. | | 11. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
. | | 12. paren(Vertices(the_graph);s1)
. | | 13. dfs(the_obj;s;u) = (s1 @ s)
. | | 14. s' : traversal(the_graph)
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. | | 15. ∀i:Vertices(the_graph).
. | |     (i ∈ v)
. | |     ⇒
. | |     (inl(i) ∈ s')
. | |     ∨ (inl(i) ∈ dfs(the_obj;s;u))
. | |     ∨ (inr(i) ∈ dfs(the_obj;s;u))
. | | 16. l_disjoint(Vertices(the_graph)+
. | |     Vertices(the_graph);s';dfs(the_obj;s;u))
. | | 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | | 18. paren(Vertices(the_graph);s')
. | | 19. list_accum(s',j.dfs(the_obj;s';j);dfs(the_obj;s;u);v)
. | |     =
. | |     (s' @ dfs(the_obj;s;u))
. | | 20. i : Vertices(the_graph)
. | | 21. i = u
. | | ⊢  (inl(i) ∈ s') ∨ (inl(i) ∈ s1) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. | |   by ObviousFrom [-1;8]
. | |
. | .\
. | . 1. the_graph : Graph
. | . 2. the_obj : GraphObject(the_graph)
. | . 3. Vertices(the_graph) List
. | . 4. u : Vertices(the_graph)
. | . 5. v : Vertices(the_graph) List
. | . 6. ∀s:traversal(the_graph).
. | . |  ∃s':traversal(the_graph).
. | . |  (∀i:Vertices(the_graph).
. | . |   (i ∈ v) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
. | . |  & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. | . |  & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | . |  & paren(Vertices(the_graph);s')
. | . |  & list_accum(s',j.dfs(the_obj;s';j);s;v) = (s' @ s)
. | . 7. s : traversal(the_graph)
. | . 8. s1 : traversal(the_graph)
. | . 9. (inl(u) ∈ s1) ∨ (inl(u) ∈ s) ∨ (inr(u) ∈ s)
. | . 10. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s1;s)
. | . 11. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
. | . 12. paren(Vertices(the_graph);s1)
. | . 13. dfs(the_obj;s;u) = (s1 @ s)
. | . 14. s' : traversal(the_graph)
. | . 15. ∀i:Vertices(the_graph).
. | . |   (i ∈ v)
. | . |   ⇒
. | . |   (inl(i) ∈ s')
. | . |   ∨ (inl(i) ∈ dfs(the_obj;s;u))
. | . |   ∨ (inr(i) ∈ dfs(the_obj;s;u))
. | . 16. l_disjoint(Vertices(the_graph)+
. | . |   Vertices(the_graph);s';dfs(the_obj;s;u))
. | . 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | . 18. paren(Vertices(the_graph);s')
. | . 19. list_accum(s',j.dfs(the_obj;s';j);dfs(the_obj;s;u);v)
. | . |   =
. | . |   (s' @ dfs(the_obj;s;u))
. | . 20. i : Vertices(the_graph)
. | . 21. (i ∈ v)
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. | . ⊢  (inl(i) ∈ s') ∨ (inl(i) ∈ s1) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. | . | by (InstHyp [i] -7) THEN (SubstFor dfs(the_obj;s;u) -1)
. | . |    THEN
. | . |    (RWO
. | . |     Thm* ∀x:T, l1,l2:T List.
. | . |          (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
. | . |     {member_append}
. | . |     -1)
. | . |
. | . 1. the_graph : Graph
. | . 2. the_obj : GraphObject(the_graph)
. | . 3. Vertices(the_graph) List
. | . 4. u : Vertices(the_graph)
. | . 5. v : Vertices(the_graph) List
. | . 6. ∀s:traversal(the_graph).
. | . |  ∃s':traversal(the_graph).
. | . |  (∀i:Vertices(the_graph).
. | . |   (i ∈ v) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
. | . |  & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. | . |  & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | . |  & paren(Vertices(the_graph);s')
. | . |  & list_accum(s',j.dfs(the_obj;s';j);s;v) = (s' @ s)
. | . 7. s : traversal(the_graph)
. | . 8. s1 : traversal(the_graph)
. | . 9. (inl(u) ∈ s1) ∨ (inl(u) ∈ s) ∨ (inr(u) ∈ s)
. | . 10. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s1;s)
. | . 11. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
. | . 12. paren(Vertices(the_graph);s1)
. | . 13. dfs(the_obj;s;u) = (s1 @ s)
. | . 14. s' : traversal(the_graph)
. | . 15. ∀i:Vertices(the_graph).
. | . |   (i ∈ v)
. | . |   ⇒
. | . |   (inl(i) ∈ s')
. | . |   ∨ (inl(i) ∈ dfs(the_obj;s;u))
. | . |   ∨ (inr(i) ∈ dfs(the_obj;s;u))
. | . 16. l_disjoint(Vertices(the_graph)+
. | . |   Vertices(the_graph);s';dfs(the_obj;s;u))
. | . 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | . 18. paren(Vertices(the_graph);s')
. | . 19. list_accum(s',j.dfs(the_obj;s';j);dfs(the_obj;s;u);v)
. | . |   =
. | . |   (s' @ dfs(the_obj;s;u))
. | . 20. i : Vertices(the_graph)
. | . 21. (i ∈ v)
. | . 22. (inl(i) ∈ s')
. | . |   ∨ (inl(i) ∈ s1) ∨ (inl(i) ∈ s)
. | . |   ∨ (inr(i) ∈ s1)
. | . |   ∨ (inr(i) ∈ s)
. | . ⊢  (inl(i) ∈ s') ∨ (inl(i) ∈ s1) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. | . | by AssertBY ((inr(i) ∈ s1) ⇒ (inl(i) ∈ s1))
. | . |    (Auto
. | . |     THEN
. | . |     (Inst
. | . |      Thm* ∀s':(T+T) List.
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. | . |           paren(T;s')
. | . |           ⇒
. | . |           (∀i:T. (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
. | . |      {paren_balance2}
. | . |      [Vertices(the_graph);s1;i]))
. | . |
. | . 1. the_graph : Graph
. | . 2. the_obj : GraphObject(the_graph)
. | . 3. Vertices(the_graph) List
. | . 4. u : Vertices(the_graph)
. | . 5. v : Vertices(the_graph) List
. | . 6. ∀s:traversal(the_graph).
. | .    ∃s':traversal(the_graph).
. | .    (∀i:Vertices(the_graph).
. | .     (i ∈ v) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
. | .    & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. | .    & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | .    & paren(Vertices(the_graph);s')
. | .    & list_accum(s',j.dfs(the_obj;s';j);s;v) = (s' @ s)
. | . 7. s : traversal(the_graph)
. | . 8. s1 : traversal(the_graph)
. | . 9. (inl(u) ∈ s1) ∨ (inl(u) ∈ s) ∨ (inr(u) ∈ s)
. | . 10. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s1;s)
. | . 11. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
. | . 12. paren(Vertices(the_graph);s1)
. | . 13. dfs(the_obj;s;u) = (s1 @ s)
. | . 14. s' : traversal(the_graph)
. | . 15. ∀i:Vertices(the_graph).
. | .     (i ∈ v)
. | .     ⇒
. | .     (inl(i) ∈ s')
. | .     ∨ (inl(i) ∈ dfs(the_obj;s;u))
. | .     ∨ (inr(i) ∈ dfs(the_obj;s;u))
. | . 16. l_disjoint(Vertices(the_graph)+
. | .     Vertices(the_graph);s';dfs(the_obj;s;u))
. | . 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | . 18. paren(Vertices(the_graph);s')
. | . 19. list_accum(s',j.dfs(the_obj;s';j);dfs(the_obj;s;u);v)
. | .     =
. | .     (s' @ dfs(the_obj;s;u))
. | . 20. i : Vertices(the_graph)
. | . 21. (i ∈ v)
. | . 22. (inl(i) ∈ s')
. | .     ∨ (inl(i) ∈ s1) ∨ (inl(i) ∈ s)
. | .     ∨ (inr(i) ∈ s1)
. | .     ∨ (inr(i) ∈ s)
. | . 23. (inr(i) ∈ s1) ⇒ (inl(i) ∈ s1)
. | . ⊢  (inl(i) ∈ s') ∨ (inl(i) ∈ s1) ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. | .   by ObviousFrom [-1;-2]
. |
. |\
. | 1. the_graph : Graph
. | 2. the_obj : GraphObject(the_graph)
. | 3. Vertices(the_graph) List
. | 4. u : Vertices(the_graph)
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. | 5. v : Vertices(the_graph) List
. | 6. ∀s:traversal(the_graph).
. |    ∃s':traversal(the_graph).
. |    (∀i:Vertices(the_graph).
. |     (i ∈ v) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
. |    & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. |    & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. |    & paren(Vertices(the_graph);s')
. |    & list_accum(s',j.dfs(the_obj;s';j);s;v) = (s' @ s)
. | 7. s : traversal(the_graph)
. | 8. s1 : traversal(the_graph)
. | 9. (inl(u) ∈ s1) ∨ (inl(u) ∈ s) ∨ (inr(u) ∈ s)
. | 10. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s1;s)
. | 11. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
. | 12. paren(Vertices(the_graph);s1)
. | 13. dfs(the_obj;s;u) = (s1 @ s)
. | 14. s' : traversal(the_graph)
. | 15. ∀i:Vertices(the_graph).
. |     (i ∈ v)
. |     ⇒
. |     (inl(i) ∈ s')
. |     ∨ (inl(i) ∈ dfs(the_obj;s;u))
. |     ∨ (inr(i) ∈ dfs(the_obj;s;u))
. | 16. l_disjoint(Vertices(the_graph)+
. |     Vertices(the_graph);s';dfs(the_obj;s;u))
. | 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | 18. paren(Vertices(the_graph);s')
. | 19. list_accum(s',j.dfs(the_obj;s';j);dfs(the_obj;s;u);v)
. |     =
. |     (s' @ dfs(the_obj;s;u))
. | ⊢  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s' @ s1;s)
. |   by (RWO
. |       Thm* ∀a,b,c:T List.
. |            l_disjoint(T;a @ b;c)
. |            ⟺
. |            l_disjoint(T;a;c) & l_disjoint(T;b;c)
. |       {l_disjoint_append}
. |       0)
. |      THEN
. |      (SubstFor dfs(the_obj;s;u) -4)
. |      THEN
. |      (RWO
. |       Thm* ∀a,b,c:T List.
. |            l_disjoint(T;c;a @ b)
. |            ⟺
. |            l_disjoint(T;c;a) & l_disjoint(T;c;b)
. |       {l_disjoint_append2}
. |       -4)
. |
. |\
. | 1. the_graph : Graph
. | 2. the_obj : GraphObject(the_graph)
. | 3. Vertices(the_graph) List
. | 4. u : Vertices(the_graph)
. | 5. v : Vertices(the_graph) List
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. | 6. ∀s:traversal(the_graph).
. |    ∃s':traversal(the_graph).
. |    (∀i:Vertices(the_graph).
. |     (i ∈ v) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
. |    & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. |    & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. |    & paren(Vertices(the_graph);s')
. |    & list_accum(s',j.dfs(the_obj;s';j);s;v) = (s' @ s)
. | 7. s : traversal(the_graph)
. | 8. s1 : traversal(the_graph)
. | 9. (inl(u) ∈ s1) ∨ (inl(u) ∈ s) ∨ (inr(u) ∈ s)
. | 10. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s1;s)
. | 11. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
. | 12. paren(Vertices(the_graph);s1)
. | 13. dfs(the_obj;s;u) = (s1 @ s)
. | 14. s' : traversal(the_graph)
. | 15. ∀i:Vertices(the_graph).
. |     (i ∈ v)
. |     ⇒
. |     (inl(i) ∈ s')
. |     ∨ (inl(i) ∈ dfs(the_obj;s;u))
. |     ∨ (inr(i) ∈ dfs(the_obj;s;u))
. | 16. l_disjoint(Vertices(the_graph)+
. |     Vertices(the_graph);s';dfs(the_obj;s;u))
. | 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | 18. paren(Vertices(the_graph);s')
. | 19. list_accum(s',j.dfs(the_obj;s';j);dfs(the_obj;s;u);v)
. |     =
. |     (s' @ dfs(the_obj;s;u))
. | ⊢  no_repeats(Vertices(the_graph)+Vertices(the_graph);s' @ s1)
. |   by (RWO
. |       Thm* ∀l1,l2:T List.
. |            no_repeats(T;l1 @ l2)
. |            ⟺
. |            l_disjoint(T;l1;l2)
. |            & no_repeats(T;l1)
. |            & no_repeats(T;l2)
. |       {no_repeats_append_iff}
. |       0)
. |      THEN
. |      (SubstFor dfs(the_obj;s;u) -4)
. |      THEN
. |      (RWO
. |       Thm* ∀a,b,c:T List.
. |            l_disjoint(T;c;a @ b)
. |            ⟺
. |            l_disjoint(T;c;a) & l_disjoint(T;c;b)
. |       {l_disjoint_append2}
. |       -4)
. |
. |\
. | 1. the_graph : Graph
. | 2. the_obj : GraphObject(the_graph)
. | 3. Vertices(the_graph) List
. | 4. u : Vertices(the_graph)
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. | 5. v : Vertices(the_graph) List
. | 6. ∀s:traversal(the_graph).
. |    ∃s':traversal(the_graph).
. |    (∀i:Vertices(the_graph).
. |     (i ∈ v) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
. |    & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. |    & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. |    & paren(Vertices(the_graph);s')
. |    & list_accum(s',j.dfs(the_obj;s';j);s;v) = (s' @ s)
. | 7. s : traversal(the_graph)
. | 8. s1 : traversal(the_graph)
. | 9. (inl(u) ∈ s1) ∨ (inl(u) ∈ s) ∨ (inr(u) ∈ s)
. | 10. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s1;s)
. | 11. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
. | 12. paren(Vertices(the_graph);s1)
. | 13. dfs(the_obj;s;u) = (s1 @ s)
. | 14. s' : traversal(the_graph)
. | 15. ∀i:Vertices(the_graph).
. |     (i ∈ v)
. |     ⇒
. |     (inl(i) ∈ s')
. |     ∨ (inl(i) ∈ dfs(the_obj;s;u))
. |     ∨ (inr(i) ∈ dfs(the_obj;s;u))
. | 16. l_disjoint(Vertices(the_graph)+
. |     Vertices(the_graph);s';dfs(the_obj;s;u))
. | 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | 18. paren(Vertices(the_graph);s')
. | 19. list_accum(s',j.dfs(the_obj;s';j);dfs(the_obj;s;u);v)
. |     =
. |     (s' @ dfs(the_obj;s;u))
. | ⊢  paren(Vertices(the_graph);s' @ s1)
. |   by BackThru
. |      Thm* ∀s',s3:(T+T) List.
. |           paren(T;s') ⇒ paren(T;s3) ⇒ paren(T;s3 @ s')
. |      {append_paren}
. |
. .\
. . 1. the_graph : Graph
. . 2. the_obj : GraphObject(the_graph)
. . 3. Vertices(the_graph) List
. . 4. u : Vertices(the_graph)
. . 5. v : Vertices(the_graph) List
. . 6. ∀s:traversal(the_graph).
. .    ∃s':traversal(the_graph).
. .    (∀i:Vertices(the_graph).
. .     (i ∈ v) ⇒ (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s))
. .    & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. .    & no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. .    & paren(Vertices(the_graph);s')
. .    & list_accum(s',j.dfs(the_obj;s';j);s;v) = (s' @ s)
. . 7. s : traversal(the_graph)
. . 8. s1 : traversal(the_graph)
. . 9. (inl(u) ∈ s1) ∨ (inl(u) ∈ s) ∨ (inr(u) ∈ s)
. . 10. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s1;s)
. . 11. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
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. . 12. paren(Vertices(the_graph);s1)
. . 13. dfs(the_obj;s;u) = (s1 @ s)
. . 14. s' : traversal(the_graph)
. . 15. ∀i:Vertices(the_graph).
. .     (i ∈ v)
. .     ⇒
. .     (inl(i) ∈ s')

. .     ∨ (inl(i) ∈ dfs(the_obj;s;u))
. .     ∨ (inr(i) ∈ dfs(the_obj;s;u))
. . 16. l_disjoint(Vertices(the_graph)+
. .     Vertices(the_graph);s';dfs(the_obj;s;u))
. . 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. . 18. paren(Vertices(the_graph);s')
. . 19. list_accum(s',j.dfs(the_obj;s';j);dfs(the_obj;s;u);v)
. .     =
. .     (s' @ dfs(the_obj;s;u))
. . ⊢  (s' @ s1 @ s) = ((s' @ s1) @ s)
. .   by RWO
. .      Thm* ∀as,bs,cs:T List. ((as @ bs) @ cs) = (as @ (bs @ cs))
. .      {append_assoc}
. .      0
END
```