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dfs_member  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), s:Traversal, i:V.
|  ∃s':Traversal.
|  (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
|  & l_disjoint(V+V;s';s) & no_repeats(V+V;s')
|  & paren(V;s')
|  & dfs(the_obj;s;i) = (s' @ s)
| by Auto
|    THEN
|    (Inst Thm: dfs_induction2
|     [the_graph
|     ;the_obj
|     ;λ2i,s1,s2. (inl(i) ∈ s2) ∨ (inl(i) ∈ s1) ∨ (inr(i) ∈ s1)
|     ;s
|     ;i])
|
|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. traversal(the_graph)
| 4. Vertices(the_graph)
| 5. s1 : traversal(the_graph)
| 6. i1 : Vertices(the_graph)
| 7. (inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1)
| ⊢  (inl(i1) ∈ nil) ∨ (inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1)  by Obvious
|
|\
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. traversal(the_graph)
| 4. Vertices(the_graph)
| 5. s1 : traversal(the_graph)
| 6. s2 : traversal(the_graph)
| 7. s3 : traversal(the_graph)
| 8. i1 : Vertices(the_graph)
| 9. j : Vertices(the_graph)
| 10. i1-the_graph->j
| 11. paren(Vertices(the_graph);s2)
| 12. paren(Vertices(the_graph);s3)
| 13. (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
| 14. (inl(i1) ∈ s3) ∨ (inl(i1) ∈ s2 @ s1) ∨ (inr(i1) ∈ s2 @ s1)
| ⊢  (inl(i1) ∈ s3 @ s2) ∨ (inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1)
| | by All
| |    (RWO
| |     Thm* ∀x:T, l1,l2:T List. (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| |     {member_append})
| |
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. traversal(the_graph)
| 4. Vertices(the_graph)
| 5. s1 : traversal(the_graph)
| 6. s2 : traversal(the_graph)
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| 7. s3 : traversal(the_graph)
| 8. i1 : Vertices(the_graph)
| 9. j : Vertices(the_graph)
| 10. i1-the_graph->j
| 11. paren(Vertices(the_graph);s2)
| 12. paren(Vertices(the_graph);s3)
| 13. (inl(j) ∈ s2) ∨ (inl(j) ∈ s1) ∨ (inr(j) ∈ s1)
| 14. (inl(i1) ∈ s3)
| |   ∨ (inl(i1) ∈ s2) ∨ (inl(i1) ∈ s1)
| |   ∨ (inr(i1) ∈ s2)
| |   ∨ (inr(i1) ∈ s1)
| ⊢  (inl(i1) ∈ s3) ∨ (inl(i1) ∈ s2) ∨ (inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1)
| | by (Thin -2) THEN SplitOrHyps THEN (Try (ObviousFrom [-1]))
| |
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. traversal(the_graph)
| 4. Vertices(the_graph)
| 5. s1 : traversal(the_graph)
| 6. s2 : traversal(the_graph)
| 7. s3 : traversal(the_graph)
| 8. i1 : Vertices(the_graph)
| 9. j : Vertices(the_graph)
| 10. i1-the_graph->j
| 11. paren(Vertices(the_graph);s2)
| 12. paren(Vertices(the_graph);s3)
| 13. (inr(i1) ∈ s2)
| ⊢  (inl(i1) ∈ s3) ∨ (inl(i1) ∈ s2) ∨ (inl(i1) ∈ s1) ∨ (inr(i1) ∈ s1)
|   by (FwdThru
|       Thm* ∀s':(T+T) List.
|            paren(T;s') ⇒ (∀i:T. (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
|       {paren_balance2}
|       [-1])
|      THEN
|      (ObviousFrom [-1])
|
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. 1. the_graph : Graph
. 2. GraphObject(the_graph)
. 3. traversal(the_graph)
. 4. Vertices(the_graph)
. 5. s1 : traversal(the_graph)
. 6. s2 : traversal(the_graph)
. 7. i1 : Vertices(the_graph)
. 8. ¬(inl(i1) ∈ s1)
. 9. ¬(inr(i1) ∈ s1)
. 10. (inl(i1) ∈ s2)

.     ∨ (inl(i1) ∈ [inr(i1) / s1])
.     ∨ (inr(i1) ∈ [inr(i1) / s1])
. ⊢  (inl(i1) ∈ [inl(i1) / (s2 @ [inr(i1)])])
.    ∨ (inl(i1) ∈ s1)
.    ∨ (inr(i1) ∈ s1)  by ObviousConcl
END
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