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dfs_induction3  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), P:(V→Traversal→Traversal→Prop)
|  , s:Traversal, i:V.
|  (∀s1,s2:Traversal, i:V.
|   P(i,s1,s2) ⇒ l_disjoint(V+V;s2;s1) & no_repeats(V+V;s2))
|  ⇒
|  (∀s:Traversal, i:V.
|   member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil))
|  ⇒
|  (∀s1,s2,s3:Traversal, i,j:V.
|   i-the_graph->j
|   ⇒
|   P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2))
|  ⇒
|  (∀s1,s2:Traversal, i:V.
|   ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
|   ⇒
|   (∀j:V.
|    i-the_graph->j
|    ⇒
|    ¬j = i ⇒ (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
|   ⇒
|   P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])]))
|  ⇒
|  (∃s':Traversal. P(i,s,s') & dfs(the_obj;s;i) = (s' @ s))
| by Auto THEN (Inst Thm: dfs_DASH_measure [the_graph;the_obj])
|    THEN
|    ExRepDHyps
|    THEN
|    (Assert
|     (∀d:ℕ, s:traversal(the_graph), i:Vertices(the_graph).
|      M(s)≤d
|      ⇒
|      (∃s':traversal(the_graph).
|       M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s))))
|
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| .....assertion .....
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. P :
| |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
| |  Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |  P(i,s1,s2)
| |  ⇒
| |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| 7. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |  P(i,s1,s2)
| |  ⇒
| |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
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| 8. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |  member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| 9. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |  i-the_graph->j
| |  ⇒
| |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 10. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |   ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| |   ⇒
| |   (∀j:Vertices(the_graph).
| |    i-the_graph->j
| |    ⇒
| |    ¬j = i
| |    ⇒
| |    (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| |   ⇒
| |   P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 11. M : traversal(the_graph)→ℕ
| 12. ∀i:Vertices(the_graph), s:traversal(the_graph).
| |   M([inl(i) / s])≤M(s)
| 13. ∀i:Vertices(the_graph), s:traversal(the_graph).
| |   ¬member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ M([inr(i) / s])<M(s)
| ⊢  ∀d:ℕ, s:traversal(the_graph), i:Vertices(the_graph).
| |  M(s)≤d
| |  ⇒
| |  (∃s':traversal(the_graph).
| |   M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s))
| | by (Thin 5) THEN (Thin 4) THEN CompleteInductionOnNat
| |    THEN
| |    (RecUnfold `dfs` 0)
| |    THEN
| |    SplitOnConclITE
| |    THEN
| |    (Try
| |     ((InstConcl [nil]) THEN (Reduce 0) THEN (Complete EasyHyp)))
| |
| .....falsecase .....
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. P :
| |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
| |  Prop
| 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |  P(i,s1,s2)
| |  ⇒
| |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |  P(i,s1,s2)
| |  ⇒
| |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |  member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |  i-the_graph->j
| |  ⇒
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| |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |  ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| |  ⇒
| |  (∀j:Vertices(the_graph).
| |   i-the_graph->j
| |   ⇒
| |   ¬j = i
| |   ⇒
| |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| |  ⇒
| |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. M : traversal(the_graph)→ℕ
| 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| |   M([inl(i) / s])≤M(s)
| 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| |   ¬member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ M([inr(i) / s])<M(s)
| 12. d : ℕ
| 13. ∀d1:ℕ.
| |   d1<d
| |   ⇒
| |   (∀s:traversal(the_graph), i:Vertices(the_graph).
| |    M(s)≤d1
| |    ⇒
| |    (∃s':traversal(the_graph).
| |     M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s)))
| 14. s : traversal(the_graph)
| 15. i : Vertices(the_graph)
| 16. M(s)≤d
| 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| ⊢  ∃s':traversal(the_graph).
| |  M(s' @ s)≤M(s)
| |  & P(i,s,s')
| |  & [inl(i) /
| |     (the_obj.eacc((λs',j. dfs(the_obj;s';j)),[inr(i) / s],i))]
| |    =
| |    (s' @ s)
| | by (Inst Thm: graphobj_DASH_properties [the_graph;the_obj])
| |    THEN
| |    (Analyze -1)
| |    THEN
| |    (Analyze -1)
| |    THEN
| |    (Assert
| |     (∀L:Vertices(the_graph) List.
| |      (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| |      ⇒
| |      (∀s2:traversal(the_graph).
| |       M(s2)<d
| |       ⇒
| |       member-paren(x,y.the_obj.eq(x,y);i;s2)
| |       ⇒
| |       (∃s':traversal(the_graph).
| |        M(s' @ s2)≤M(s2)
| |        & P(i,s2,s')
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| |        & list_accum(s',x'.dfs(the_obj;s';x');s2;L)
| |          =
| |          (s' @ s2)))))
| |
| |\
| | .....assertion .....
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. P :
| | |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| | |  →Prop
| | 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | |  P(i,s1,s2)
| | |  ⇒
| | |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | |  P(i,s1,s2)
| | |  ⇒
| | |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | |  member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | |  i-the_graph->j
| | |  ⇒
| | |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | |  ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| | |  ⇒
| | |  (∀j:Vertices(the_graph).
| | |   i-the_graph->j
| | |   ⇒
| | |   ¬j = i
| | |   ⇒
| | |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| | |  ⇒
| | |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. M : traversal(the_graph)→ℕ
| | 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | |   M([inl(i) / s])≤M(s)
| | 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | |   ¬member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ M([inr(i) / s])<M(s)
| | 12. d : ℕ
| | 13. ∀d1:ℕ.
| | |   d1<d
| | |   ⇒
| | |   (∀s:traversal(the_graph), i:Vertices(the_graph).
| | |    M(s)≤d1
| | |    ⇒
| | |    (∃s':traversal(the_graph).
| | |     M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s)))
| | 14. s : traversal(the_graph)
| | 15. i : Vertices(the_graph)
| | 16. M(s)≤d
| | 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
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| | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| | |   , f:(T→Vertices(the_graph)→T).
| | |   ∃L:Vertices(the_graph) List.
| | |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | |   ∃L:Vertices(the_graph) List.
| | |   no_repeats(Vertices(the_graph);L)
| | |   & (∀y:Vertices(the_graph). (y ∈ L))
| | |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | ⊢  ∀L:Vertices(the_graph) List.
| | |  (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| | |  ⇒
| | |  (∀s2:traversal(the_graph).
| | |   M(s2)<d
| | |   ⇒
| | |   member-paren(x,y.the_obj.eq(x,y);i;s2)
| | |   ⇒
| | |   (∃s':traversal(the_graph).
| | |    M(s' @ s2)≤M(s2)
| | |    & P(i,s2,s')
| | |    & list_accum(s',x'.dfs(the_obj;s';x');s2;L) = (s' @ s2)))
| | | by InductionOnList THEN (Reduce 0)
| | |
| | |\
| | | 1. the_graph : Graph
| | | 2. the_obj : GraphObject(the_graph)
| | | 3. P :
| | |    Vertices(the_graph)→traversal(the_graph)→
| | |    traversal(the_graph)→Prop
| | | 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | |    P(i,s1,s2)
| | |    ⇒
| | |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | | 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | |    P(i,s1,s2)
| | |    ⇒
| | |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | |    member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| | | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | |    i-the_graph->j
| | |    ⇒
| | |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | |    ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| | |    ⇒
| | |    (∀j:Vertices(the_graph).
| | |     i-the_graph->j
| | |     ⇒
| | |     ¬j = i
| | |     ⇒
| | |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| | |    ⇒
| | |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
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| | | 9. M : traversal(the_graph)→ℕ
| | | 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | |     M([inl(i) / s])≤M(s)
| | | 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | |     ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | |     ⇒
| | |     M([inr(i) / s])<M(s)
| | | 12. d : ℕ
| | | 13. ∀d1:ℕ.
| | |     d1<d
| | |     ⇒
| | |     (∀s:traversal(the_graph), i:Vertices(the_graph).
| | |      M(s)≤d1
| | |      ⇒
| | |      (∃s':traversal(the_graph).
| | |       M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s)))
| | | 14. s : traversal(the_graph)
| | | 15. i : Vertices(the_graph)
| | | 16. M(s)≤d
| | | 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| | |     , f:(T→Vertices(the_graph)→T).
| | |     ∃L:Vertices(the_graph) List.
| | |     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | |     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | |     ∃L:Vertices(the_graph) List.
| | |     no_repeats(Vertices(the_graph);L)
| | |     & (∀y:Vertices(the_graph). (y ∈ L))
| | |     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | | 21. Vertices(the_graph) List
| | | ⊢  (∀y:Vertices(the_graph). (y ∈ nil) ⇒ i-the_graph->y)
| | |    ⇒
| | |    (∀s2:traversal(the_graph).
| | |     M(s2)<d
| | |     ⇒
| | |     member-paren(x,y.the_obj.eq(x,y);i;s2)
| | |     ⇒
| | |     (∃s':traversal(the_graph).
| | |      M(s' @ s2)≤M(s2) & P(i,s2,s') & s2 = (s' @ s2)))
| | |   by Auto THEN (InstConcl [nil]) THEN (Reduce 0) THEN EasyHyp
| | |
| | .\
| | . 1. the_graph : Graph
| | . 2. the_obj : GraphObject(the_graph)
| | . 3. P :
| | . |  Vertices(the_graph)→traversal(the_graph)→
| | . |  traversal(the_graph)→Prop
| | . 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . |  P(i,s1,s2)
| | . |  ⇒
| | . |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | . 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . |  P(i,s1,s2)
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| | . |  ⇒
| | . |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | . |  member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| | . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | . |  i-the_graph->j
| | . |  ⇒
| | . |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . |  ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| | . |  ⇒
| | . |  (∀j:Vertices(the_graph).
| | . |   i-the_graph->j
| | . |   ⇒
| | . |   ¬j = i
| | . |   ⇒
| | . |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| | . |  ⇒
| | . |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | . 9. M : traversal(the_graph)→ℕ
| | . 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . |   M([inl(i) / s])≤M(s)
| | . 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . |   ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . |   ⇒
| | . |   M([inr(i) / s])<M(s)
| | . 12. d : ℕ
| | . 13. ∀d1:ℕ.
| | . |   d1<d
| | . |   ⇒
| | . |   (∀s:traversal(the_graph), i:Vertices(the_graph).
| | . |    M(s)≤d1
| | . |    ⇒
| | . |    (∃s':traversal(the_graph).
| | . |     M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s)))
| | . 14. s : traversal(the_graph)
| | . 15. i : Vertices(the_graph)
| | . 16. M(s)≤d
| | . 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | . 19. ∀T:Type, s:T, x:Vertices(the_graph)
| | . |   , f:(T→Vertices(the_graph)→T).
| | . |   ∃L:Vertices(the_graph) List.
| | . |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | . |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | . 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | . |   ∃L:Vertices(the_graph) List.
| | . |   no_repeats(Vertices(the_graph);L)
| | . |   & (∀y:Vertices(the_graph). (y ∈ L))
| | . |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | . 21. Vertices(the_graph) List
| | . 22. u : Vertices(the_graph)
| | . 23. v : Vertices(the_graph) List
| | . 24. (∀y:Vertices(the_graph). (y ∈ v) ⇒ i-the_graph->y)
| | . |   ⇒
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| | . |   (∀s2:traversal(the_graph).
| | . |    M(s2)<d
| | . |    ⇒
| | . |    member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . |    ⇒
| | . |    (∃s':traversal(the_graph).
| | . |     M(s' @ s2)≤M(s2)
| | . |     & P(i,s2,s')
| | . |     & list_accum(s',x'.dfs(the_obj;s';x');s2;v) = (s' @ s2)))
| | . ⊢  (∀y:Vertices(the_graph). (y ∈ [u / v]) ⇒ i-the_graph->y)
| | . |  ⇒
| | . |  (∀s2:traversal(the_graph).
| | . |   M(s2)<d
| | . |   ⇒
| | . |   member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . |   ⇒
| | . |   (∃s':traversal(the_graph).
| | . |    M(s' @ s2)≤M(s2)
| | . |    & P(i,s2,s')
| | . |    & list_accum(s',x'.dfs(the_obj;s';x');dfs(the_obj;s2;u);v)
| | . |      =
| | . |      (s' @ s2)))  by ParallelOp -1
| | . |
| | . |\
| | . | .....antecedent .....
| | . | 1. the_graph : Graph
| | . | 2. the_obj : GraphObject(the_graph)
| | . | 3. P :
| | . |    Vertices(the_graph)→traversal(the_graph)→
| | . |    traversal(the_graph)→Prop
| | . | 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . |    P(i,s1,s2)
| | . |    ⇒
| | . |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | . | 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . |    P(i,s1,s2)
| | . |    ⇒
| | . |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | . | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | . |    member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| | . | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | . |    i-the_graph->j
| | . |    ⇒
| | . |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | . | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . |    ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| | . |    ⇒
| | . |    (∀j:Vertices(the_graph).
| | . |     i-the_graph->j
| | . |     ⇒
| | . |     ¬j = i
| | . |     ⇒
| | . |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| | . |    ⇒
| | . |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
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| | . | 9. M : traversal(the_graph)→ℕ
| | . | 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . |     M([inl(i) / s])≤M(s)
| | . | 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . |     ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . |     ⇒
| | . |     M([inr(i) / s])<M(s)
| | . | 12. d : ℕ
| | . | 13. ∀d1:ℕ.
| | . |     d1<d
| | . |     ⇒
| | . |     (∀s:traversal(the_graph), i:Vertices(the_graph).
| | . |      M(s)≤d1
| | . |      ⇒
| | . |      (∃s':traversal(the_graph).
| | . |       M(s' @ s)≤M(s)
| | . |       & P(i,s,s')
| | . |       & dfs(the_obj;s;i) = (s' @ s)))
| | . | 14. s : traversal(the_graph)
| | . | 15. i : Vertices(the_graph)
| | . | 16. M(s)≤d
| | . | 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | . | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| | . |     , f:(T→Vertices(the_graph)→T).
| | . |     ∃L:Vertices(the_graph) List.
| | . |     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | . |     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | . | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | . |     ∃L:Vertices(the_graph) List.
| | . |     no_repeats(Vertices(the_graph);L)
| | . |     & (∀y:Vertices(the_graph). (y ∈ L))
| | . |     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | . | 21. Vertices(the_graph) List
| | . | 22. u : Vertices(the_graph)
| | . | 23. v : Vertices(the_graph) List
| | . | 24. ∀y:Vertices(the_graph). (y ∈ [u / v]) ⇒ i-the_graph->y
| | . | ⊢  ∀y:Vertices(the_graph). (y ∈ v) ⇒ i-the_graph->y
| | . |   by (RepeatFor 2 (ParallelOp -1)) THEN (ObviousFrom [-1])
| | . |
| | . .\
| | . . 1. the_graph : Graph
| | . . 2. the_obj : GraphObject(the_graph)
| | . . 3. P :
| | . . |  Vertices(the_graph)→traversal(the_graph)→
| | . . |  traversal(the_graph)→Prop
| | . . 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . |  P(i,s1,s2)
| | . . |  ⇒
| | . . |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | . . 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . |  P(i,s1,s2)
| | . . |  ⇒
| | . . |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | . . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
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| | . . |  member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| | . . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | . . |  i-the_graph->j
| | . . |  ⇒
| | . . |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | . . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . |  ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| | . . |  ⇒
| | . . |  (∀j:Vertices(the_graph).
| | . . |   i-the_graph->j
| | . . |   ⇒
| | . . |   ¬j = i
| | . . |   ⇒
| | . . |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| | . . |  ⇒
| | . . |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | . . 9. M : traversal(the_graph)→ℕ
| | . . 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . |   M([inl(i) / s])≤M(s)
| | . . 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . |   ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . |   ⇒
| | . . |   M([inr(i) / s])<M(s)
| | . . 12. d : ℕ
| | . . 13. ∀d1:ℕ.
| | . . |   d1<d
| | . . |   ⇒
| | . . |   (∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . |    M(s)≤d1
| | . . |    ⇒
| | . . |    (∃s':traversal(the_graph).
| | . . |     M(s' @ s)≤M(s)
| | . . |     & P(i,s,s')
| | . . |     & dfs(the_obj;s;i) = (s' @ s)))
| | . . 14. s : traversal(the_graph)
| | . . 15. i : Vertices(the_graph)
| | . . 16. M(s)≤d
| | . . 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | . . 19. ∀T:Type, s:T, x:Vertices(the_graph)
| | . . |   , f:(T→Vertices(the_graph)→T).
| | . . |   ∃L:Vertices(the_graph) List.
| | . . |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | . . |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | . . 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | . . |   ∃L:Vertices(the_graph) List.
| | . . |   no_repeats(Vertices(the_graph);L)
| | . . |   & (∀y:Vertices(the_graph). (y ∈ L))
| | . . |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | . . 21. Vertices(the_graph) List
| | . . 22. u : Vertices(the_graph)
| | . . 23. v : Vertices(the_graph) List
| | . . 24. ∀y:Vertices(the_graph). (y ∈ [u / v]) ⇒ i-the_graph->y
| | . . 25. ∀s2:traversal(the_graph).
| | . . |   M(s2)<d
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| | . . |   ⇒
| | . . |   member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . |   ⇒
| | . . |   (∃s':traversal(the_graph).
| | . . |    M(s' @ s2)≤M(s2)
| | . . |    & P(i,s2,s')
| | . . |    & list_accum(s',x'.dfs(the_obj;s';x');s2;v) = (s' @ s2))
| | . ⊢  ∀s2:traversal(the_graph).
| | . . | M(s2)<d
| | . . | ⇒
| | . . | member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . | ⇒
| | . . | (∃s':traversal(the_graph).
| | . . |  M(s' @ s2)≤M(s2)
| | . . |  & P(i,s2,s')
| | . . |  & list_accum(s',x'.dfs(the_obj;s';x');dfs(the_obj;s2;u);v)
| | . . |    =
| | . . |    (s' @ s2))  by Auto
| | . . |
| | . . 1. the_graph : Graph
| | . . 2. the_obj : GraphObject(the_graph)
| | . . 3. P :
| | . . | Vertices(the_graph)→traversal(the_graph)→
| | . . | traversal(the_graph)→Prop
| | . . 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . | P(i,s1,s2)
| | . . | ⇒
| | . . | l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | . . 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . | P(i,s1,s2)
| | . . | ⇒
| | . . | no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | . . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . | member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| | . . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | . . | i-the_graph->j
| | . . | ⇒
| | . . | P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | . . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . | ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| | . . | ⇒
| | . . | (∀j:Vertices(the_graph).
| | . . |  i-the_graph->j
| | . . |  ⇒
| | . . |  ¬j = i
| | . . |  ⇒
| | . . |  (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| | . . | ⇒
| | . . | P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | . . 9. M : traversal(the_graph)→ℕ
| | . . 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . |  M([inl(i) / s])≤M(s)
| | . . 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . |  ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . |  ⇒
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| | . . |   M([inr(i) / s])<M(s)
| | . . 12. d : ℕ
| | . . 13. ∀d1:ℕ.
| | . . |   d1<d
| | . . |   ⇒
| | . . |   (∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . |    M(s)≤d1
| | . . |    ⇒
| | . . |    (∃s':traversal(the_graph).
| | . . |     M(s' @ s)≤M(s)
| | . . |     & P(i,s,s')
| | . . |     & dfs(the_obj;s;i) = (s' @ s)))
| | . . 14. s : traversal(the_graph)
| | . . 15. i : Vertices(the_graph)
| | . . 16. M(s)≤d
| | . . 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | . . 19. ∀T:Type, s:T, x:Vertices(the_graph)
| | . . |   , f:(T→Vertices(the_graph)→T).
| | . . |   ∃L:Vertices(the_graph) List.
| | . . |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | . . |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | . . 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | . . |   ∃L:Vertices(the_graph) List.
| | . . |   no_repeats(Vertices(the_graph);L)
| | . . |   & (∀y:Vertices(the_graph). (y ∈ L))
| | . . |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | . . 21. Vertices(the_graph) List
| | . . 22. u : Vertices(the_graph)
| | . . 23. v : Vertices(the_graph) List
| | . . 24. ∀y:Vertices(the_graph). (y ∈ [u / v]) ⇒ i-the_graph->y
| | . . 25. ∀s2:traversal(the_graph).
| | . . |   M(s2)<d
| | . . |   ⇒
| | . . |   member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . |   ⇒
| | . . |   (∃s':traversal(the_graph).
| | . . |    M(s' @ s2)≤M(s2)
| | . . |    & P(i,s2,s')
| | . . |    & list_accum(s',x'.dfs(the_obj;s';x');s2;v) = (s' @ s2))
| | . . 26. s2 : traversal(the_graph)
| | . . 27. M(s2)<d
| | . . 28. member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . ⊢  ∃s':traversal(the_graph).
| | . . |  M(s' @ s2)≤M(s2)
| | . . |  & P(i,s2,s')
| | . . |  & list_accum(s',x'.dfs(the_obj;s';x');dfs(the_obj;s2;u);v)
| | . . |    =
| | . . |    (s' @ s2)  by InstHyp [d-1;s2;u] 13
| | . . |
| | . . 1. the_graph : Graph
| | . . 2. the_obj : GraphObject(the_graph)
| | . . 3. P :
| | . . |  Vertices(the_graph)→traversal(the_graph)→
| | . . |  traversal(the_graph)→Prop
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| | . . 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . |  P(i,s1,s2)
| | . . |  ⇒
| | . . |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | . . 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . |  P(i,s1,s2)
| | . . |  ⇒
| | . . |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | . . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . |  member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| | . . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | . . |  i-the_graph->j
| | . . |  ⇒
| | . . |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | . . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . |  ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| | . . |  ⇒
| | . . |  (∀j:Vertices(the_graph).
| | . . |   i-the_graph->j
| | . . |   ⇒
| | . . |   ¬j = i
| | . . |   ⇒
| | . . |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| | . . |  ⇒
| | . . |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | . . 9. M : traversal(the_graph)→ℕ
| | . . 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . |   M([inl(i) / s])≤M(s)
| | . . 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . |   ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . |   ⇒
| | . . |   M([inr(i) / s])<M(s)
| | . . 12. d : ℕ
| | . . 13. ∀d1:ℕ.
| | . . |   d1<d
| | . . |   ⇒
| | . . |   (∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . |    M(s)≤d1
| | . . |    ⇒
| | . . |    (∃s':traversal(the_graph).
| | . . |     M(s' @ s)≤M(s)
| | . . |     & P(i,s,s')
| | . . |     & dfs(the_obj;s;i) = (s' @ s)))
| | . . 14. s : traversal(the_graph)
| | . . 15. i : Vertices(the_graph)
| | . . 16. M(s)≤d
| | . . 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | . . 19. ∀T:Type, s:T, x:Vertices(the_graph)
| | . . |   , f:(T→Vertices(the_graph)→T).
| | . . |   ∃L:Vertices(the_graph) List.
| | . . |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | . . |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | . . 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | . . |   ∃L:Vertices(the_graph) List.
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| | . . |   no_repeats(Vertices(the_graph);L)
| | . . |   & (∀y:Vertices(the_graph). (y ∈ L))
| | . . |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | . . 21. Vertices(the_graph) List
| | . . 22. u : Vertices(the_graph)
| | . . 23. v : Vertices(the_graph) List
| | . . 24. ∀y:Vertices(the_graph). (y ∈ [u / v]) ⇒ i-the_graph->y
| | . . 25. ∀s2:traversal(the_graph).
| | . . |   M(s2)<d
| | . . |   ⇒
| | . . |   member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . |   ⇒
| | . . |   (∃s':traversal(the_graph).
| | . . |    M(s' @ s2)≤M(s2)
| | . . |    & P(i,s2,s')
| | . . |    & list_accum(s',x'.dfs(the_obj;s';x');s2;v) = (s' @ s2))
| | . . 26. s2 : traversal(the_graph)
| | . . 27. M(s2)<d
| | . . 28. member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . 29. ∃s':traversal(the_graph).
| | . . |   M(s' @ s2)≤M(s2)
| | . . |   & P(u,s2,s')
| | . . |   & dfs(the_obj;s2;u) = (s' @ s2)
| | . . ⊢  ∃s':traversal(the_graph).
| | . . |  M(s' @ s2)≤M(s2)
| | . . |  & P(i,s2,s')
| | . . |  & list_accum(s',x'.dfs(the_obj;s';x');dfs(the_obj;s2;u);v)
| | . . |    =
| | . . |    (s' @ s2)
| | . . | by ExRepD THEN (HypSubst -1 0) THEN (InstHyp [s' @ s2] -8)
| | . . |
| | . . |\
| | . . | .....antecedent .....
| | . . | 1. the_graph : Graph
| | . . | 2. the_obj : GraphObject(the_graph)
| | . . | 3. P :
| | . . |    Vertices(the_graph)→traversal(the_graph)→
| | . . |    traversal(the_graph)→Prop
| | . . | 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . |    P(i,s1,s2)
| | . . |    ⇒
| | . . |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | . . | 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . |    P(i,s1,s2)
| | . . |    ⇒
| | . . |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | . . | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . |    member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| | . . | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | . . |    i-the_graph->j
| | . . |    ⇒
| | . . |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | . . | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . |    ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| | . . |    ⇒
```

```
| | . . |    (∀j:Vertices(the_graph).
| | . . |     i-the_graph->j
| | . . |     ⇒
| | . . |     ¬j = i
| | . . |     ⇒
| | . . |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| | . . |    ⇒
| | . . |    P(i,[inr(i) / s1],s2)
| | . . |    ⇒
| | . . |    P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | . . | 9. M : traversal(the_graph)→ℕ
| | . . | 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . |     M([inl(i) / s])≤M(s)
| | . . | 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . |     ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . |     ⇒
| | . . |     M([inr(i) / s])<M(s)
| | . . | 12. d : ℕ
| | . . | 13. ∀d1:ℕ.
| | . . |     d1<d
| | . . |     ⇒
| | . . |     (∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . |      M(s)≤d1
| | . . |      ⇒
| | . . |      (∃s':traversal(the_graph).
| | . . |       M(s' @ s)≤M(s)
| | . . |       & P(i,s,s')
| | . . |       & dfs(the_obj;s;i) = (s' @ s)))
| | . . | 14. s : traversal(the_graph)
| | . . | 15. i : Vertices(the_graph)
| | . . | 16. M(s)≤d
| | . . | 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | . . | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| | . . |     , f:(T→Vertices(the_graph)→T).
| | . . |     ∃L:Vertices(the_graph) List.
| | . . |     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | . . |     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | . . | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | . . |     ∃L:Vertices(the_graph) List.
| | . . |     no_repeats(Vertices(the_graph);L)
| | . . |     & (∀y:Vertices(the_graph). (y ∈ L))
| | . . |     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | . . | 21. Vertices(the_graph) List
| | . . | 22. u : Vertices(the_graph)
| | . . | 23. v : Vertices(the_graph) List
| | . . | 24. ∀y:Vertices(the_graph). (y ∈ [u / v]) ⇒ i-the_graph->y
| | . . | 25. ∀s2:traversal(the_graph).
| | . . |     M(s2)<d
| | . . |     ⇒
| | . . |     member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . |     ⇒
| | . . |     (∃s':traversal(the_graph).
| | . . |      M(s' @ s2)≤M(s2)
| | . . |      & P(i,s2,s')
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```
| | . . |      & list_accum(s',x'.dfs(the_obj;s';x');s2;v)
| | . . |        =
| | . . |        (s' @ s2))
| | . . | 26. s2 : traversal(the_graph)
| | . . | 27. M(s2)<d
| | . . | 28. member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . | 29. s' : traversal(the_graph)
| | . . | 30. M(s' @ s2)≤M(s2)
| | . . | 31. P(u,s2,s')
| | . . | 32. dfs(the_obj;s2;u) = (s' @ s2)
| | . . | ⊢  member-paren(x,y.the_obj.eq(x,y);i;s' @ s2)
| | . . |   by (MoveToConcl -5)
| | . . |      THEN
| | . . |      (RWO
| | . . |       Thm* ∀E:(T→T→ℙ).
| | . . |            (∀x,y:T. E(x,y) ⟺ x = y)
| | . . |            ⇒
| | . . |            (∀i:T, s:(T+T) List.
| | . . |             member-paren(x,y.E(x,y);i;s)
| | . . |             ⟺
| | . . |             (inl(i) ∈ s) ∨ (inr(i) ∈ s))
| | . . |       {assert_DASH_member_DASH_paren}
| | . . |       0)
| | . . |      THEN
| | . . |      ObviousConcl
| | . . |
| | . . .\
| | . . . 1. the_graph : Graph
| | . . . 2. the_obj : GraphObject(the_graph)
| | . . . 3. P :
| | . . . |  Vertices(the_graph)→traversal(the_graph)→
| | . . . |  traversal(the_graph)→Prop
| | . . . 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . . |  P(i,s1,s2)
| | . . . |  ⇒
| | . . . |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | . . . 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . . |  P(i,s1,s2)
| | . . . |  ⇒
| | . . . |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | . . . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . . |  member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| | . . . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | . . . |  i-the_graph->j
| | . . . |  ⇒
| | . . . |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | . . . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . . |  ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| | . . . |  ⇒
| | . . . |  (∀j:Vertices(the_graph).
| | . . . |   i-the_graph->j
| | . . . |   ⇒
| | . . . |   ¬j = i
| | . . . |   ⇒
| | . . . |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
```

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| | . . . |  ⇒
| | . . . |  P(i,[inr(i) / s1],s2)
| | . . . |  ⇒
| | . . . |  P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | . . . 9. M : traversal(the_graph)→ℕ
| | . . . 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . . |   M([inl(i) / s])≤M(s)
| | . . . 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . . |   ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . . |   ⇒
| | . . . |   M([inr(i) / s])<M(s)
| | . . . 12. d : ℕ
| | . . . 13. ∀d1:ℕ.
| | . . . |   d1<d
| | . . . |   ⇒
| | . . . |   (∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . . |    M(s)≤d1
| | . . . |    ⇒
| | . . . |    (∃s':traversal(the_graph).
| | . . . |     M(s' @ s)≤M(s)
| | . . . |     & P(i,s,s')
| | . . . |     & dfs(the_obj;s;i) = (s' @ s)))
| | . . . 14. s : traversal(the_graph)
| | . . . 15. i : Vertices(the_graph)
| | . . . 16. M(s)≤d
| | . . . 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . . 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | . . . 19. ∀T:Type, s:T, x:Vertices(the_graph)
| | . . . |   , f:(T→Vertices(the_graph)→T).
| | . . . |   ∃L:Vertices(the_graph) List.
| | . . . |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | . . . |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | . . . 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | . . . |   ∃L:Vertices(the_graph) List.
| | . . . |   no_repeats(Vertices(the_graph);L)
| | . . . |   & (∀y:Vertices(the_graph). (y ∈ L))
| | . . . |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | . . . 21. Vertices(the_graph) List
| | . . . 22. u : Vertices(the_graph)
| | . . . 23. v : Vertices(the_graph) List
| | . . . 24. ∀y:Vertices(the_graph). (y ∈ [u / v]) ⇒ i-the_graph->y
| | . . . 25. ∀s2:traversal(the_graph).
| | . . . |   M(s2)<d
| | . . . |   ⇒
| | . . . |   member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . . |   ⇒
| | . . . |   (∃s':traversal(the_graph).
| | . . . |    M(s' @ s2)≤M(s2)
| | . . . |    & P(i,s2,s')
| | . . . |    & list_accum(s',x'.dfs(the_obj;s';x');s2;v)
| | . . . |      =
| | . . . |      (s' @ s2))
| | . . . 26. s2 : traversal(the_graph)
| | . . . 27. M(s2)<d
| | . . . 28. member-paren(x,y.the_obj.eq(x,y);i;s2)
```

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| | . . . 29. s' : traversal(the_graph)
| | . . . 30. M(s' @ s2)≤M(s2)
| | . . . 31. P(u,s2,s')
| | . . . 32. dfs(the_obj;s2;u) = (s' @ s2)
| | . . . 33. ∃s'@0:traversal(the_graph).
| | . . . |   M(s'@0 @ s' @ s2)≤M(s' @ s2)
| | . . . |   & P(i,s' @ s2,s'@0)
| | . . . |   & list_accum(s',x'.dfs(the_obj;s';x');s' @ s2;v)
| | . . . |     =
| | . . . |     (s'@0 @ s' @ s2)
| | . . . ⊢  ∃s'@0:traversal(the_graph).
| | . . . |  M(s'@0 @ s2)≤M(s2)
| | . . . |  & P(i,s2,s'@0)
| | . . . |  & list_accum(s',x'.dfs(the_obj;s';x');s' @ s2;v)
| | . . . |    =
| | . . . |    (s'@0 @ s2)
| | . . . | by ExRepD THEN (HypSubst -1 0) THEN (Thin -1)
| | . . . |
| | . . . 1. the_graph : Graph
| | . . . 2. the_obj : GraphObject(the_graph)
| | . . . 3. P :
| | . . . |  Vertices(the_graph)→traversal(the_graph)→
| | . . . |  traversal(the_graph)→Prop
| | . . . 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . . |  P(i,s1,s2)
| | . . . |  ⇒
| | . . . |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | . . . 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . . |  P(i,s1,s2)
| | . . . |  ⇒
| | . . . |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | . . . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . . |  member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| | . . . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | . . . |  i-the_graph->j
| | . . . |  ⇒
| | . . . |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | . . . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . . |  ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| | . . . |  ⇒
| | . . . |  (∀j:Vertices(the_graph).
| | . . . |   i-the_graph->j
| | . . . |   ⇒
| | . . . |   ¬j = i
| | . . . |   ⇒
| | . . . |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| | . . . |  ⇒
| | . . . |  P(i,[inr(i) / s1],s2)
| | . . . |  ⇒
| | . . . |  P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | . . . 9. M : traversal(the_graph)→ℕ
| | . . . 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . . |   M([inl(i) / s])≤M(s)
| | . . . 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . . |   ¬member-paren(x,y.the_obj.eq(x,y);i;s)
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| | . . . |   ⇒
| | . . . |   M([inr(i) / s])<M(s)
| | . . 12. d : ℕ
| | . . 13. ∀d1:ℕ.
| | . . . |   d1<d
| | . . . |   ⇒
| | . . . |   (∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . . |    M(s)≤d1
| | . . . |    ⇒
| | . . . |    (∃s':traversal(the_graph).
| | . . . |     M(s' @ s)≤M(s)
| | . . . |     & P(i,s,s')
| | . . . |     & dfs(the_obj;s;i) = (s' @ s)))
| | . . 14. s : traversal(the_graph)
| | . . 15. i : Vertices(the_graph)
| | . . 16. M(s)≤d
| | . . 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | . . 19. ∀T:Type, s:T, x:Vertices(the_graph)
| | . . . |   , f:(T→Vertices(the_graph)→T).
| | . . . |   ∃L:Vertices(the_graph) List.
| | . . . |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | . . . |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | . . 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | . . . |   ∃L:Vertices(the_graph) List.
| | . . . |   no_repeats(Vertices(the_graph);L)
| | . . . |   & (∀y:Vertices(the_graph). (y ∈ L))
| | . . . |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | . . 21. Vertices(the_graph) List
| | . . 22. u : Vertices(the_graph)
| | . . 23. v : Vertices(the_graph) List
| | . . 24. ∀y:Vertices(the_graph). (y ∈ [u / v]) ⇒ i-the_graph->y
| | . . 25. ∀s2:traversal(the_graph).
| | . . . |   M(s2)<d
| | . . . |   ⇒
| | . . . |   member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . . |   ⇒
| | . . . |   (∃s':traversal(the_graph).
| | . . . |    M(s' @ s2)≤M(s2)
| | . . . |    & P(i,s2,s')
| | . . . |    & list_accum(s',x'.dfs(the_obj;s';x');s2;v)
| | . . . |      =
| | . . . |      (s' @ s2))
| | . . 26. s2 : traversal(the_graph)
| | . . 27. M(s2)<d
| | . . 28. member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . 29. s' : traversal(the_graph)
| | . . 30. M(s' @ s2)≤M(s2)
| | . . 31. P(u,s2,s')
| | . . 32. dfs(the_obj;s2;u) = (s' @ s2)
| | . . 33. s'@0 : traversal(the_graph)
| | . . 34. M(s'@0 @ s' @ s2)≤M(s' @ s2)
| | . . 35. P(i,s' @ s2,s'@0)
| | . . ⊢  ∃s'@1:traversal(the_graph).
| | . . . |  M(s'@1 @ s2)≤M(s2)
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```
| | . . . |  & P(i,s2,s'@1)
| | . . . |  & (s'@0 @ s' @ s2) = (s'@1 @ s2)
| | . . . | by (InstConcl [s'@0 @ s'])
| | . . . |    THEN
| | . . . |    (RWO
| | . . . |     Thm* ∀a,b,c:Top List. ((a @ b) @ c) ∼ (a @ b @ c)
| | . . . |     {append_assoc_sq}
| | . . . |     0)
| | . . . |
| | . . . 1. the_graph : Graph
| | . . . 2. the_obj : GraphObject(the_graph)
| | . . . 3. P :
| | . . .    Vertices(the_graph)→traversal(the_graph)→
| | . . .    traversal(the_graph)→Prop
| | . . . 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . .    P(i,s1,s2)
| | . . .    ⇒
| | . . .    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | . . . 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . .    P(i,s1,s2)
| | . . .    ⇒
| | . . .    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | . . . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . .    member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| | . . . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | . . .    i-the_graph->j
| | . . .    ⇒
| | . . .    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | . . . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | . . .    ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| | . . .    ⇒
| | . . .    (∀j:Vertices(the_graph).
| | . . .     i-the_graph->j
| | . . .     ⇒
| | . . .     ¬j = i
| | . . .     ⇒
| | . . .     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| | . . .    ⇒
| | . . .    P(i,[inr(i) / s1],s2)
| | . . .    ⇒
| | . . .    P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | . . . 9. M : traversal(the_graph)→ℕ
| | . . . 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . .     M([inl(i) / s])≤M(s)
| | . . . 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| | . . .     ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . .     ⇒
| | . . .     M([inr(i) / s])<M(s)
| | . . . 12. d : ℕ
| | . . . 13. ∀d1:ℕ.
| | . . .     d1<d
| | . . .     ⇒
| | . . .     (∀s:traversal(the_graph), i:Vertices(the_graph).
| | . . .      M(s)≤d1
| | . . .      ⇒
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```
| | . . .      (∃s':traversal(the_graph).
| | . . .       M(s' @ s)≤M(s)
| | . . .       & P(i,s,s')
| | . . .       & dfs(the_obj;s;i) = (s' @ s)))
| | . . . 14. s : traversal(the_graph)
| | . . . 15. i : Vertices(the_graph)
| | . . . 16. M(s)≤d
| | . . . 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| | . . . 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | . . . 19. ∀T:Type, s:T, x:Vertices(the_graph)
| | . . .     , f:(T→Vertices(the_graph)→T).
| | . . .     ∃L:Vertices(the_graph) List.
| | . . .     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | . . .     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | . . . 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | . . .     ∃L:Vertices(the_graph) List.
| | . . .     no_repeats(Vertices(the_graph);L)
| | . . .     & (∀y:Vertices(the_graph). (y ∈ L))
| | . . .     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | . . . 21. Vertices(the_graph) List
| | . . . 22. u : Vertices(the_graph)
| | . . . 23. v : Vertices(the_graph) List
| | . . . 24. ∀y:Vertices(the_graph). (y ∈ [u / v]) ⇒ i-the_graph->y
| | . . . 25. ∀s2:traversal(the_graph).
| | . . .     M(s2)<d
| | . . .     ⇒
| | . . .     member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . .     ⇒
| | . . .     (∃s':traversal(the_graph).
| | . . .      M(s' @ s2)≤M(s2)
| | . . .      & P(i,s2,s')
| | . . .      & list_accum(s',x'.dfs(the_obj;s';x');s2;v)
| | . . .        =
| | . . .        (s' @ s2))
| | . . . 26. s2 : traversal(the_graph)
| | . . . 27. M(s2)<d
| | . . . 28. member-paren(x,y.the_obj.eq(x,y);i;s2)
| | . . . 29. s' : traversal(the_graph)
| | . . . 30. M(s' @ s2)≤M(s2)
| | . . . 31. P(u,s2,s')
| | . . . 32. dfs(the_obj;s2;u) = (s' @ s2)
| | . . . 33. s'@0 : traversal(the_graph)
| | . . . 34. M(s'@0 @ s' @ s2)≤M(s' @ s2)
| | . . . 35. P(i,s' @ s2,s'@0)
| | . . . ⊢  P(i,s2,s'@0 @ s')
| | . . .   by ((InstHyp [u] -12) THENA (Auto THEN ObviousConcl))
| | . . .      THEN
| | . . .      (BackThruHyp' 7)
| |
| .\
| . 1. the_graph : Graph
| . 2. the_obj : GraphObject(the_graph)
| . 3. P :
| . |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| . |  →Prop
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| . 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . |  P(i,s1,s2)
| . |  ⇒
| . |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . |  P(i,s1,s2)
| . |  ⇒
| . |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . |  member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . |  i-the_graph->j
| . |  ⇒
| . |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . |  ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| . |  ⇒
| . |  (∀j:Vertices(the_graph).
| . |   i-the_graph->j
| . |   ⇒
| . |   ¬j = i
| . |   ⇒
| . |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| . |  ⇒
| . |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . 9. M : traversal(the_graph)→ℕ
| . 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . |   M([inl(i) / s])≤M(s)
| . 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . |   ¬member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ M([inr(i) / s])<M(s)
| . 12. d : ℕ
| . 13. ∀d1:ℕ.
| . |   d1<d
| . |   ⇒
| . |   (∀s:traversal(the_graph), i:Vertices(the_graph).
| . |    M(s)≤d1
| . |    ⇒
| . |    (∃s':traversal(the_graph).
| . |     M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s)))
| . 14. s : traversal(the_graph)
| . 15. i : Vertices(the_graph)
| . 16. M(s)≤d
| . 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| . 19. ∀T:Type, s:T, x:Vertices(the_graph)
| . |   , f:(T→Vertices(the_graph)→T).
| . |   ∃L:Vertices(the_graph) List.
| . |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| . |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| . |   ∃L:Vertices(the_graph) List.
| . |   no_repeats(Vertices(the_graph);L)
| . |   & (∀y:Vertices(the_graph). (y ∈ L))
| . |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . 21. ∀L:Vertices(the_graph) List.
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| . |   (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| . |   ⇒
| . |   (∀s2:traversal(the_graph).
| . |    M(s2)<d
| . |    ⇒
| . |    member-paren(x,y.the_obj.eq(x,y);i;s2)
| . |    ⇒
| . |    (∃s':traversal(the_graph).
| . |     M(s' @ s2)≤M(s2)
| . |     & P(i,s2,s')
| . |     & list_accum(s',x'.dfs(the_obj;s';x');s2;L) = (s' @ s2)))
| . ⊢  ∃s':traversal(the_graph).
| . |  M(s' @ s)≤M(s)
| . |  & P(i,s,s')
| . |  & [inl(i) /
| . |     (the_obj.eacc((λs',j. dfs(the_obj;s';j)),[inr(i) / s],i))]
| . |    =
| . |    (s' @ s)
| . | by (((InstHyp
| . |       [traversal(the_graph)
| . |       ;[inr(i) / s]
| . |       ;i
| . |       ;λ₂s',j. dfs(the_obj;s';j)]
| . |       -3)
| . |      THEN
| . |      ExRepD
| . |      THEN
| . |      (InstHyp [L;[inr(i) / s]] -4))
| . |     THENA
| . |     (Auto THEN EasyHyp
| . |      THEN
| . |      (Try
| . |       ((RWO
| . |         Thm* ∀E:(T→T→ℙ).
| . |              (∀x,y:T. E(x,y) ⟺ x = y)
| . |              ⇒
| . |              (∀i:T, s:(T+T) List.
| . |               member-paren(x,y.E(x,y);i;s)
| . |               ⟺
| . |               (inl(i) ∈ s) ∨ (inr(i) ∈ s))
| . |         {assert_DASH_member_DASH_paren}
| . |         0)
| . |        THEN
| . |        ObviousConcl))
| . |      THEN
| . |      (AutoInst [i;s])))
| . |    THEN
| . |    ExRepD
| . |    THEN
| . |    (InstConcl [[inl(i)] @ s' @ [inr(i)]])
| . |    THEN
| . |    (Reduce 0)
| . |    THEN
| . |    EasyHyp
| . |    THEN
```

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| . |    (Try
| . |     ((RWO
| . |       Thm* ∀a,b,c:Top List. ((a @ b) @ c) ∼ (a @ b @ c)
| . |       {append_assoc_sq}
| . |       0)
| . |      THEN
| . |      (Reduce 0)
| . |      THEN
| . |      Analyze))
| . |
| . |\
| . | 1. the_graph : Graph
| . | 2. the_obj : GraphObject(the_graph)
| . | 3. P :
| . |    Vertices(the_graph)→traversal(the_graph)→
| . |    traversal(the_graph)→Prop
| . | 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . |    P(i,s1,s2)
| . |    ⇒
| . |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . | 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . |    P(i,s1,s2)
| . |    ⇒
| . |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . |    member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| . | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . |    i-the_graph->j
| . |    ⇒
| . |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . |    ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| . |    ⇒
| . |    (∀j:Vertices(the_graph).
| . |     i-the_graph->j
| . |     ⇒
| . |     ¬j = i
| . |     ⇒
| . |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| . |    ⇒
| . |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . | 9. M : traversal(the_graph)→ℕ
| . | 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . |     M([inl(i) / s])≤M(s)
| . | 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . |     ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . |     ⇒
| . |     M([inr(i) / s])<M(s)
| . | 12. d : ℕ
| . | 13. ∀d1:ℕ.
| . |     d1<d
| . |     ⇒
| . |     (∀s:traversal(the_graph), i:Vertices(the_graph).
| . |      M(s)≤d1
| . |      ⇒
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| . |      (∃s':traversal(the_graph).
| . |       M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s)))
| . | 14. s : traversal(the_graph)
| . | 15. i : Vertices(the_graph)
| . | 16. M(s)≤d
| . | 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| . | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| . |     , f:(T→Vertices(the_graph)→T).
| . |     ∃L:Vertices(the_graph) List.
| . |     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| . |     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| . |     ∃L:Vertices(the_graph) List.
| . |     no_repeats(Vertices(the_graph);L)
| . |     & (∀y:Vertices(the_graph). (y ∈ L))
| . |     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . | 21. ∀L:Vertices(the_graph) List.
| . |     (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| . |     ⇒
| . |     (∀s2:traversal(the_graph).
| . |      M(s2)<d
| . |      ⇒
| . |      member-paren(x,y.the_obj.eq(x,y);i;s2)
| . |      ⇒
| . |      (∃s':traversal(the_graph).
| . |       M(s' @ s2)≤M(s2)
| . |       & P(i,s2,s')
| . |       & list_accum(s',x'.dfs(the_obj;s';x');s2;L) = (s' @ s2)))
| . | 22. L : Vertices(the_graph) List
| . | 23. ∀y:Vertices(the_graph). i-the_graph->y ⟺ (y ∈ L)
| . | 24. the_obj.eacc((λ2s',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . |     =
| . |     list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | 25. s' : traversal(the_graph)
| . | 26. M(s' @ [inr(i) / s])≤M([inr(i) / s])
| . | 27. P(i,[inr(i) / s],s')
| . | 28. list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . |     =
| . |     (s' @ [inr(i) / s])
| . | ⊢  M([inl(i) / ((s' @ [inr(i)]) @ s)])≤M(s)
| . |   by (RW0
| . |       Thm* ∀a,b,c:Top List. ((a @ b) @ c) ∼ (a @ b @ c)
| . |       {append_assoc_sq}
| . |       0)
| . |      THEN
| . |      (Reduce 0)
| . |      THEN
| . |      (InstHyp [i;(s' @ [inr(i)]) @ s] 10)
| . |      THEN
| . |      (RW0
| . |       Thm* ∀a,b,c:Top List. ((a @ b) @ c) ∼ (a @ b @ c)
| . |       {append_assoc_sq}
| . |       -1)
| . |      THEN
```

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| . |      (Reduce -1)
| . |      THEN
| . |      (InstHyp [i;s] 11)
| . |
| . |\
| . | 1. the_graph : Graph
| . | 2. the_obj : GraphObject(the_graph)
| . | 3. P :
| . | | Vertices(the_graph)→traversal(the_graph)→
| . | | traversal(the_graph)→Prop
| . | 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | P(i,s1,s2)
| . | | ⇒
| . | | l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . | 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | P(i,s1,s2)
| . | | ⇒
| . | | no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . | | member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| . | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . | | i-the_graph->j
| . | | ⇒
| . | | P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| . | | ⇒
| . | | (∀j:Vertices(the_graph).
| . | |  i-the_graph->j
| . | |  ⇒
| . | |  ¬j = i
| . | |  ⇒
| . | |  (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| . | | ⇒
| . | | P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . | 9. M : traversal(the_graph)→ℕ
| . | 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | |  M([inl(i) / s])≤M(s)
| . | 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | |  ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | |  ⇒
| . | |  M([inr(i) / s])<M(s)
| . | 12. d : ℕ
| . | 13. ∀d1:ℕ.
| . | |  d1<d
| . | |  ⇒
| . | |  (∀s:traversal(the_graph), i:Vertices(the_graph).
| . | |   M(s)≤d1
| . | |   ⇒
| . | |   (∃s':traversal(the_graph).
| . | |    M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s)))
| . | 14. s : traversal(the_graph)
| . | 15. i : Vertices(the_graph)
| . | 16. M(s)≤d
| . | 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
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| . | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| . | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| . | |   , f:(T→Vertices(the_graph)→T).
| . | |   ∃L:Vertices(the_graph) List.
| . | |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| . | |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| . | |   ∃L:Vertices(the_graph) List.
| . | |   no_repeats(Vertices(the_graph);L)
| . | |   & (∀y:Vertices(the_graph). (y ∈ L))
| . | |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . | 21. ∀L:Vertices(the_graph) List.
| . | |   (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| . | |   ⇒
| . | |   (∀s2:traversal(the_graph).
| . | |    M(s2)<d
| . | |    ⇒
| . | |    member-paren(x,y.the_obj.eq(x,y);i;s2)
| . | |    ⇒
| . | |    (∃s':traversal(the_graph).
| . | |     M(s' @ s2)≤M(s2)
| . | |     & P(i,s2,s')
| . | |     & list_accum(s',x'.dfs(the_obj;s';x');s2;L) = (s' @ s2)))
| . | 22. L : Vertices(the_graph) List
| . | 23. ∀y:Vertices(the_graph). i-the_graph->y ⟺ (y ∈ L)
| . | 24. the_obj.eacc((λ2s',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . | |   =
| . | |   list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | 25. s' : traversal(the_graph)
| . | 26. M(s' @ [inr(i) / s])≤M([inr(i) / s])
| . | 27. P(i,[inr(i) / s],s')
| . | 28. list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | |   =
| . | |   (s' @ [inr(i) / s])
| . | ⊢  P(i,s,[inl(i) / (s' @ [inr(i)])])
| . | | by (BackThru 8) THEN (AssertBY (j ∈ L) EasyHyp)
| . | |
| . | 1. the_graph : Graph
| . | 2. the_obj : GraphObject(the_graph)
| . | 3. P :
| . | |  Vertices(the_graph)→traversal(the_graph)→
| . | |  traversal(the_graph)→Prop
| . | 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | |  P(i,s1,s2)
| . | |  ⇒
| . | |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . | 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | |  P(i,s1,s2)
| . | |  ⇒
| . | |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . | |  member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| . | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . | |  i-the_graph->j
| . | |  ⇒
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| . | |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | |  ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| . | |  ⇒
| . | |  (∀j:Vertices(the_graph).
| . | |   i-the_graph->j
| . | |   ⇒
| . | |   ¬j = i
| . | |   ⇒
| . | |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| . | |  ⇒
| . | |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . | 9. M : traversal(the_graph)→ℕ
| . | 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | |   M([inl(i) / s])≤M(s)
| . | 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | |   ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | |   ⇒
| . | |   M([inr(i) / s])<M(s)
| . | 12. d : ℕ
| . | 13. ∀d1:ℕ.
| . | |   d1<d
| . | |   ⇒
| . | |   (∀s:traversal(the_graph), i:Vertices(the_graph).
| . | |    M(s)≤d1
| . | |    ⇒
| . | |    (∃s':traversal(the_graph).
| . | |     M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s)))
| . | 14. s : traversal(the_graph)
| . | 15. i : Vertices(the_graph)
| . | 16. M(s)≤d
| . | 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| . | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| . | |   , f:(T→Vertices(the_graph)→T).
| . | |   ∃L:Vertices(the_graph) List.
| . | |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| . | |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| . | |   ∃L:Vertices(the_graph) List.
| . | |   no_repeats(Vertices(the_graph);L)
| . | |   & (∀y:Vertices(the_graph). (y ∈ L))
| . | |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . | 21. ∀L:Vertices(the_graph) List.
| . | |   (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| . | |   ⇒
| . | |   (∀s2:traversal(the_graph).
| . | |    M(s2)<d
| . | |    ⇒
| . | |    member-paren(x,y.the_obj.eq(x,y);i;s2)
| . | |    ⇒
| . | |    (∃s':traversal(the_graph).
| . | |     M(s' @ s2)≤M(s2)
| . | |     & P(i,s2,s')
| . | |     & list_accum(s',x'.dfs(the_obj;s';x');s2;L) = (s' @ s2)))
```

```
| . | 22. L : Vertices(the_graph) List
| . | 23. ∀y:Vertices(the_graph). i-the_graph->y ⟺ (y ∈ L)
| . | 24. the_obj.eacc((λ2s',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . | |   =
| . | |   list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | 25. s' : traversal(the_graph)
| . | 26. M(s' @ [inr(i) / s])≤M([inr(i) / s])
| . | 27. P(i,[inr(i) / s],s')
| . | 28. list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | |   =
| . | |   (s' @ [inr(i) / s])
| . | 29. j : Vertices(the_graph)
| . | 30. i-the_graph->j
| . | 31. ¬j = i
| . | 32. (j ∈ L)
| . | ⊢  (inl(j) ∈ s') ∨ member-paren(x,y.the_obj.eq(x,y);j;s)
| . | | by (Inst Thm: dfs_accum_member
| . | |     [the_graph;the_obj;L;[inr(i) / s]])
| . | |    THEN
| . | |    ExRepD
| . | |    THEN
| . | |    (HypSubst -1 0)
| . | |    THEN
| . | |    (InstHyp [j] -5)
| . | |    THEN
| . | |    (ParallelOp -1)
| . | |
| . | |\
| . | | 1. the_graph : Graph
| . | | 2. the_obj : GraphObject(the_graph)
| . | | 3. P :
| . | | |  Vertices(the_graph)→traversal(the_graph)→
| . | | |  traversal(the_graph)→Prop
| . | | 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | |  P(i,s1,s2)
| . | | |  ⇒
| . | | |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . | | 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | |  P(i,s1,s2)
| . | | |  ⇒
| . | | |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . | | |  member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| . | | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . | | |  i-the_graph->j
| . | | |  ⇒
| . | | |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . | | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | |  ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| . | | |  ⇒
| . | | |  (∀j:Vertices(the_graph).
| . | | |   i-the_graph->j
| . | | |   ⇒
| . | | |   ¬j = i
| . | | |   ⇒
```

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| . | | |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| . | | |  ⇒
| . | | |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . | | 9. M : traversal(the_graph)→ℕ
| . | | 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | | |   M([inl(i) / s])≤M(s)
| . | | 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | | |   ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | | |   ⇒
| . | | |   M([inr(i) / s])<M(s)
| . | | 12. d : ℕ
| . | | 13. ∀d1:ℕ.
| . | | |   d1<d
| . | | |   ⇒
| . | | |   (∀s:traversal(the_graph), i:Vertices(the_graph).
| . | | |    M(s)≤d1
| . | | |    ⇒
| . | | |    (∃s':traversal(the_graph).
| . | | |     M(s' @ s)≤M(s)
| . | | |     & P(i,s,s')
| . | | |     & dfs(the_obj;s;i) = (s' @ s)))
| . | | 14. s : traversal(the_graph)
| . | | 15. i : Vertices(the_graph)
| . | | 16. M(s)≤d
| . | | 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| . | | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| . | | |   , f:(T→Vertices(the_graph)→T).
| . | | |   ∃L:Vertices(the_graph) List.
| . | | |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| . | | |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . | | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| . | | |   ∃L:Vertices(the_graph) List.
| . | | |   no_repeats(Vertices(the_graph);L)
| . | | |   & (∀y:Vertices(the_graph). (y ∈ L))
| . | | |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . | | 21. ∀L:Vertices(the_graph) List.
| . | | |   (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| . | | |   ⇒
| . | | |   (∀s2:traversal(the_graph).
| . | | |    M(s2)<d
| . | | |    ⇒
| . | | |    member-paren(x,y.the_obj.eq(x,y);i;s2)
| . | | |    ⇒
| . | | |    (∃s':traversal(the_graph).
| . | | |     M(s' @ s2)≤M(s2)
| . | | |     & P(i,s2,s')
| . | | |     & list_accum(s',x'.dfs(the_obj;s';x');s2;L)
| . | | |       =
| . | | |       (s' @ s2)))
| . | | 22. L : Vertices(the_graph) List
| . | | 23. ∀y:Vertices(the_graph). i-the_graph->y ⟺ (y ∈ L)
| . | | 24. the_obj.eacc((λ₂s',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . | | |   =
| . | | |   list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
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| . | | 25. s' : traversal(the_graph)
| . | | 26. M(s' @ [inr(i) / s])≤M([inr(i) / s])
| . | | 27. P(i,[inr(i) / s],s')
| . | | 28. list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | | |   =
| . | | |   (s' @ [inr(i) / s])
| . | | 29. j : Vertices(the_graph)
| . | | 30. i-the_graph->j
| . | | 31. ¬j = i
| . | | 32. (j ∈ L)
| . | | 33. s1 : traversal(the_graph)
| . | | 34. ∀i@0:Vertices(the_graph).
| . | | |   (i@0 ∈ L)
| . | | |   ⇒
| . | | |   (inl(i@0) ∈ s1)
| . | | |   ∨ (inl(i@0) ∈ [inr(i) / s])
| . | | |   ∨ (inr(i@0) ∈ [inr(i) / s])
| . | | 35. l_disjoint(Vertices(the_graph)+
| . | | |   Vertices(the_graph);s1;[inr(i) / s])
| . | | 36. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
| . | | 37. paren(Vertices(the_graph);s1)
| . | | 38. list_accum(s',j.dfs(the_obj;s';j);[inr(i) / s];L)
| . | | |   =
| . | | |   (s1 @ [inr(i) / s])
| . | | 39. (inl(j) ∈ s1)
| . | | ⊢  (inl(j) ∈ s')
| . | | | by Inst
| . | | |    Thm* ∀x,z,y,w:T List.
| . | | |         ||x|| = ||z|| ∈ ℤ ∨ ||y|| = ||w|| ∈ ℤ
| . | | |         ⇒
| . | | |         (x @ y) = (z @ w) ⇒ x = z & y = w
| . | | |    {append_one_one}
| . | | |    [Vertices(the_graph)+Vertices(the_graph)
| . | | |    ;s1
| . | | |    ;s'
| . | | |    ;[inr(i) / s]
| . | | |    ;[inr(i) / s]]
| . | | |
| . | | |\
| . | | | .....antecedent .....
| . | | | 1. the_graph : Graph
| . | | | 2. the_obj : GraphObject(the_graph)
| . | | | 3. P :
| . | | |    Vertices(the_graph)→traversal(the_graph)→
| . | | |    traversal(the_graph)→Prop
| . | | | 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | |    P(i,s1,s2)
| . | | |    ⇒
| . | | |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . | | | 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | |    P(i,s1,s2)
| . | | |    ⇒
| . | | |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | | | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . | | |    member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
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| . | | | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . | | |    i-the_graph->j
| . | | |    ⇒
| . | | |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . | | | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | |    ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| . | | |    ⇒
| . | | |    (∀j:Vertices(the_graph).
| . | | |     i-the_graph->j
| . | | |     ⇒
| . | | |     ¬j = i
| . | | |     ⇒
| . | | |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| . | | |    ⇒
| . | | |    P(i,[inr(i) / s1],s2)
| . | | |    ⇒
| . | | |    P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . | | | 9. M : traversal(the_graph)→ℕ
| . | | | 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | | |     M([inl(i) / s])≤M(s)
| . | | | 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | | |     ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | | |     ⇒
| . | | |     M([inr(i) / s])<M(s)
| . | | | 12. d : ℕ
| . | | | 13. ∀d1:ℕ.
| . | | |     d1<d
| . | | |     ⇒
| . | | |     (∀s:traversal(the_graph), i:Vertices(the_graph).
| . | | |      M(s)≤d1
| . | | |      ⇒
| . | | |      (∃s':traversal(the_graph).
| . | | |       M(s' @ s)≤M(s)
| . | | |       & P(i,s,s')
| . | | |       & dfs(the_obj;s;i) = (s' @ s)))
| . | | | 14. s : traversal(the_graph)
| . | | | 15. i : Vertices(the_graph)
| . | | | 16. M(s)≤d
| . | | | 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | | | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| . | | | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| . | | |     , f:(T→Vertices(the_graph)→T).
| . | | |     ∃L:Vertices(the_graph) List.
| . | | |     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| . | | |     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . | | | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| . | | |     ∃L:Vertices(the_graph) List.
| . | | |     no_repeats(Vertices(the_graph);L)
| . | | |     & (∀y:Vertices(the_graph). (y ∈ L))
| . | | |     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . | | | 21. ∀L:Vertices(the_graph) List.
| . | | |     (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| . | | |     ⇒
| . | | |     (∀s2:traversal(the_graph).
| . | | |      M(s2)<d
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| . | | | |      ⇒
| . | | | |      member-paren(x,y.the_obj.eq(x,y);i;s2)
| . | | | |      ⇒
| . | | | |      (∃s':traversal(the_graph).
| . | | | |       M(s' @ s2)≤M(s2)
| . | | | |       & P(i,s2,s')
| . | | | |       & list_accum(s',x'.dfs(the_obj;s';x');s2;L)
| . | | | |         =
| . | | | |         (s' @ s2)))
| . | | | | 22. L : Vertices(the_graph) List
| . | | | | 23. ∀y:Vertices(the_graph). i-the_graph->y ⟺ (y ∈ L)
| . | | | | 24. the_obj.eacc((λ2s',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . | | | |     =
| . | | | |     list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | | | | 25. s' : traversal(the_graph)
| . | | | | 26. M(s' @ [inr(i) / s])≤M([inr(i) / s])
| . | | | | 27. P(i,[inr(i) / s],s')
| . | | | | 28. list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | | | |     =
| . | | | |     (s' @ [inr(i) / s])
| . | | | | 29. j : Vertices(the_graph)
| . | | | | 30. i-the_graph->j
| . | | | | 31. ¬j = i
| . | | | | 32. (j ∈ L)
| . | | | | 33. s1 : traversal(the_graph)
| . | | | | 34. ∀i@0:Vertices(the_graph).
| . | | | |     (i@0 ∈ L)
| . | | | |     ⇒
| . | | | |     (inl(i@0) ∈ s1)
| . | | | |     ∨ (inl(i@0) ∈ [inr(i) / s])
| . | | | |     ∨ (inr(i@0) ∈ [inr(i) / s])
| . | | | | 35. l_disjoint(Vertices(the_graph)+
| . | | | |     Vertices(the_graph);s1;[inr(i) / s])
| . | | | | 36. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
| . | | | | 37. paren(Vertices(the_graph);s1)
| . | | | | 38. list_accum(s',j.dfs(the_obj;s';j);[inr(i) / s];L)
| . | | | |     =
| . | | | |     (s1 @ [inr(i) / s])
| . | | | | 39. (inl(j) ∈ s1)
| . | | | | ⊢  ||s1|| = ||s'|| ∨ ||[inr(i) / s]|| = ||[inr(i) / s]||
| . | | | |   by ObviousConcl
| . | | | |

| . | | | |\
| . | | | | .....antecedent .....
| . | | | | 1. the_graph : Graph
| . | | | | 2. the_obj : GraphObject(the_graph)
| . | | | | 3. P :
| . | | | |    Vertices(the_graph)→traversal(the_graph)→
| . | | | |    traversal(the_graph)→Prop
| . | | | | 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | | |    P(i,s1,s2)
| . | | | |    ⇒
| . | | | |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . | | | | 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | | |    P(i,s1,s2)
```

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| . | | | |    ⇒
| . | | | |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | | | | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . | | | |    member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| . | | | | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . | | | |    i-the_graph->j
| . | | | |    ⇒
| . | | | |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . | | | | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | | |    ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| . | | | |    ⇒
| . | | | |    (∀j:Vertices(the_graph).
| . | | | |     i-the_graph->j
| . | | | |     ⇒
| . | | | |     ¬j = i
| . | | | |     ⇒
| . | | | |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| . | | | |    ⇒
| . | | | |    P(i,[inr(i) / s1],s2)
| . | | | |    ⇒
| . | | | |    P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . | | | | 9. M : traversal(the_graph)→ℕ
| . | | | | 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | | | |     M([inl(i) / s])≤M(s)
| . | | | | 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | | | |     ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | | | |     ⇒
| . | | | |     M([inr(i) / s])<M(s)
| . | | | | 12. d : ℕ
| . | | | | 13. ∀d1:ℕ.
| . | | | |     d1<d
| . | | | |     ⇒
| . | | | |     (∀s:traversal(the_graph), i:Vertices(the_graph).
| . | | | |      M(s)≤d1
| . | | | |      ⇒
| . | | | |      (∃s':traversal(the_graph).
| . | | | |       M(s' @ s)≤M(s)
| . | | | |       & P(i,s,s')
| . | | | |       & dfs(the_obj;s;i) = (s' @ s)))
| . | | | | 14. s : traversal(the_graph)
| . | | | | 15. i : Vertices(the_graph)
| . | | | | 16. M(s)≤d
| . | | | | 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | | | | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| . | | | | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| . | | | |     , f:(T→Vertices(the_graph)→T).
| . | | | |     ∃L:Vertices(the_graph) List.
| . | | | |     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| . | | | |     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . | | | | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| . | | | |     ∃L:Vertices(the_graph) List.
| . | | | |     no_repeats(Vertices(the_graph);L)
| . | | | |     & (∀y:Vertices(the_graph). (y ∈ L))
| . | | | |     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . | | | | 21. ∀L:Vertices(the_graph) List.
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| . | | | |     (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| . | | | |     ⇒
| . | | | |     (∀s2:traversal(the_graph).
| . | | | |      M(s2)<d
| . | | | |      ⇒
| . | | | |      member-paren(x,y.the_obj.eq(x,y);i;s2)
| . | | | |      ⇒
| . | | | |      (∃s':traversal(the_graph).
| . | | | |       M(s' @ s2)≤M(s2)
| . | | | |       & P(i,s2,s')
| . | | | |       & list_accum(s',x'.dfs(the_obj;s';x');s2;L)
| . | | | |         =
| . | | | |         (s' @ s2)))
| . | | | | 22. L : Vertices(the_graph) List
| . | | | | 23. ∀y:Vertices(the_graph). i-the_graph->y ⟺ (y ∈ L)
| . | | | | 24. the_obj.eacc((λ2s',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . | | | |     =
| . | | | |     list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | | | | 25. s' : traversal(the_graph)
| . | | | | 26. M(s' @ [inr(i) / s])≤M([inr(i) / s])
| . | | | | 27. P(i,[inr(i) / s],s')
| . | | | | 28. list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | | | |     =
| . | | | |     (s' @ [inr(i) / s])
| . | | | | 29. j : Vertices(the_graph)
| . | | | | 30. i-the_graph->j
| . | | | | 31. ¬j = i
| . | | | | 32. (j ∈ L)
| . | | | | 33. s1 : traversal(the_graph)
| . | | | | 34. ∀i@0:Vertices(the_graph).
| . | | | |     (i@0 ∈ L)
| . | | | |     ⇒
| . | | | |     (inl(i@0) ∈ s1)
| . | | | |     ∨ (inl(i@0) ∈ [inr(i) / s])
| . | | | |     ∨ (inr(i@0) ∈ [inr(i) / s])
| . | | | | 35. l_disjoint(Vertices(the_graph)+
| . | | | |     Vertices(the_graph);s1;[inr(i) / s])
| . | | | | 36. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
| . | | | | 37. paren(Vertices(the_graph);s1)
| . | | | | 38. list_accum(s',j.dfs(the_obj;s';j);[inr(i) / s];L)
| . | | | |     =
| . | | | |     (s1 @ [inr(i) / s])
| . | | | | 39. (inl(j) ∈ s1)
| . | | | | ⊢  (s1 @ [inr(i) / s]) = (s' @ [inr(i) / s])
| . | | | |   by Fold `traversal` 0
| . | | | |
| . | | | .\
| . | | | . 1. the_graph : Graph
| . | | | . 2. the_obj : GraphObject(the_graph)
| . | | | . 3. P :
| . | | | .    Vertices(the_graph)→traversal(the_graph)→
| . | | | .    traversal(the_graph)→Prop
| . | | | . 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | | .    P(i,s1,s2)
| . | | | .    ⇒
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| . | | .    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . | | . 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | .    P(i,s1,s2)
| . | | .    ⇒
| . | | .    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | | . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . | | .    member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| . | | . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . | | .    i-the_graph->j
| . | | .    ⇒
| . | | .    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . | | . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | | .    ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| . | | .    ⇒
| . | | .    (∀j:Vertices(the_graph).
| . | | .     i-the_graph->j
| . | | .     ⇒
| . | | .     ¬j = i
| . | | .     ⇒
| . | | .     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| . | | .    ⇒
| . | | .    P(i,[inr(i) / s1],s2)
| . | | .    ⇒
| . | | .    P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . | | . 9. M : traversal(the_graph)→ℕ
| . | | . 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | | .     M([inl(i) / s])≤M(s)
| . | | . 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | | .     ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | | .     ⇒
| . | | .     M([inr(i) / s])<M(s)
| . | | . 12. d : ℕ
| . | | . 13. ∀d1:ℕ.
| . | | .     d1<d
| . | | .     ⇒
| . | | .     (∀s:traversal(the_graph), i:Vertices(the_graph).
| . | | .      M(s)≤d1
| . | | .      ⇒
| . | | .      (∃s':traversal(the_graph).
| . | | .       M(s' @ s)≤M(s)
| . | | .       & P(i,s,s')
| . | | .       & dfs(the_obj;s;i) = (s' @ s)))
| . | | . 14. s : traversal(the_graph)
| . | | . 15. i : Vertices(the_graph)
| . | | . 16. M(s)≤d
| . | | . 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | | . 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| . | | . 19. ∀T:Type, s:T, x:Vertices(the_graph)
| . | | .     , f:(T→Vertices(the_graph)→T).
| . | | .     ∃L:Vertices(the_graph) List.
| . | | .     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| . | | .     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . | | . 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| . | | .     ∃L:Vertices(the_graph) List.
| . | | .     no_repeats(Vertices(the_graph);L)
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| . | | .     & (∀y:Vertices(the_graph). (y ∈ L))
| . | | .     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . | | . 21. ∀L:Vertices(the_graph) List.
| . | | .     (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| . | | .     ⇒
| . | | .     (∀s2:traversal(the_graph).
| . | | .      M(s2)<d
| . | | .      ⇒
| . | | .      member-paren(x,y.the_obj.eq(x,y);i;s2)
| . | | .      ⇒
| . | | .      (∃s':traversal(the_graph).
| . | | .       M(s' @ s2)≤M(s2)
| . | | .       & P(i,s2,s')
| . | | .       & list_accum(s',x'.dfs(the_obj;s';x');s2;L)
| . | | .         =
| . | | .         (s' @ s2)))
| . | | . 22. L : Vertices(the_graph) List
| . | | . 23. ∀y:Vertices(the_graph). i-the_graph->y ⟺ (y ∈ L)
| . | | . 24. the_obj.eacc((λ2s',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . | | .     =
| . | | .     list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | | . 25. s' : traversal(the_graph)
| . | | . 26. M(s' @ [inr(i) / s])≤M([inr(i) / s])
| . | | . 27. P(i,[inr(i) / s],s')
| . | | . 28. list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | | .     =
| . | | .     (s' @ [inr(i) / s])
| . | | . 29. j : Vertices(the_graph)
| . | | . 30. i-the_graph->j
| . | | . 31. ¬j = i
| . | | . 32. (j ∈ L)
| . | | . 33. s1 : traversal(the_graph)
| . | | . 34. ∀i@0:Vertices(the_graph).
| . | | .     (i@0 ∈ L)
| . | | .     ⇒
| . | | .     (inl(i@0) ∈ s1)
| . | | .     ∨ (inl(i@0) ∈ [inr(i) / s])
| . | | .     ∨ (inr(i@0) ∈ [inr(i) / s])
| . | | . 35. l_disjoint(Vertices(the_graph)+
| . | | .     Vertices(the_graph);s1;[inr(i) / s])
| . | | . 36. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
| . | | . 37. paren(Vertices(the_graph);s1)
| . | | . 38. list_accum(s',j.dfs(the_obj;s';j);[inr(i) / s];L)
| . | | .     =
| . | | .     (s1 @ [inr(i) / s])
| . | | . 39. (inl(j) ∈ s1)
| . | | . 40. s1 = s'
| . | | . 41. [inr(i) / s] = [inr(i) / s]
| . | | . ⊢  (inl(j) ∈ s')  by HypSubst -2 -3
| . | |
| . | .\
| . | . 1. the_graph : Graph
| . | . 2. the_obj : GraphObject(the_graph)
| . | . 3. P :
| . | .    Vertices(the_graph)→traversal(the_graph)→
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| . | .    traversal(the_graph)→Prop
| . | . 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | .    P(i,s1,s2)
| . | .    ⇒
| . | .    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . | . 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | .    P(i,s1,s2)
| . | .    ⇒
| . | .    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . | .    member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| . | . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . | .    i-the_graph->j
| . | .    ⇒
| . | .    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . | . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . | .    ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| . | .    ⇒
| . | .    (∀j:Vertices(the_graph).
| . | .     i-the_graph->j
| . | .     ⇒
| . | .     ¬j = i
| . | .     ⇒
| . | .     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| . | .    ⇒
| . | .    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . | . 9. M : traversal(the_graph)→ℕ
| . | . 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | .     M([inl(i) / s])≤M(s)
| . | . 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | .     ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | .     ⇒
| . | .     M([inr(i) / s])<M(s)
| . | . 12. d : ℕ
| . | . 13. ∀d1:ℕ.
| . | .     d1<d
| . | .     ⇒
| . | .     (∀s:traversal(the_graph), i:Vertices(the_graph).
| . | .      M(s)≤d1
| . | .      ⇒
| . | .      (∃s':traversal(the_graph).
| . | .       M(s' @ s)≤M(s)
| . | .       & P(i,s,s')
| . | .       & dfs(the_obj;s;i) = (s' @ s)))
| . | . 14. s : traversal(the_graph)
| . | . 15. i : Vertices(the_graph)
| . | . 16. M(s)≤d
| . | . 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | . 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| . | . 19. ∀T:Type, s:T, x:Vertices(the_graph)
| . | .     , f:(T→Vertices(the_graph)→T).
| . | .     ∃L:Vertices(the_graph) List.
| . | .     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| . | .     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . | . 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
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| . | .     ∃L:Vertices(the_graph) List.
| . | .     no_repeats(Vertices(the_graph);L)
| . | .     & (∀y:Vertices(the_graph). (y ∈ L))
| . | .     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . | . 21. ∀L:Vertices(the_graph) List.
| . | .     (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| . | .     ⇒
| . | .     (∀s2:traversal(the_graph).
| . | .      M(s2)<d
| . | .      ⇒
| . | .      member-paren(x,y.the_obj.eq(x,y);i;s2)
| . | .      ⇒
| . | .      (∃s':traversal(the_graph).
| . | .       M(s' @ s2)≤M(s2)
| . | .       & P(i,s2,s')
| . | .       & list_accum(s',x'.dfs(the_obj;s';x');s2;L)
| . | .         =
| . | .         (s' @ s2)))
| . | . 22. L : Vertices(the_graph) List
| . | . 23. ∀y:Vertices(the_graph). i-the_graph->y ⟺ (y ∈ L)
| . | . 24. the_obj.eacc((λ2s',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . | .     =
| . | .     list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | . 25. s' : traversal(the_graph)
| . | . 26. M(s' @ [inr(i) / s])≤M([inr(i) / s])
| . | . 27. P(i,[inr(i) / s],s')
| . | . 28. list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | .     =
| . | .     (s' @ [inr(i) / s])
| . | . 29. j : Vertices(the_graph)
| . | . 30. i-the_graph->j
| . | . 31. ¬j = i
| . | . 32. (j ∈ L)
| . | . 33. s1 : traversal(the_graph)
| . | . 34. ∀i@0:Vertices(the_graph).
| . | .     (i@0 ∈ L)
| . | .     ⇒
| . | .     (inl(i@0) ∈ s1)
| . | .     ∨ (inl(i@0) ∈ [inr(i) / s])
| . | .     ∨ (inr(i@0) ∈ [inr(i) / s])
| . | . 35. l_disjoint(Vertices(the_graph)+
| . | .     Vertices(the_graph);s1;[inr(i) / s])
| . | . 36. no_repeats(Vertices(the_graph)+Vertices(the_graph);s1)
| . | . 37. paren(Vertices(the_graph);s1)
| . | . 38. list_accum(s',j.dfs(the_obj;s';j);[inr(i) / s];L)
| . | .     =
| . | .     (s1 @ [inr(i) / s])
| . | . 39. (inl(j) ∈ [inr(i) / s]) ∨ (inr(j) ∈ [inr(i) / s])
| . | . ⊢  member-paren(x,y.the_obj.eq(x,y);j;s)
| . | .   by (RW0
| . | .       Thm* ∀E:(T→T→𝔹).
| . | .            (∀x,y:T. E(x,y) ⟺ x = y)
| . | .            ⇒
| . | .            (∀i:T, s:(T+T) List.
| . | .             member-paren(x,y.E(x,y);i;s)
```

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| . | .             ⟺
| . | .             (inl(i) ∈ s) ∨ (inr(i) ∈ s))
| . | .       {assert_DASH_member_DASH_paren}
| . | .       0)
| . | .      THEN
| . | .      (ObviousFrom [-1;-9])
| . |
| . |\
| . | .....subterm .....
| . | 1. the_graph : Graph
| . | 2. the_obj : GraphObject(the_graph)
| . | 3. P :
| . |    Vertices(the_graph)→traversal(the_graph)→
| . |    traversal(the_graph)→Prop
| . | 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . |    P(i,s1,s2)
| . |    ⇒
| . |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . | 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . |    P(i,s1,s2)
| . |    ⇒
| . |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . |    member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| . | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . |    i-the_graph->j
| . |    ⇒
| . |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . |    ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| . |    ⇒
| . |    (∀j:Vertices(the_graph).
| . |     i-the_graph->j
| . |     ⇒
| . |     ¬j = i
| . |     ⇒
| . |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| . |    ⇒
| . |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . | 9. M : traversal(the_graph)→ℕ
| . | 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . |     M([inl(i) / s])≤M(s)
| . | 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . |     ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . |     ⇒
| . |     M([inr(i) / s])<M(s)
| . | 12. d : ℕ
| . | 13. ∀d1:ℕ.
| . |     d1<d
| . |     ⇒
| . |     (∀s:traversal(the_graph), i:Vertices(the_graph).
| . |      M(s)≤d1
| . |      ⇒
| . |      (∃s':traversal(the_graph).
| . |       M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s)))
```

```
| . | 14. s : traversal(the_graph)
| . | 15. i : Vertices(the_graph)
| . | 16. M(s)≤d
| . | 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| . | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| . |     , f:(T→Vertices(the_graph)→T).
| . |     ∃L:Vertices(the_graph) List.
| . |     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| . |     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| . |     ∃L:Vertices(the_graph) List.
| . |     no_repeats(Vertices(the_graph);L)
| . |     & (∀y:Vertices(the_graph). (y ∈ L))
| . |     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . | 21. ∀L:Vertices(the_graph) List.
| . |     (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| . |     ⇒
| . |     (∀s2:traversal(the_graph).
| . |      M(s2)<d
| . |      ⇒
| . |      member-paren(x,y.the_obj.eq(x,y);i;s2)
| . |      ⇒
| . |      (∃s':traversal(the_graph).
| . |       M(s' @ s2)≤M(s2)
| . |       & P(i,s2,s')
| . |       & list_accum(s',x'.dfs(the_obj;s';x');s2;L) = (s' @ s2)))
| . | 22. L : Vertices(the_graph) List
| . | 23. ∀y:Vertices(the_graph). i-the_graph->y ⟺ (y ∈ L)
| . | 24. the_obj.eacc((λ2s',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . |     =
| . |     list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . | 25. s' : traversal(the_graph)
| . | 26. M(s' @ [inr(i) / s])≤M([inr(i) / s])
| . | 27. P(i,[inr(i) / s],s')
| . | 28. list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . |     =
| . |     (s' @ [inr(i) / s])
| . | ⊢  the_obj.eacc((λs',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . |    =
| . |    (s' @ [inr(i) / s])
| . |   by (All (Unfold `so_lambda`)) THEN (Fold `traversal` 0)
| . |
| . .\
| . . 1. the_graph : Graph
| . . 2. the_obj : GraphObject(the_graph)
| . . 3. P :
| . .    Vertices(the_graph)→traversal(the_graph)→
| . .    traversal(the_graph)→Prop
| . . 4. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . .    P(i,s1,s2)
| . .    ⇒
| . .    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . . 5. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . .    P(i,s1,s2)
```

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| . .    ⇒
| . .    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . .    member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
| . . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . .    i-the_graph->j
| . .    ⇒
| . .    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . .    ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
| . .    ⇒
| . .    (∀j:Vertices(the_graph).
| . .     i-the_graph->j
| . .     ⇒
| . .     ¬j = i
| . .     ⇒
| . .     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
| . .    ⇒
| . .    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . . 9. M : traversal(the_graph)→ℕ
| . . 10. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . .     M([inl(i) / s])≤M(s)
| . . 11. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . .     ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . .     ⇒
| . .     M([inr(i) / s])<M(s)
| . . 12. d : ℕ
| . . 13. ∀d1:ℕ.
| . .     d1<d
| . .     ⇒
| . .     (∀s:traversal(the_graph), i:Vertices(the_graph).
| . .      M(s)≤d1
| . .      ⇒
| . .      (∃s':traversal(the_graph).
| . .       M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s)))
| . . 14. s : traversal(the_graph)
| . . 15. i : Vertices(the_graph)
| . . 16. M(s)≤d
| . . 17. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . . 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| . . 19. ∀T:Type, s:T, x:Vertices(the_graph)
| . .     , f:(T→Vertices(the_graph)→T).
| . .     ∃L:Vertices(the_graph) List.
| . .     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| . .     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . . 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| . .     ∃L:Vertices(the_graph) List.
| . .     no_repeats(Vertices(the_graph);L)
| . .     & (∀y:Vertices(the_graph). (y ∈ L))
| . .     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . . 21. ∀L:Vertices(the_graph) List.
| . .     (∀y:Vertices(the_graph). (y ∈ L) ⇒ i-the_graph->y)
| . .     ⇒
| . .     (∀s2:traversal(the_graph).
| . .      M(s2)<d
```

```
| . .      ⇒
| . .      member-paren(x,y.the_obj.eq(x,y);i;s2)
| . .      ⇒
| . .      (∃s':traversal(the_graph).
| . .       M(s' @ s2)≤M(s2)
| . .       & P(i,s2,s')
| . .       & list_accum(s',x'.dfs(the_obj;s';x');s2;L) = (s' @ s2)))
| . . 22. L : Vertices(the_graph) List
| . . 23. ∀y:Vertices(the_graph). i-the_graph->y ⟺ (y ∈ L)
| . . 24. the_obj.eacc((λ2s',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . .     =
| . .     list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . . 25. s' : traversal(the_graph)
| . . 26. M(s' @ [inr(i) / s])≤M([inr(i) / s])
| . . 27. P(i,[inr(i) / s],s')
| . . 28. list_accum(s',x'.dfs(the_obj;s';x');[inr(i) / s];L)
| . .     =
| . .     (s' @ [inr(i) / s])
| . . 29. traversal(the_graph)
| . . ⊢  the_obj.eacc((λs',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . .    ∈ (Vertices(the_graph)+Vertices(the_graph)) List
| . .   by (Fold `traversal` 0)
| . .      THEN
| . .      (Inst
| . .       Thm* For any graph
| . .            ∀t:GraphObject(the_graph), T:Type.
| . .            t.eacc ∈ (T→V→T)→T→V→T
| . .       {gro_eacc_wf}
| . .       [the_graph;the_obj;traversal(the_graph)])
|
.\
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. P :
.    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
.    Prop
. 4. s : traversal(the_graph)
. 5. i : Vertices(the_graph)
. 6. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
.    P(i,s1,s2)
.    ⇒
.    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. 7. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
.    P(i,s1,s2)
.    ⇒
.    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. 8. ∀s:traversal(the_graph), i:Vertices(the_graph).
.    member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ P(i,s,nil)
. 9. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
.    i-the_graph->j
.    ⇒
.    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
. 10. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
.     ¬member-paren(x,y.the_obj.eq(x,y);i;s1)
.     ⇒
```

```
.     (∀j:Vertices(the_graph).
.      i-the_graph->j
.      ⇒
.      ¬j = i
.      ⇒
.      (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1))
.     ⇒
.     P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
. 11. M : traversal(the_graph)→ℕ
. 12. ∀i:Vertices(the_graph), s:traversal(the_graph).
.     M([inl(i) / s])≤M(s)
. 13. ∀i:Vertices(the_graph), s:traversal(the_graph).
.     ¬member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ M([inr(i) / s])<M(s)
. 14. ∀d:ℕ, s:traversal(the_graph), i:Vertices(the_graph).
.     M(s)≤d
.     ⇒
.     (∃s':traversal(the_graph).
.      M(s' @ s)≤M(s) & P(i,s,s') & dfs(the_obj;s;i) = (s' @ s))
. ⊢  ∃s':traversal(the_graph). P(i,s,s') & dfs(the_obj;s;i) = (s' @ s)
.   by (InstHyp [M(s);s;i] -1) THEN (ParallelOp -1)
END
```