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dfs_induction4  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), P:(V→Traversal→Traversal→Prop)
|  , s:Traversal, i:V.
|  (∀s:Traversal, i:V. (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil))
|  ⇒
|  (∀s1,s2,s3:Traversal, i,j:V.
|   i-the_graph->j
|   ⇒
|   paren(V;s2)
|   ⇒
|   (∀k:V. (inr(k) ∈ s2) ⇒ j-the_graph->*k)
|   ⇒
|   paren(V;s3) ⇒ P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2))
|  ⇒
|  (∀s1,s2:Traversal, i:V.
|   ¬(inr(i) ∈ s1)
|   ⇒
|   ¬(inl(i) ∈ s1)
|   ⇒
|   paren(V;s2)
|   ⇒
|   l_disjoint(V+V;s2;s1)
|   ⇒
|   no_repeats(V+V;s2)
|   ⇒
|   (∀j:V. (inr(j) ∈ s2) ⇒ i-the_graph->*j)
|   ⇒
|   (∀j:V.
|    i-the_graph->j
|    ⇒
|    ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
|   ⇒
|   P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])]))
|  ⇒
|  (∃s':Traversal.
|   P(i,s,s')
|   & l_disjoint(V+V;s';s) & no_repeats(V+V;s')
|   & paren(V;s')
|   & (∀j:V. (inr(j) ∈ s') ⇒ i-the_graph->*j)
|   & dfs(the_obj;s;i) = (s' @ s))
| by Auto
|    THEN
|    (Inst Thm: dfs_induction3
|     [the_graph
|     ;the_obj
|     ;λ₂i,s1,s2.
|      P(i,s1,s2)
|      & l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
|      & no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
|      & paren(Vertices(the_graph);s2)
|      & (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
|     ;s
|     ;i])
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|    THEN
|    EasyHyp
|
|\
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. P :
|    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
|    Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
|    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
|    i-the_graph->j
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
|    ⇒
|    paren(Vertices(the_graph);s3)
|    ⇒
|    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
|    ¬(inr(i) ∈ s1)
|    ⇒
|    ¬(inl(i) ∈ s1)
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
|    ⇒
|    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
|    ⇒
|    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
|    ⇒
|    (∀j:Vertices(the_graph).
|     i-the_graph->j
|     ⇒
|     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
|    ⇒
|    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. i1 : Vertices(the_graph)
| 11. member-paren(x,y.the_obj.eq(x,y);i1;s1)
| ⊢  P(i1,s1,nil)
|   by (Inst Thm: graphobj_DASH_properties [the_graph;the_obj])
|
|      THEN
|      (RWO
|       Thm* ∀E:(T→T→ℙ).
|            (∀x,y:T. E(x,y) ⟺ x = y)
|            ⇒
|            (∀i:T, s:(T+T) List.
|             member-paren(x,y.E(x,y);i;s)
|             ⟺
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|             (inl(i) ∈ s) ∨ (inr(i) ∈ s))
|       {assert_DASH_member_DASH_paren}
|       -2)
|      THEN
|      EasyHyp
|
|\
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. P :
|    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
|    Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
|    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
|    i-the_graph->j
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
|    ⇒
|    paren(Vertices(the_graph);s3)
|    ⇒
|    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
|    ¬(inr(i) ∈ s1)
|    ⇒
|    ¬(inl(i) ∈ s1)
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
|    ⇒
|    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
|    ⇒
|    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
|    ⇒
|    (∀j:Vertices(the_graph).
|     i-the_graph->j
|     ⇒
|     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
|    ⇒
|    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. i1 : Vertices(the_graph)
| 11. member-paren(x,y.the_obj.eq(x,y);i1;s1)
| ⊢  l_disjoint(Vertices(the_graph)+Vertices(the_graph);nil;s1)
|   by ObviousConcl
|
|\
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. P :
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|    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
|    Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
|    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
|    i-the_graph->j
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
|    ⇒
|    paren(Vertices(the_graph);s3)
|    ⇒
|    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
|    ¬(inr(i) ∈ s1)
|    ⇒
|    ¬(inl(i) ∈ s1)
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
|    ⇒
|    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
|    ⇒
|    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
|    ⇒
|    (∀j:Vertices(the_graph).
|     i-the_graph->j
|     ⇒
|     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
|    ⇒
|    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. i1 : Vertices(the_graph)
|
| 11. member-paren(x,y.the_obj.eq(x,y);i1;s1)
| ⊢  no_repeats(Vertices(the_graph)+Vertices(the_graph);nil)
|   by ObviousConcl
|
|\
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. P :
|    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
|    Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
|    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
|    i-the_graph->j
|    ⇒
|    paren(Vertices(the_graph);s2)
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|    ⇒
|    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
|    ⇒
|    paren(Vertices(the_graph);s3)
|    ⇒
|    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
|    ¬(inr(i) ∈ s1)
|    ⇒
|    ¬(inl(i) ∈ s1)
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
|    ⇒
|    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
|    ⇒
|    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
|    ⇒
|    (∀j:Vertices(the_graph).
|     i-the_graph->j
|     ⇒
|     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
|    ⇒
|    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. i1 : Vertices(the_graph)
| 11. member-paren(x,y.the_obj.eq(x,y);i1;s1)
| ⊢  paren(Vertices(the_graph);nil)  by ObviousConcl
|
|\
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. P :
|    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
|    Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
|    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
|    i-the_graph->j
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
|    ⇒
|    paren(Vertices(the_graph);s3)
|    ⇒
|    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
|    ¬(inr(i) ∈ s1)
|    ⇒
|    ¬(inl(i) ∈ s1)
|    ⇒
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|    paren(Vertices(the_graph);s2)
|    ⇒
|    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
|    ⇒
|    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
|    ⇒
|    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
|    ⇒
|    (∀j:Vertices(the_graph).
|     i-the_graph->j
|     ⇒
|     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
|    ⇒
|    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. i1 : Vertices(the_graph)
| 11. member-paren(x,y.the_obj.eq(x,y);i1;s1)
| 12. j : Vertices(the_graph)
| 13. (inr(j) ∈ nil)
| ⊢  i1-the_graph->*j  by ObviousContradiction [-1]
|
|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. P :
|    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
|    Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
|    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
|    i-the_graph->j
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
|    ⇒
|    paren(Vertices(the_graph);s3)
|    ⇒
|    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
|    ¬(inr(i) ∈ s1)
|    ⇒
|    ¬(inl(i) ∈ s1)
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
|    ⇒
|    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
|    ⇒
|    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
|    ⇒
|    (∀j:Vertices(the_graph).
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|     i-the_graph->j
|     ⇒
|     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
|    ⇒
|    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. s2 : traversal(the_graph)
| 11. s3 : traversal(the_graph)
| 12. i1 : Vertices(the_graph)
| 13. j : Vertices(the_graph)
| 14. i1-the_graph->j
| 15. P(j,s1,s2)
| 16. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| 18. paren(Vertices(the_graph);s2)
| 19. ∀j@0:Vertices(the_graph). (inr(j@0) ∈ s2) ⇒ j-the_graph->*j@0
| 20. P(i1,s2 @ s1,s3)
| 21. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s3;s2 @ s1)
| 22. no_repeats(Vertices(the_graph)+Vertices(the_graph);s3)
| 23. paren(Vertices(the_graph);s3)
| 24. ∀j:Vertices(the_graph). (inr(j) ∈ s3) ⇒ i1-the_graph->*j
| ⊢  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s3 @ s2;s1)
|   by (RWO
|       Thm* ∀a,b,c:T List.
|            l_disjoint(T;c;a @ b)
|            ⟺
|            l_disjoint(T;c;a) & l_disjoint(T;c;b)
|       {l_disjoint_append2}
|       -4)
|      THEN
|      (RWO
|       Thm* ∀a,b,c:T List.
|            l_disjoint(T;a @ b;c)
|            ⟺
|            l_disjoint(T;a;c) & l_disjoint(T;b;c)
|       {l_disjoint_append}
|       0)
|
|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. P :
|    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
|    Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
|    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
|    i-the_graph->j
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
|    ⇒
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|    paren(Vertices(the_graph);s3)
|    ⇒
|    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
|    ¬(inr(i) ∈ s1)
|    ⇒
|    ¬(inl(i) ∈ s1)
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
|    ⇒
|    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
|    ⇒
|    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
|    ⇒
|    (∀j:Vertices(the_graph).
|     i-the_graph->j
|     ⇒
|     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
|    ⇒
|    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. s2 : traversal(the_graph)
| 11. s3 : traversal(the_graph)
| 12. i1 : Vertices(the_graph)
| 13. j : Vertices(the_graph)
| 14. i1-the_graph->j
| 15. P(j,s1,s2)
| 16. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| 18. paren(Vertices(the_graph);s2)
| 19. ∀j@0:Vertices(the_graph). (inr(j@0) ∈ s2) ⇒ j-the_graph->*j@0
| 20. P(i1,s2 @ s1,s3)
| 21. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s3;s2 @ s1)
| 22. no_repeats(Vertices(the_graph)+Vertices(the_graph);s3)
| 23. paren(Vertices(the_graph);s3)
| 24. ∀j:Vertices(the_graph). (inr(j) ∈ s3) ⇒ i1-the_graph->*j
| ⊢  no_repeats(Vertices(the_graph)+Vertices(the_graph);s3 @ s2)
|   by (RWO
|       Thm* ∀a,b,c:T List.
|            l_disjoint(T;c;a @ b)
|            ⟺
|            l_disjoint(T;c;a) & l_disjoint(T;c;b)
|       {l_disjoint_append2}
|       -4)
|      THEN
|      (RWO
|       Thm* ∀l1,l2:T List.
|            no_repeats(T;l1 @ l2)
|            ⟺
|            l_disjoint(T;l1;l2) & no_repeats(T;l1) & no_repeats(T;l2)
|       {no_repeats_append_iff}
|       0)
|
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|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. P :
|    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
|    Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
|    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
|    i-the_graph->j
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
|    ⇒
|    paren(Vertices(the_graph);s3)
|    ⇒
|    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
|    ¬(inr(i) ∈ s1)
|    ⇒
|    ¬(inl(i) ∈ s1)
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
|    ⇒
|    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
|    ⇒
|    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
|    ⇒
|    (∀j:Vertices(the_graph).
|     i-the_graph->j
|     ⇒
|     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
|    ⇒
|    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. s2 : traversal(the_graph)
| 11. s3 : traversal(the_graph)
| 12. i1 : Vertices(the_graph)
| 13. j : Vertices(the_graph)
| 14. i1-the_graph->j
| 15. P(j,s1,s2)
| 16. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| 18. paren(Vertices(the_graph);s2)
| 19. ∀j@0:Vertices(the_graph). (inr(j@0) ∈ s2) ⇒ j-the_graph->*j@0
| 20. P(i1,s2 @ s1,s3)
| 21. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s3;s2 @ s1)
| 22. no_repeats(Vertices(the_graph)+Vertices(the_graph);s3)
| 23. paren(Vertices(the_graph);s3)
| 24. ∀j:Vertices(the_graph). (inr(j) ∈ s3) ⇒ i1-the_graph->*j
```

```
| ⊢  paren(Vertices(the_graph);s3 @ s2)
|   by BackThru
|      Thm* ∀s',s3:(T+T) List.
|           paren(T;s') ⇒ paren(T;s3) ⇒ paren(T;s3 @ s')
|      {append_paren}
|
|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. P :
| |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
| |  Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |  (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |  i-the_graph->j
| |  ⇒
| |  paren(Vertices(the_graph);s2)
| |  ⇒
| |  (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| |  ⇒
| |  paren(Vertices(the_graph);s3)
| |  ⇒
| |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |  ¬(inr(i) ∈ s1)
| |  ⇒
| |  ¬(inl(i) ∈ s1)
| |  ⇒
| |  paren(Vertices(the_graph);s2)
| |  ⇒
| |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| |  ⇒
| |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| |  ⇒
| |  (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| |  ⇒
| |  (∀j:Vertices(the_graph).
| |   i-the_graph->j
| |   ⇒
| |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| |  ⇒
| |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. s2 : traversal(the_graph)
| 11. s3 : traversal(the_graph)
| 12. i1 : Vertices(the_graph)
| 13. j : Vertices(the_graph)
| 14. i1-the_graph->j
| 15. P(j,s1,s2)
| 16. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| 18. paren(Vertices(the_graph);s2)
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```
| 19. ∀j@0:Vertices(the_graph). (inr(j@0) ∈ s2) ⇒ j-the_graph->*j@0
| 20. P(i1,s2 @ s1,s3)
| 21. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s3;s2 @ s1)
| 22. no_repeats(Vertices(the_graph)+Vertices(the_graph);s3)
| 23. paren(Vertices(the_graph);s3)
| 24. ∀j:Vertices(the_graph). (inr(j) ∈ s3) ⇒ i1-the_graph->*j
| 25. j1 : Vertices(the_graph)
| 26. (inr(j1) ∈ s3 @ s2)
| ⊢  i1-the_graph->*j1
| | by (RWO
| |     Thm* ∀x:T, l1,l2:T List. (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| |     {member_append}
| |     -1)
| |    THEN
| |    SplitOrHyps
| |
| |\
| | 1. the_graph : Graph
| | 2. GraphObject(the_graph)
| | 3. P :
| |    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| |    →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |    i-the_graph->j
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| |    ⇒
| |    paren(Vertices(the_graph);s3)
| |    ⇒
| |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |    ¬(inr(i) ∈ s1)
| |    ⇒
| |    ¬(inl(i) ∈ s1)
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| |    ⇒
| |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| |    ⇒
| |    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| |    ⇒
| |    (∀j:Vertices(the_graph).
| |     i-the_graph->j
| |     ⇒
| |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| |    ⇒
| |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
```

```
| | 9. s1 : traversal(the_graph)
| | 10. s2 : traversal(the_graph)
| | 11. s3 : traversal(the_graph)
| | 12. i1 : Vertices(the_graph)
| | 13. j : Vertices(the_graph)
| | 14. i1-the_graph->j
| | 15. P(j,s1,s2)
| | 16. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 18. paren(Vertices(the_graph);s2)
| | 19. ∀j@0:Vertices(the_graph). (inr(j@0) ∈ s2) ⇒ j-the_graph->*j@0
| | 20. P(i1,s2 @ s1,s3)
| | 21. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s3;s2 @ s1)
| | 22. no_repeats(Vertices(the_graph)+Vertices(the_graph);s3)
| | 23. paren(Vertices(the_graph);s3)
| | 24. ∀j:Vertices(the_graph). (inr(j) ∈ s3) ⇒ i1-the_graph->*j
| | 25. j1 : Vertices(the_graph)
| | 26. (inr(j1) ∈ s3)
| | ⊢  i1-the_graph->*j1  by EasyHyp
| |
| .\
| . 1. the_graph : Graph
| . 2. GraphObject(the_graph)
| . 3. P :
| .    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| .    →Prop
| . 4. traversal(the_graph)
| . 5. Vertices(the_graph)
| . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| .    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| .    i-the_graph->j
| .    ⇒
| .    paren(Vertices(the_graph);s2)
| .    ⇒
| .    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| .    ⇒
| .    paren(Vertices(the_graph);s3)
| .    ⇒
| .    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| .    ¬(inr(i) ∈ s1)
| .    ⇒
| .    ¬(inl(i) ∈ s1)
| .    ⇒
| .    paren(Vertices(the_graph);s2)
| .    ⇒
| .    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| .    ⇒
| .    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| .    ⇒
| .    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| .    ⇒
| .    (∀j:Vertices(the_graph).
| .     i-the_graph->j
```

```
| .     ⇒
| .     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| .    ⇒
| .    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . 9. s1 : traversal(the_graph)
| . 10. s2 : traversal(the_graph)
| . 11. s3 : traversal(the_graph)
| . 12. i1 : Vertices(the_graph)
| . 13. j : Vertices(the_graph)
| . 14. i1-the_graph->j
| . 15. P(j,s1,s2)
| . 16. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . 18. paren(Vertices(the_graph);s2)
| . 19. ∀j@0:Vertices(the_graph). (inr(j@0) ∈ s2) ⇒ j-the_graph->*j@0
| . 20. P(i1,s2 @ s1,s3)
| . 21. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s3;s2 @ s1)
| . 22. no_repeats(Vertices(the_graph)+Vertices(the_graph);s3)
| . 23. paren(Vertices(the_graph);s3)
| . 24. ∀j:Vertices(the_graph). (inr(j) ∈ s3) ⇒ i1-the_graph->*j
| . 25. j1 : Vertices(the_graph)
| . 26. (inr(j1) ∈ s2)
| . ⊢  i1-the_graph->*j1
| .   by (AssertBY j-the_graph->*j1 EasyHyp) THEN Connect
|
|\
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. P :
| |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
| |  Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |  (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |  i-the_graph->j
| |  ⇒
| |  paren(Vertices(the_graph);s2)
| |  ⇒
| |  (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| |  ⇒
| |  paren(Vertices(the_graph);s3)
| |  ⇒
| |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |  ¬(inr(i) ∈ s1)
| |  ⇒
| |  ¬(inl(i) ∈ s1)
| |  ⇒
| |  paren(Vertices(the_graph);s2)
| |  ⇒
| |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| |  ⇒
| |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
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| |  ⇒
| |  (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| |  ⇒
| |  (∀j:Vertices(the_graph).
| |   i-the_graph->j
| |   ⇒
| |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| |  ⇒
| |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. s2 : traversal(the_graph)
| 11. i1 : Vertices(the_graph)
| 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
| 13. ∀j:Vertices(the_graph).
| |   i1-the_graph->j
| |   ⇒
| |   ¬j = i1
| |   ⇒
| |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| 14. P(i1,[inr(i1) / s1],s2)
| 15. l_disjoint(Vertices(the_graph)+
| |   Vertices(the_graph);s2;[inr(i1) / s1])
| 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| 17. paren(Vertices(the_graph);s2)
| 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| ⊢  P(i1,s1,[inl(i1) / (s2 @ [inr(i1)])])
| | by (Inst Thm: graphobj_DASH_properties [the_graph;the_obj])
| |    THEN
| |    (RWO
| |     Thm* ∀E:(T→T→ℙ).
| |          (∀x,y:T. E(x,y) ⟺ x = y)
| |          ⇒
| |          (∀i:T, s:(T+T) List.
| |           member-paren(x,y.E(x,y);i;s)
| |           ⟺
| |           (inl(i) ∈ s) ∨ (inr(i) ∈ s))
| |     {assert_DASH_member_DASH_paren}
| |     -8)
| |    THEN
| |    (BackThru 8)
| |
| |\
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. P :
| | |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| | |  →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | |  (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | |  i-the_graph->j
| | |  ⇒
| | |  paren(Vertices(the_graph);s2)
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| | |  ⇒
| | |  (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| | |  ⇒
| | |  paren(Vertices(the_graph);s3)
| | |  ⇒
| | |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | |  ¬(inr(i) ∈ s1)
| | |  ⇒
| | |  ¬(inl(i) ∈ s1)
| | |  ⇒
| | |  paren(Vertices(the_graph);s2)
| | |  ⇒
| | |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | |  ⇒
| | |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | |  ⇒
| | |  (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| | |  ⇒
| | |  (∀j:Vertices(the_graph).
| | |   i-the_graph->j
| | |   ⇒
| | |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| | |  ⇒
| | |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. s1 : traversal(the_graph)
| | 10. s2 : traversal(the_graph)
| | 11. i1 : Vertices(the_graph)
| | 12. ¬((inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1))
| | 13. ∀j:Vertices(the_graph).
| | |   i1-the_graph->j
| | |   ⇒
| | |   ¬j = i1
| | |   ⇒
| | |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| | 14. P(i1,[inr(i1) / s1],s2)
| | 15. l_disjoint(Vertices(the_graph)+
| | |   Vertices(the_graph);s2;[inr(i1) / s1])
| | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 17. paren(Vertices(the_graph);s2)
| | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | 19. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | 20. ∀T:Type, s:T, x:Vertices(the_graph)
| | |   , f:(T→Vertices(the_graph)→T).
| | |   ∃L:Vertices(the_graph) List.
| | |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | 21. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | |   ∃L:Vertices(the_graph) List.
| | |   no_repeats(Vertices(the_graph);L)
| | |   & (∀y:Vertices(the_graph). (y ∈ L))
| | |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | ⊢  ¬(inr(i1) ∈ s1)  by AllHyps (λh.ParallelOp h)
| | |
| | 1. the_graph : Graph
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```
| | 2. the_obj : GraphObject(the_graph)
| | 3. P :
| |    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| |    →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |    i-the_graph->j
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| |    ⇒
| |    paren(Vertices(the_graph);s3)
| |    ⇒
| |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |    ¬(inr(i) ∈ s1)
| |    ⇒
| |    ¬(inl(i) ∈ s1)
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| |    ⇒
| |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| |    ⇒
| |    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| |    ⇒
| |    (∀j:Vertices(the_graph).
| |     i-the_graph->j
| |     ⇒
| |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| |    ⇒
| |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. s1 : traversal(the_graph)
| | 10. s2 : traversal(the_graph)
| | 11. i1 : Vertices(the_graph)
| | 12. ∀j:Vertices(the_graph).
| |     i1-the_graph->j
| |     ⇒
| |     ¬j = i1
| |     ⇒
| |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| | 13. P(i1,[inr(i1) / s1],s2)
| | 14. l_disjoint(Vertices(the_graph)+
| |     Vertices(the_graph);s2;[inr(i1) / s1])
| | 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 16. paren(Vertices(the_graph);s2)
| | 17. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| |     , f:(T→Vertices(the_graph)→T).
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| |     ∃L:Vertices(the_graph) List.
| |     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| |     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| |     ∃L:Vertices(the_graph) List.
| |     no_repeats(Vertices(the_graph);L)
| |     & (∀y:Vertices(the_graph). (y ∈ L))
| |     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | 21. (inr(i1) ∈ s1)
| | ⊢  (inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1)  by ObviousFrom [-1]
| |
| |\
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. P :
| | |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| | |  →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | |  (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | |  i-the_graph->j
| | |  ⇒
| | |  paren(Vertices(the_graph);s2)
| | |  ⇒
| | |  (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| | |  ⇒
| | |  paren(Vertices(the_graph);s3)
| | |  ⇒
| | |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | |  ¬(inr(i) ∈ s1)
| | |  ⇒
| | |  ¬(inl(i) ∈ s1)
| | |  ⇒
| | |  paren(Vertices(the_graph);s2)
| | |  ⇒
| | |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | |  ⇒
| | |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | |  ⇒
| | |  (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| | |  ⇒
| | |  (∀j:Vertices(the_graph).
| | |   i-the_graph->j
| | |   ⇒
| | |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| | |  ⇒
| | |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. s1 : traversal(the_graph)
| | 10. s2 : traversal(the_graph)
| | 11. i1 : Vertices(the_graph)
| | 12. ¬((inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1))
| | 13. ∀j:Vertices(the_graph).
```

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| | |   i1-the_graph->j
| | |   ⇒
| | |   ¬j = i1
| | |   ⇒
| | |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| | 14. P(i1,[inr(i1) / s1],s2)
| | 15. l_disjoint(Vertices(the_graph)+
| | |   Vertices(the_graph);s2;[inr(i1) / s1])
| | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 17. paren(Vertices(the_graph);s2)
| | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | 19. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | 20. ∀T:Type, s:T, x:Vertices(the_graph)
| | |   , f:(T→Vertices(the_graph)→T).
| | |   ∃L:Vertices(the_graph) List.
| | |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | 21. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | |   ∃L:Vertices(the_graph) List.
| | |   no_repeats(Vertices(the_graph);L)
| | |   & (∀y:Vertices(the_graph). (y ∈ L))
| | |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | ⊢  ¬(inl(i1) ∈ s1)  by AllHyps (λh.ParallelOp h)
| | |
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. P :
| |    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| |    →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |    i-the_graph->j
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| |    ⇒
| |    paren(Vertices(the_graph);s3)
| |    ⇒
| |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |    ¬(inr(i) ∈ s1)
| |    ⇒
| |    ¬(inl(i) ∈ s1)
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| |    ⇒
| |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| |    ⇒
| |    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
```

```
| |    ⇒
| |    (∀j:Vertices(the_graph).
| |     i-the_graph->j
| |     ⇒
| |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| |    ⇒
| |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. s1 : traversal(the_graph)
| | 10. s2 : traversal(the_graph)
| | 11. i1 : Vertices(the_graph)
| | 12. ∀j:Vertices(the_graph).
| |     i1-the_graph->j
| |     ⇒
| |     ¬j = i1
| |     ⇒
| |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| | 13. P(i1,[inr(i1) / s1],s2)
| | 14. l_disjoint(Vertices(the_graph)+
| |     Vertices(the_graph);s2;[inr(i1) / s1])
| | 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 16. paren(Vertices(the_graph);s2)
| | 17. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | 18. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | 19. ∀T:Type, s:T, x:Vertices(the_graph)
| |     , f:(T→Vertices(the_graph)→T).
| |     ∃L:Vertices(the_graph) List.
| |     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| |     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | 20. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| |     ∃L:Vertices(the_graph) List.
| |     no_repeats(Vertices(the_graph);L)
| |     & (∀y:Vertices(the_graph). (y ∈ L))
| |     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | 21. (inl(i1) ∈ s1)
| | ⊢  (inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1)  by ObviousFrom [-1]
| |
| |\
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. P :
| |    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| |    →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |    i-the_graph->j
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| |    ⇒
| |    paren(Vertices(the_graph);s3)
| |    ⇒
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| |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |    ¬(inr(i) ∈ s1)
| |    ⇒
| |    ¬(inl(i) ∈ s1)
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| |    ⇒
| |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| |    ⇒
| |    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| |    ⇒
| |    (∀j:Vertices(the_graph).
| |     i-the_graph->j
| |     ⇒
| |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| |    ⇒
| |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. s1 : traversal(the_graph)
| | 10. s2 : traversal(the_graph)
| | 11. i1 : Vertices(the_graph)
| | 12. ¬((inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1))
| | 13. ∀j:Vertices(the_graph).
| |     i1-the_graph->j
| |     ⇒
| |     ¬j = i1
| |     ⇒
| |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| | 14. P(i1,[inr(i1) / s1],s2)
| | 15. l_disjoint(Vertices(the_graph)+
| |     Vertices(the_graph);s2;[inr(i1) / s1])
| | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 17. paren(Vertices(the_graph);s2)
| | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | 19. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | 20. ∀T:Type, s:T, x:Vertices(the_graph)
| |     , f:(T→Vertices(the_graph)→T).
| |     ∃L:Vertices(the_graph) List.
| |     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| |     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | 21. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| |     ∃L:Vertices(the_graph) List.
| |     no_repeats(Vertices(the_graph);L)
| |     & (∀y:Vertices(the_graph). (y ∈ L))
| |     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | ⊢  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| |   by RWO
| |      Thm* ∀a,b:T List, t:T.
| |           l_disjoint(T;b;[t / a]) ⟺ ¬(t ∈ b) & l_disjoint(T;b;a)
| |      {l_disjoint_cons2}
| |      -7
| |
| .\
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| . 1. the_graph : Graph
| . 2. the_obj : GraphObject(the_graph)
| . 3. P :
| .    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| .    →Prop
| . 4. traversal(the_graph)
| . 5. Vertices(the_graph)
| . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| .    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| .    i-the_graph->j
| .    ⇒
| .    paren(Vertices(the_graph);s2)
| .    ⇒
| .    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| .    ⇒
| .    paren(Vertices(the_graph);s3)
| .    ⇒
| .    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| .    ¬(inr(i) ∈ s1)
| .    ⇒
| .    ¬(inl(i) ∈ s1)
| .    ⇒
| .    paren(Vertices(the_graph);s2)
| .    ⇒
| .    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| .    ⇒
| .    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| .    ⇒
| .    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| .    ⇒
| .    (∀j:Vertices(the_graph).
| .     i-the_graph->j
| .     ⇒
| .     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| .    ⇒
| .    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . 9. s1 : traversal(the_graph)
| . 10. s2 : traversal(the_graph)
| . 11. i1 : Vertices(the_graph)
| . 12. ¬((inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1))
| . 13. ∀j:Vertices(the_graph).
| .     i1-the_graph->j
| .     ⇒
| .     ¬j = i1
| .     ⇒
| .     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| . 14. P(i1,[inr(i1) / s1],s2)
| . 15. l_disjoint(Vertices(the_graph)+
| .     Vertices(the_graph);s2;[inr(i1) / s1])
| . 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . 17. paren(Vertices(the_graph);s2)
| . 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| . 19. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
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| . 20. ∀T:Type, s:T, x:Vertices(the_graph)
| .     , f:(T→Vertices(the_graph)→T).
| .     ∃L:Vertices(the_graph) List.
| .     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| .     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . 21. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| .     ∃L:Vertices(the_graph) List.
| .     no_repeats(Vertices(the_graph);L)
| .     & (∀y:Vertices(the_graph). (y ∈ L))
| .     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . 22. j : Vertices(the_graph)
| . 23. i1-the_graph->j
| . 24. ¬j = i1
| . ⊢  (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
| .   by AllHyps
| .      (λh.
| .       (InstHyp [j] h)
| .       THEN
| .       (RWO
| .        Thm* ∀E:(T→T→ℙ).
| .             (∀x,y:T. E(x,y) ⟺ x = y)
| .             ⇒
| .             (∀i:T, s:(T+T) List.
| .              member-paren(x,y.E(x,y);i;s)
| .              ⟺
| .              (inl(i) ∈ s) ∨ (inr(i) ∈ s))
| .        {assert_DASH_member_DASH_paren}
| .        -1))
|
|\
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. P :
| |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
| |  Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |  (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |  i-the_graph->j
| |  ⇒
| |  paren(Vertices(the_graph);s2)
| |  ⇒
| |  (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| |  ⇒
| |  paren(Vertices(the_graph);s3)
| |  ⇒
| |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |  ¬(inr(i) ∈ s1)
| |  ⇒
| |  ¬(inl(i) ∈ s1)
| |  ⇒
| |  paren(Vertices(the_graph);s2)
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| |  ⇒
| |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| |  ⇒
| |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| |  ⇒
| |  (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| |  ⇒
| |  (∀j:Vertices(the_graph).
| |   i-the_graph->j
| |   ⇒
| |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| |  ⇒
| |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. s2 : traversal(the_graph)
| 11. i1 : Vertices(the_graph)
| 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
| 13. ∀j:Vertices(the_graph).
| |   i1-the_graph->j
| |   ⇒
| |   ¬j = i1
| |   ⇒
| |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| 14. P(i1,[inr(i1) / s1],s2)
| 15. l_disjoint(Vertices(the_graph)+
| |   Vertices(the_graph);s2;[inr(i1) / s1])
| 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| 17. paren(Vertices(the_graph);s2)
| 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| ⊢  l_disjoint(Vertices(the_graph)+
| |  Vertices(the_graph);[inl(i1) / (s2 @ [inr(i1)])];s1)
| | by ((Inst Thm: graphobj_DASH_properties [the_graph;the_obj])
| |     THEN
| |     (RWO
| |      Thm* ∀E:(T→T→ℙ).
| |           (∀x,y:T. E(x,y) ⟺ x = y)
| |           ⇒
| |           (∀i:T, s:(T+T) List.
| |            member-paren(x,y.E(x,y);i;s)
| |            ⟺
| |            (inl(i) ∈ s) ∨ (inr(i) ∈ s))
| |      {assert_DASH_member_DASH_paren}
| |      -8)
| |     THEN
| |     (RWO
| |      Thm* ∀a,b:T List, t:T.
| |           l_disjoint(T;b;[t / a]) ⟺ ¬(t ∈ b) & l_disjoint(T;b;a)
| |      {l_disjoint_cons2}
| |      -5))
| |    THEN
| |    (RWO
| |     Thm* ∀a,b:T List, t:T.
| |          l_disjoint(T;[t / a];b) ⟺ ¬(t ∈ b) & l_disjoint(T;a;b)
| |     {l_disjoint_cons}
| |     0)
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| |    THEN
| |    (RWO
| |     Thm* ∀a,b,c:T List.
| |          l_disjoint(T;a @ b;c)
| |          ⟺
| |          l_disjoint(T;a;c) & l_disjoint(T;b;c)
| |     {l_disjoint_append}
| |     0)
| |    THEN
| |    (RWO
| |     Thm* ∀a,b:T List, t:T.
| |          l_disjoint(T;[t / a];b) ⟺ ¬(t ∈ b) & l_disjoint(T;a;b)
| |     {l_disjoint_cons}
| |     0)
| |
| |\
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. P :
| | |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| | |  →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | |  (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | |  i-the_graph->j
| | |  ⇒
| | |  paren(Vertices(the_graph);s2)
| | |  ⇒
| | |  (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| | |  ⇒
| | |  paren(Vertices(the_graph);s3)
| | |  ⇒
| | |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | |  ¬(inr(i) ∈ s1)
| | |  ⇒
| | |  ¬(inl(i) ∈ s1)
| | |  ⇒
| | |  paren(Vertices(the_graph);s2)
| | |  ⇒
| | |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | |  ⇒
| | |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | |  ⇒
| | |  (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| | |  ⇒
| | |  (∀j:Vertices(the_graph).
| | |   i-the_graph->j
| | |   ⇒
| | |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| | |  ⇒
| | |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. s1 : traversal(the_graph)
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| | 10. s2 : traversal(the_graph)
| | 11. i1 : Vertices(the_graph)
| | 12. ¬((inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1))
| | 13. ∀j:Vertices(the_graph).
| | |   i1-the_graph->j
| | |   ⇒
| | |   ¬j = i1
| | |   ⇒
| | |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| | 14. P(i1,[inr(i1) / s1],s2)
| | 15. ¬(inr(i1) ∈ s2)
| | 16. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 18. paren(Vertices(the_graph);s2)
| | 19. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | 20. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | 21. ∀T:Type, s:T, x:Vertices(the_graph)
| | |   , f:(T→Vertices(the_graph)→T).
| | |   ∃L:Vertices(the_graph) List.
| | |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | 22. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | |   ∃L:Vertices(the_graph) List.
| | |   no_repeats(Vertices(the_graph);L)
| | |   & (∀y:Vertices(the_graph). (y ∈ L))
| | |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | ⊢  ¬(inl(i1) ∈ s1)  by AllHyps (λh.ParallelOp h)
| | |
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. P :
| |    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| |    →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |    i-the_graph->j
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| |    ⇒
| |    paren(Vertices(the_graph);s3)
| |    ⇒
| |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |    ¬(inr(i) ∈ s1)
| |    ⇒
| |    ¬(inl(i) ∈ s1)
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
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| |    ⇒
| |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| |    ⇒
| |    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| |    ⇒
| |    (∀j:Vertices(the_graph).
| |     i-the_graph->j
| |     ⇒
| |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| |    ⇒
| |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. s1 : traversal(the_graph)
| | 10. s2 : traversal(the_graph)
| | 11. i1 : Vertices(the_graph)
| | 12. ¬((inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1))
| | 13. ∀j:Vertices(the_graph).
| |     i1-the_graph->j
| |     ⇒
| |     ¬j = i1
| |     ⇒
| |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| | 14. P(i1,[inr(i1) / s1],s2)
| | 15. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 17. paren(Vertices(the_graph);s2)
| | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | 19. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | 20. ∀T:Type, s:T, x:Vertices(the_graph)
| |     , f:(T→Vertices(the_graph)→T).
| |     ∃L:Vertices(the_graph) List.
| |     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| |     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | 21. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| |     ∃L:Vertices(the_graph) List.
| |     no_repeats(Vertices(the_graph);L)
| |     & (∀y:Vertices(the_graph). (y ∈ L))
| |     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | 22. (inl(i1) ∈ s1)
| | ⊢  (inr(i1) ∈ s2)  by Obvious
| |
| |\
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. P :
| | |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| | |  →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | |  (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | |  i-the_graph->j
| | |  ⇒
| | |  paren(Vertices(the_graph);s2)
| | |  ⇒
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| | |  (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| | |  ⇒
| | |  paren(Vertices(the_graph);s3)
| | |  ⇒
| | |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | |  ¬(inr(i) ∈ s1)
| | |  ⇒
| | |  ¬(inl(i) ∈ s1)
| | |  ⇒
| | |  paren(Vertices(the_graph);s2)
| | |  ⇒
| | |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | |  ⇒
| | |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | |  ⇒
| | |  (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| | |  ⇒
| | |  (∀j:Vertices(the_graph).
| | |   i-the_graph->j
| | |   ⇒
| | |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| | |  ⇒
| | |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. s1 : traversal(the_graph)
| | 10. s2 : traversal(the_graph)
| | 11. i1 : Vertices(the_graph)
| | 12. ¬((inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1))
| | 13. ∀j:Vertices(the_graph).
| | |   i1-the_graph->j
| | |   ⇒
| | |   ¬j = i1
| | |   ⇒
| | |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| | 14. P(i1,[inr(i1) / s1],s2)
| | 15. ¬(inr(i1) ∈ s2)
| | 16. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 18. paren(Vertices(the_graph);s2)
| | 19. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | 20. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | 21. ∀T:Type, s:T, x:Vertices(the_graph)
| | |   , f:(T→Vertices(the_graph)→T).
| | |   ∃L:Vertices(the_graph) List.
| | |   (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| | |   & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | 22. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| | |   ∃L:Vertices(the_graph) List.
| | |   no_repeats(Vertices(the_graph);L)
| | |   & (∀y:Vertices(the_graph). (y ∈ L))
| | |   & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | ⊢  ¬(inr(i1) ∈ s1)  by AllHyps (λh.ParallelOp h)
| | |
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
```

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| | 3. P :
| |    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| |    →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |    i-the_graph->j
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| |    ⇒
| |    paren(Vertices(the_graph);s3)
| |    ⇒
| |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |    ¬(inr(i) ∈ s1)
| |    ⇒
| |    ¬(inl(i) ∈ s1)
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| |    ⇒
| |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| |    ⇒
| |    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| |    ⇒
| |    (∀j:Vertices(the_graph).
| |     i-the_graph->j
| |     ⇒
| |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| |    ⇒
| |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. s1 : traversal(the_graph)
| | 10. s2 : traversal(the_graph)
| | 11. i1 : Vertices(the_graph)
| | 12. ¬((inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1))
| | 13. ∀j:Vertices(the_graph).
| |     i1-the_graph->j
| |     ⇒
| |     ¬j = i1
| |     ⇒
| |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| | 14. P(i1,[inr(i1) / s1],s2)
| | 15. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 17. paren(Vertices(the_graph);s2)
| | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | 19. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| | 20. ∀T:Type, s:T, x:Vertices(the_graph)
| |     , f:(T→Vertices(the_graph)→T).
| |     ∃L:Vertices(the_graph) List.
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| |     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| |     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| | 21. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| |     ∃L:Vertices(the_graph) List.
| |     no_repeats(Vertices(the_graph);L)
| |     & (∀y:Vertices(the_graph). (y ∈ L))
| |     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| | 22. (inr(i1) ∈ s1)
| | ⊢  (inr(i1) ∈ s2)  by Obvious
| |
| .\
| . 1. the_graph : Graph
| . 2. the_obj : GraphObject(the_graph)
| . 3. P :
| .    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| .    →Prop
| . 4. traversal(the_graph)
| . 5. Vertices(the_graph)
| . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| .    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| .    i-the_graph->j
| .    ⇒
| .    paren(Vertices(the_graph);s2)
| .    ⇒
| .    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| .    ⇒
| .    paren(Vertices(the_graph);s3)
| .    ⇒
| .    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| .    ¬(inr(i) ∈ s1)
| .    ⇒
| .    ¬(inl(i) ∈ s1)
| .    ⇒
| .    paren(Vertices(the_graph);s2)
| .    ⇒
| .    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| .    ⇒
| .    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| .    ⇒
| .    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| .    ⇒
| .    (∀j:Vertices(the_graph).
| .     i-the_graph->j
| .     ⇒
| .     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| .    ⇒
| .    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . 9. s1 : traversal(the_graph)
| . 10. s2 : traversal(the_graph)
| . 11. i1 : Vertices(the_graph)
| . 12. ¬((inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1))
| . 13. ∀j:Vertices(the_graph).
| .     i1-the_graph->j
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| .     ⇒
| .     ¬j = i1
| .     ⇒
| .     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| . 14. P(i1,[inr(i1) / s1],s2)
| . 15. ¬(inr(i1) ∈ s2)
| . 16. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . 18. paren(Vertices(the_graph);s2)
| . 19. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| . 20. ∀x,y:Vertices(the_graph). the_obj.eq(x,y) ⟺ x = y
| . 21. ∀T:Type, s:T, x:Vertices(the_graph)
| .     , f:(T→Vertices(the_graph)→T).
| .     ∃L:Vertices(the_graph) List.
| .     (∀y:Vertices(the_graph). x-the_graph->y ⟺ (y ∈ L))
| .     & the_obj.eacc(f,s,x) = list_accum(s',x'.f(s',x');s;L)
| . 22. ∀T:Type, s:T, f:(T→Vertices(the_graph)→T).
| .     ∃L:Vertices(the_graph) List.
| .     no_repeats(Vertices(the_graph);L)
| .     & (∀y:Vertices(the_graph). (y ∈ L))
| .     & the_obj.vacc(f,s) = list_accum(s',x'.f(s',x');s;L)
| . ⊢  l_disjoint(Vertices(the_graph)+Vertices(the_graph);nil;s1)
| .   by ObviousConcl
|
|\
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. P :
| |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
| |  Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |  (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |  i-the_graph->j
| |  ⇒
| |  paren(Vertices(the_graph);s2)
| |  ⇒
| |  (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| |  ⇒
| |  paren(Vertices(the_graph);s3)
| |  ⇒
| |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |  ¬(inr(i) ∈ s1)
| |  ⇒
| |  ¬(inl(i) ∈ s1)
| |  ⇒
| |  paren(Vertices(the_graph);s2)
| |  ⇒
| |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| |  ⇒
| |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| |  ⇒
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```
| |  (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| |  ⇒
| |  (∀j:Vertices(the_graph).
| |   i-the_graph->j
| |   ⇒
| |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| |  ⇒
| |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. s2 : traversal(the_graph)
| 11. i1 : Vertices(the_graph)
| 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
| 13. ∀j:Vertices(the_graph).
| |   i1-the_graph->j
| |   ⇒
| |   ¬j = i1
| |   ⇒
| |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| 14. P(i1,[inr(i1) / s1],s2)
| 15. l_disjoint(Vertices(the_graph)+
| |   Vertices(the_graph);s2;[inr(i1) / s1])
| 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| 17. paren(Vertices(the_graph);s2)
| 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| ⊢  no_repeats(Vertices(the_graph)+
| |  Vertices(the_graph);[inl(i1) / (s2 @ [inr(i1)])])
| | by (RWO
| |     Thm* ∀l:T List, x:T.
| |          no_repeats(T;[x / l]) ⟺ no_repeats(T;l) & ¬(x ∈ l)
| |     {no_repeats_cons}
| |     0)
| |    THEN
| |    (Try
| |     (RWO
| |      Thm* ∀l1,l2:T List.
| |           no_repeats(T;l1 @ l2)
| |           ⟺
| |           l_disjoint(T;l1;l2)
| |           & no_repeats(T;l1)
| |           & no_repeats(T;l2)
| |      {no_repeats_append_iff}
| |      0))
| |    THEN
| |    (Try
| |     (RWO
| |      Thm* ∀x:T, l1,l2:T List. (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| |      {member_append}
| |      0))
| |    THEN
| |    (Try
| |     (RWO Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a    {member_singleton}
| |      0))

| |
| |\
| | 1. the_graph : Graph
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| | 2. the_obj : GraphObject(the_graph)
| | 3. P :
| | |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| | |  →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| | |  (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| | |  i-the_graph->j
| | |  ⇒
| | |  paren(Vertices(the_graph);s2)
| | |  ⇒
| | |  (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| | |  ⇒
| | |  paren(Vertices(the_graph);s3)
| | |  ⇒
| | |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| | |  ¬(inr(i) ∈ s1)
| | |  ⇒
| | |  ¬(inl(i) ∈ s1)
| | |  ⇒
| | |  paren(Vertices(the_graph);s2)
| | |  ⇒
| | |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | |  ⇒
| | |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | |  ⇒
| | |  (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| | |  ⇒
| | |  (∀j:Vertices(the_graph).
| | |   i-the_graph->j
| | |   ⇒
| | |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| | |  ⇒
| | |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. s1 : traversal(the_graph)
| | 10. s2 : traversal(the_graph)
| | 11. i1 : Vertices(the_graph)
| | 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
| | 13. ∀j:Vertices(the_graph).
| | |   i1-the_graph->j
| | |   ⇒
| | |   ¬j = i1
| | |   ⇒
| | |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| | 14. P(i1,[inr(i1) / s1],s2)
| | 15. l_disjoint(Vertices(the_graph)+
| | |   Vertices(the_graph);s2;[inr(i1) / s1])
| | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 17. paren(Vertices(the_graph);s2)
| | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | ⊢  l_disjoint(Vertices(the_graph)+
| | |  Vertices(the_graph);s2;[inr(i1)])
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| | | by All
| | |    (RWO
| | |     Thm* ∀a,b:T List, t:T.
| | |          l_disjoint(T;b;[t / a]) ⟺ ¬(t ∈ b) & l_disjoint(T;b;a)
| | |     {l_disjoint_cons2})
| | |
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. P :
| |    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| |    →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |    i-the_graph->j
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| |    ⇒
| |    paren(Vertices(the_graph);s3)
| |    ⇒
| |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |    ¬(inr(i) ∈ s1)
| |    ⇒
| |    ¬(inl(i) ∈ s1)
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| |    ⇒
| |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| |    ⇒
| |    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| |    ⇒
| |    (∀j:Vertices(the_graph).
| |     i-the_graph->j
| |     ⇒
| |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| |    ⇒
| |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. s1 : traversal(the_graph)
| | 10. s2 : traversal(the_graph)
| | 11. i1 : Vertices(the_graph)
| | 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
| | 13. ∀j:Vertices(the_graph).
| |     i1-the_graph->j
| |     ⇒
| |     ¬j = i1
| |     ⇒
| |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| | 14. P(i1,[inr(i1) / s1],s2)
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| | 15. ¬(inr(i1) ∈ s2)
| | 16. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| | 17. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 18. paren(Vertices(the_graph);s2)
| | 19. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | ⊢  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;nil)
| |   by ObviousConcl
| |
| |\
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. P :
| |    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| |    →Prop
| | 4. traversal(the_graph)
| | 5. Vertices(the_graph)
| | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| |    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| |    i-the_graph->j
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| |    ⇒
| |    paren(Vertices(the_graph);s3)
| |    ⇒
| |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| |    ¬(inr(i) ∈ s1)
| |    ⇒
| |    ¬(inl(i) ∈ s1)
| |    ⇒
| |    paren(Vertices(the_graph);s2)
| |    ⇒
| |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| |    ⇒
| |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| |    ⇒
| |    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| |    ⇒
| |    (∀j:Vertices(the_graph).
| |     i-the_graph->j
| |     ⇒
| |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| |    ⇒
| |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| | 9. s1 : traversal(the_graph)
| | 10. s2 : traversal(the_graph)
| | 11. i1 : Vertices(the_graph)
| | 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
| | 13. ∀j:Vertices(the_graph).
| |     i1-the_graph->j
| |     ⇒
| |     ¬j = i1
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| |     ⇒
| |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| | 14. P(i1,[inr(i1) / s1],s2)
| | 15. l_disjoint(Vertices(the_graph)+
| |     Vertices(the_graph);s2;[inr(i1) / s1])
| | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| | 17. paren(Vertices(the_graph);s2)
| | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| | ⊢  no_repeats(Vertices(the_graph)+Vertices(the_graph);[inr(i1)])
| |   by ObviousConcl
| |
| .\
| . 1. the_graph : Graph
| . 2. the_obj : GraphObject(the_graph)
| . 3. P :
| . |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
| . |  →Prop
| . 4. traversal(the_graph)
| . 5. Vertices(the_graph)
| . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . |  (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . |  i-the_graph->j
| . |  ⇒
| . |  paren(Vertices(the_graph);s2)
| . |  ⇒
| . |  (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| . |  ⇒
| . |  paren(Vertices(the_graph);s3)
| . |  ⇒
| . |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . |  ¬(inr(i) ∈ s1)
| . |  ⇒
| . |  ¬(inl(i) ∈ s1)
| . |  ⇒
| . |  paren(Vertices(the_graph);s2)
| . |  ⇒
| . |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . |  ⇒
| . |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . |  ⇒
| . |  (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| . |  ⇒
| . |  (∀j:Vertices(the_graph).
| . |   i-the_graph->j
| . |   ⇒
| . |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| . |  ⇒
| . |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . 9. s1 : traversal(the_graph)
| . 10. s2 : traversal(the_graph)
| . 11. i1 : Vertices(the_graph)
| . 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
| . 13. ∀j:Vertices(the_graph).
```

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| . |   i1-the_graph->j
| . |   ⇒
| . |   ¬j = i1
| . |   ⇒
| . |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| . 14. P(i1,[inr(i1) / s1],s2)
| . 15. l_disjoint(Vertices(the_graph)+
| . |   Vertices(the_graph);s2;[inr(i1) / s1])
| . 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . 17. paren(Vertices(the_graph);s2)
| . 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| . ⊢  ¬((inl(i1) ∈ s2) ∨ inl(i1) = inr(i1))
| . | by (RWO
| . |     Thm* ∀a,b:T List, t:T.
| . |          l_disjoint(T;b;[t / a]) ⟺ ¬(t ∈ b) & l_disjoint(T;b;a)
| . |     {l_disjoint_cons2}
| . |     -4)
| . |    THEN
| . |    (ParallelOp -5)
| . |    THEN
| . |    (Analyze -1)
| . |
| . |\
| . | 1. the_graph : Graph
| . | 2. the_obj : GraphObject(the_graph)
| . | 3. P :
| . |    Vertices(the_graph)→traversal(the_graph)→
| . |    traversal(the_graph)→Prop
| . | 4. traversal(the_graph)
| . | 5. Vertices(the_graph)
| . | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . |    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| . | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . |    i-the_graph->j
| . |    ⇒
| . |    paren(Vertices(the_graph);s2)
| . |    ⇒
| . |    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| . |    ⇒
| . |    paren(Vertices(the_graph);s3)
| . |    ⇒
| . |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . |    ¬(inr(i) ∈ s1)
| . |    ⇒
| . |    ¬(inl(i) ∈ s1)
| . |    ⇒
| . |    paren(Vertices(the_graph);s2)
| . |    ⇒
| . |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . |    ⇒
| . |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . |    ⇒
| . |    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| . |    ⇒
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| . |    (∀j:Vertices(the_graph).
| . |     i-the_graph->j
| . |     ⇒
| . |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| . |    ⇒
| . |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . | 9. s1 : traversal(the_graph)
| . | 10. s2 : traversal(the_graph)
| . | 11. i1 : Vertices(the_graph)
| . | 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
| . | 13. ∀j:Vertices(the_graph).
| . |     i1-the_graph->j
| . |     ⇒
| . |     ¬j = i1
| . |     ⇒
| . |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| . | 14. P(i1,[inr(i1) / s1],s2)
| . | 15. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | 17. paren(Vertices(the_graph);s2)
| . | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| . | 19. (inl(i1) ∈ s2)
| . | ⊢  (inr(i1) ∈ s2)
| . |   by BackThru
| . |      Thm* ∀s':(T+T) List.
| . |           paren(T;s') ⇒ (∀i:T. (inl(i) ∈ s') ⇒ (inr(i) ∈ s'))
| . |      {paren_balance1}
| . |
| . .\
| . . 1. the_graph : Graph
| . . 2. the_obj : GraphObject(the_graph)
| . . 3. P :
| . .    Vertices(the_graph)→traversal(the_graph)→
| . .    traversal(the_graph)→Prop
| . . 4. traversal(the_graph)
| . . 5. Vertices(the_graph)
| . . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
| . .    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| . . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
| . .    i-the_graph->j
| . .    ⇒
| . .    paren(Vertices(the_graph);s2)
| . .    ⇒
| . .    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
| . .    ⇒
| . .    paren(Vertices(the_graph);s3)
| . .    ⇒
| . .    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| . . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
| . .    ¬(inr(i) ∈ s1)
| . .    ⇒
| . .    ¬(inl(i) ∈ s1)
| . .    ⇒
| . .    paren(Vertices(the_graph);s2)
| . .    ⇒
```

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| . .    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . .    ⇒
| . .    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . .    ⇒
| . .    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| . .    ⇒
| . .    (∀j:Vertices(the_graph).
| . .     i-the_graph->j
| . .     ⇒
| . .     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
| . .    ⇒
| . .    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| . . 9. s1 : traversal(the_graph)
| . . 10. s2 : traversal(the_graph)
| . . 11. i1 : Vertices(the_graph)
| . . 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
| . . 13. ∀j:Vertices(the_graph).
| . .     i1-the_graph->j
| . .     ⇒
| . .     ¬j = i1
| . .     ⇒
| . .     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| . . 14. P(i1,[inr(i1) / s1],s2)
| . . 15. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
| . . 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . 17. paren(Vertices(the_graph);s2)
| . . 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| . . 19. inl(i1) = inr(i1)
| . . ⊢  (inr(i1) ∈ s2)  by ObviousContradiction [-1]
|
|\
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. P :
|    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
|    Prop
| 4. traversal(the_graph)
| 5. Vertices(the_graph)
| 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
|    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
| 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
|    i-the_graph->j
|    ⇒
|    paren(Vertices(the_graph);s2)
|    ⇒
|    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
|    ⇒
|    paren(Vertices(the_graph);s3)
|    ⇒
|    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
| 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
|    ¬(inr(i) ∈ s1)
|    ⇒
|    ¬(inl(i) ∈ s1)
|    ⇒
```

```
|    paren(Vertices(the_graph);s2)
|    ⇒
|    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
|    ⇒
|    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
|    ⇒
|    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
|    ⇒
|    (∀j:Vertices(the_graph).
|     i-the_graph->j
|     ⇒
|     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
|    ⇒
|    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
| 9. s1 : traversal(the_graph)
| 10. s2 : traversal(the_graph)
| 11. i1 : Vertices(the_graph)
| 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
| 13. ∀j:Vertices(the_graph).
|     i1-the_graph->j
|     ⇒
|     ¬j = i1
|     ⇒
|     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
| 14. P(i1,[inr(i1) / s1],s2)
| 15. l_disjoint(Vertices(the_graph)+
|     Vertices(the_graph);s2;[inr(i1) / s1])
| 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| 17. paren(Vertices(the_graph);s2)
| 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
| ⊢  paren(Vertices(the_graph);[inl(i1) / (s2 @ [inr(i1)])])
|   by (((RecUnfold `paren` 0) THEN OrRight) THEN OrLeft)
|      THEN
|      (InstConcl [i1;s2])
|      THEN
|      (Reduce 0)
|
.\
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. P :
. |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)→
. |  Prop
. 4. traversal(the_graph)
. 5. Vertices(the_graph)
. 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
. |  (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
. 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
. |  i-the_graph->j
. |  ⇒
. |  paren(Vertices(the_graph);s2)
. |  ⇒
. |  (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
. |  ⇒
. |  paren(Vertices(the_graph);s3)
```

```
. |  ⇒
. |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
. 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
. |  ¬(inr(i) ∈ s1)
. |  ⇒
. |  ¬(inl(i) ∈ s1)
. |  ⇒
. |  paren(Vertices(the_graph);s2)
. |  ⇒
. |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. |  ⇒
. |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. |  ⇒
. |  (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
. |  ⇒
. |  (∀j:Vertices(the_graph).
. |   i-the_graph->j
. |   ⇒
. |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
. |  ⇒
. |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
. 9. s1 : traversal(the_graph)
. 10. s2 : traversal(the_graph)
. 11. i1 : Vertices(the_graph)
. 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
. 13. ∀j:Vertices(the_graph).
. |   i1-the_graph->j
. |   ⇒
. |   ¬j = i1
. |   ⇒
. |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
. 14. P(i1,[inr(i1) / s1],s2)
. 15. l_disjoint(Vertices(the_graph)+
. |   Vertices(the_graph);s2;[inr(i1) / s1])
. 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. 17. paren(Vertices(the_graph);s2)
. 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. 19. j : Vertices(the_graph)
. 20. (inr(j) ∈ [inl(i1) / (s2 @ [inr(i1)])])
. ⊢  i1-the_graph->*j
. | by ((RWO
. |      Thm* ∀l:T List, a,x:T. (x ∈ [a / l]) ⟺ x = a ∨ (x ∈ l)
. |      {cons_member}
. |      -1)
. |     THEN
. |     (RWO
. |      Thm* ∀x:T, l1,l2:T List. (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
. |      {member_append}
. |      -1)
. |     THEN
. |     (RWO Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a    {member_singleton}
. |      -1))
. |    THEN
. |    SplitOrHyps
. |
```

```
. |\
. | 1. the_graph : Graph
. | 2. the_obj : GraphObject(the_graph)
. | 3. P :
. |    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
. |    →Prop
. | 4. traversal(the_graph)
. | 5. Vertices(the_graph)
. | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
. |    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
. | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
. |    i-the_graph->j
. |    ⇒
. |    paren(Vertices(the_graph);s2)
. |    ⇒
. |    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
. |    ⇒
. |    paren(Vertices(the_graph);s3)
. |    ⇒
. |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
. | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
. |    ¬(inr(i) ∈ s1)
. |    ⇒
. |    ¬(inl(i) ∈ s1)
. |    ⇒
. |    paren(Vertices(the_graph);s2)
. |    ⇒
. |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. |    ⇒
. |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. |    ⇒
. |    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
. |    ⇒
. |    (∀j:Vertices(the_graph).
. |     i-the_graph->j
. |     ⇒
. |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
. |    ⇒
. |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
. | 9. s1 : traversal(the_graph)
. | 10. s2 : traversal(the_graph)
. | 11. i1 : Vertices(the_graph)
. | 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
. | 13. ∀j:Vertices(the_graph).
. |     i1-the_graph->j
. |     ⇒
. |     ¬j = i1
. |     ⇒
. |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
. | 14. P(i1,[inr(i1) / s1],s2)
. | 15. l_disjoint(Vertices(the_graph)+
. |     Vertices(the_graph);s2;[inr(i1) / s1])
. | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. | 17. paren(Vertices(the_graph);s2)
. | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
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```
. | 19. j : Vertices(the_graph)
. | 20. inr(j) = inl(i1)
. | ⊢  i1-the_graph->*j  by ObviousContradiction [-1]
. |
. |\
. | 1. the_graph : Graph
. | 2. the_obj : GraphObject(the_graph)
. | 3. P :
. |    Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
. |    →Prop
. | 4. traversal(the_graph)
. | 5. Vertices(the_graph)
. | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
. |    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
. | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
. |    i-the_graph->j
. |    ⇒
. |    paren(Vertices(the_graph);s2)
. |    ⇒
. |    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
. |    ⇒
. |    paren(Vertices(the_graph);s3)
. |    ⇒
. |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
. | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
. |    ¬(inr(i) ∈ s1)
. |    ⇒
. |    ¬(inl(i) ∈ s1)
. |    ⇒
. |    paren(Vertices(the_graph);s2)
. |    ⇒
. |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. |    ⇒
. |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. |    ⇒
. |    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
. |    ⇒
. |    (∀j:Vertices(the_graph).
. |     i-the_graph->j
. |     ⇒
. |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
. |    ⇒
. |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
. | 9. s1 : traversal(the_graph)
. | 10. s2 : traversal(the_graph)
. | 11. i1 : Vertices(the_graph)
. | 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
. | 13. ∀j:Vertices(the_graph).
. |     i1-the_graph->j
. |     ⇒
. |     ¬j = i1
. |     ⇒
. |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
. | 14. P(i1,[inr(i1) / s1],s2)
. | 15. l_disjoint(Vertices(the_graph)+
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. |     Vertices(the_graph);s2;[inr(i1) / s1])
. | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. | 17. paren(Vertices(the_graph);s2)
. | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. | 19. j : Vertices(the_graph)
. | 20. (inr(j) ∈ s2)
. | ⊢  i1-the_graph->*j  by EasyHyp
. |
. .\
. . 1. the_graph : Graph
. . 2. the_obj : GraphObject(the_graph)
. . 3. P :
. . |  Vertices(the_graph)→traversal(the_graph)→traversal(the_graph)
. . |  →Prop
. . 4. traversal(the_graph)
. . 5. Vertices(the_graph)
. . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
. . |  (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
. . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
. . |  i-the_graph->j
. . |  ⇒
. . |  paren(Vertices(the_graph);s2)
. . |  ⇒
. . |  (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
. . |  ⇒
. . |  paren(Vertices(the_graph);s3)
. . |  ⇒
. . |  P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
. . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
. . |  ¬(inr(i) ∈ s1)
. . |  ⇒
. . |  ¬(inl(i) ∈ s1)
. . |  ⇒
. . |  paren(Vertices(the_graph);s2)
. . |  ⇒
. . |  l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . |  ⇒
. . |  no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . |  ⇒
. . |  (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
. . |  ⇒
. . |  (∀j:Vertices(the_graph).
. . |   i-the_graph->j
. . |   ⇒
. . |   ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
. . |  ⇒
. . |  P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
. . 9. s1 : traversal(the_graph)
. . 10. s2 : traversal(the_graph)
. . 11. i1 : Vertices(the_graph)
. . 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
. . 13. ∀j:Vertices(the_graph).
. . |   i1-the_graph->j
. . |   ⇒
. . |   ¬j = i1
```

```
. . |   ⇒
. . |   (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
. . 14. P(i1,[inr(i1) / s1],s2)
. . 15. l_disjoint(Vertices(the_graph)+
. . |   Vertices(the_graph);s2;[inr(i1) / s1])
. . 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . 17. paren(Vertices(the_graph);s2)
. . 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . 19. j : Vertices(the_graph)
. . 20. inr(j) = inr(i1)
. . ⊢  i1-the_graph->*j  by Subst (j = i1) 0
. . |
. . |\
. . | .....equality .....
. . | 1. the_graph : Graph
. . | 2. the_obj : GraphObject(the_graph)
. . | 3. P :
. . |    Vertices(the_graph)→traversal(the_graph)→
. . |    traversal(the_graph)→Prop
. . | 4. traversal(the_graph)
. . | 5. Vertices(the_graph)
. . | 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
. . |    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
. . | 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
. . |    i-the_graph->j
. . |    ⇒
. . |    paren(Vertices(the_graph);s2)
. . |    ⇒
. . |    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
. . |    ⇒
. . |    paren(Vertices(the_graph);s3)
. . |    ⇒
. . |    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
. . | 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
. . |    ¬(inr(i) ∈ s1)
. . |    ⇒
. . |    ¬(inl(i) ∈ s1)
. . |    ⇒
. . |    paren(Vertices(the_graph);s2)
. . |    ⇒
. . |    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . |    ⇒
. . |    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . |    ⇒
. . |    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
. . |    ⇒
. . |    (∀j:Vertices(the_graph).
. . |     i-the_graph->j
. . |     ⇒
. . |     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
. . |    ⇒
. . |    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
. . | 9. s1 : traversal(the_graph)
. . | 10. s2 : traversal(the_graph)
. . | 11. i1 : Vertices(the_graph)
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. . | 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
. . | 13. ∀j:Vertices(the_graph).
. . |     i1-the_graph->j
. . |     ⇒
. . |     ¬j = i1
. . |     ⇒
. . |     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
. . | 14. P(i1,[inr(i1) / s1],s2)
. . | 15. l_disjoint(Vertices(the_graph)+
. . |     Vertices(the_graph);s2;[inr(i1) / s1])
. . | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . | 17. paren(Vertices(the_graph);s2)
. . | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . | 19. j : Vertices(the_graph)
. . | 20. inr(j) = inr(i1)
. . | ⊢  j = i1  by ObviousFrom [-1]
. . |
. . .\
. . . 1. the_graph : Graph
. . . 2. the_obj : GraphObject(the_graph)
. . . 3. P :
. . .    Vertices(the_graph)→traversal(the_graph)→
. . .    traversal(the_graph)→Prop
. . . 4. traversal(the_graph)
. . . 5. Vertices(the_graph)
. . . 6. ∀s:traversal(the_graph), i:Vertices(the_graph).
. . .    (inl(i) ∈ s) ∨ (inr(i) ∈ s) ⇒ P(i,s,nil)
. . . 7. ∀s1,s2,s3:traversal(the_graph), i,j:Vertices(the_graph).
. . .    i-the_graph->j
. . .    ⇒
. . .    paren(Vertices(the_graph);s2)
. . .    ⇒
. . .    (∀k:Vertices(the_graph). (inr(k) ∈ s2) ⇒ j-the_graph->*k)
. . .    ⇒
. . .    paren(Vertices(the_graph);s3)
. . .    ⇒
. . .    P(j,s1,s2) ⇒ P(i,s2 @ s1,s3) ⇒ P(i,s1,s3 @ s2)
. . . 8. ∀s1,s2:traversal(the_graph), i:Vertices(the_graph).
. . .    ¬(inr(i) ∈ s1)
. . .    ⇒
. . .    ¬(inl(i) ∈ s1)
. . .    ⇒
. . .    paren(Vertices(the_graph);s2)
. . .    ⇒
. . .    l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s1)
. . .    ⇒
. . .    no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . .    ⇒
. . .    (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
. . .    ⇒
. . .    (∀j:Vertices(the_graph).
. . .     i-the_graph->j
. . .     ⇒
. . .     ¬j = i ⇒ (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1))
. . .    ⇒
```

```
. . .    P(i,[inr(i) / s1],s2) ⇒ P(i,s1,[inl(i) / (s2 @ [inr(i)])])
. . . 9. s1 : traversal(the_graph)
. . . 10. s2 : traversal(the_graph)
. . . 11. i1 : Vertices(the_graph)
. . . 12. ¬member-paren(x,y.the_obj.eq(x,y);i1;s1)
. . . 13. ∀j:Vertices(the_graph).
. . .     i1-the_graph->j
. . .     ⇒
. . .     ¬j = i1
. . .     ⇒
. . .     (inl(j) ∈ s2) ∨ member-paren(x,y.the_obj.eq(x,y);j;s1)
. . . 14. P(i1,[inr(i1) / s1],s2)
. . . 15. l_disjoint(Vertices(the_graph)+
. . .     Vertices(the_graph);s2;[inr(i1) / s1])
. . . 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
. . . 17. paren(Vertices(the_graph);s2)
. . . 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i1-the_graph->*j
. . . 19. j : Vertices(the_graph)
. . . 20. inr(j) = inr(i1)
. . . ⊢  i1-the_graph->*i1  by Connect
END
```