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dfs_wf  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), s:Traversal, i:V.
|  dfs(the_obj;s;i) ∈ Traversal
| by (RepeatFor 2 (Analyze 0))
|    THEN
|    (Inst Thm: dfs_DASH_measure [the_graph;the_obj])
|    THEN
|    ExRepDHyps
|    THEN
|    (Assert
|     (∀d:ℕ, s:traversal(the_graph), i:Vertices(the_graph).
|      M(s)≤d
|      ⇒
|      dfs(the_obj;s;i) ∈ {s':traversal(the_graph)| M(s')≤d }))
|
|\
| .....assertion .....
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. M : traversal(the_graph)→ℕ
| 4. ∀i:Vertices(the_graph), s:traversal(the_graph).
| |  M([inl(i) / s])≤M(s)
| 5. ∀i:Vertices(the_graph), s:traversal(the_graph).
| |  ¬member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ M([inr(i) / s])<M(s)
| ⊢  ∀d:ℕ, s:traversal(the_graph), i:Vertices(the_graph).
| |  M(s)≤d ⇒ dfs(the_obj;s;i) ∈ {s':traversal(the_graph)| M(s')≤d }
| | by CompleteInductionOnNat
| |
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. M : traversal(the_graph)→ℕ
| 4. ∀i:Vertices(the_graph), s:traversal(the_graph).
| |  M([inl(i) / s])≤M(s)
| 5. ∀i:Vertices(the_graph), s:traversal(the_graph).
| |  ¬member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ M([inr(i) / s])<M(s)
| 6. d : ℕ
| 7. ∀d1:ℕ.
| |  d1<d
| |  ⇒
| |  (∀s:traversal(the_graph), i:Vertices(the_graph).
| |   M(s)≤d1
| |   ⇒
| |   dfs(the_obj;s;i) ∈ {s':traversal(the_graph)| M(s')≤d1 })
| 8. s : traversal(the_graph)
| 9. i : Vertices(the_graph)
| 10. M(s)≤d
| ⊢  dfs(the_obj;s;i) ∈ {s':traversal(the_graph)| M(s')≤d }
| | by (RecUnfold `dfs` 0) THEN SplitOnConclITE
| |
| |\
| | .....truecase .....
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
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| | 3. M : traversal(the_graph)→ℕ
| | 4. ∀i:Vertices(the_graph), s:traversal(the_graph).
| |    M([inl(i) / s])≤M(s)
| | 5. ∀i:Vertices(the_graph), s:traversal(the_graph).
| |    ¬member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ M([inr(i) / s])<M(s)
| | 6. d : ℕ
| | 7. ∀d1:ℕ.
| |    d1<d
| |    ⇒
| |    (∀s:traversal(the_graph), i:Vertices(the_graph).
| |     M(s)≤d1
| |     ⇒
| |     dfs(the_obj;s;i) ∈ {s':traversal(the_graph)| M(s')≤d1 })
| | 8. s : traversal(the_graph)
| | 9. i : Vertices(the_graph)
| | 10. M(s)≤d
| | 11. member-paren(x,y.the_obj.eq(x,y);i;s)
| | ⊢  s ∈ {s':traversal(the_graph)| M(s')≤d }  by Analyze
| |
| .\
| . .....falsecase .....
| . 1. the_graph : Graph
| . 2. the_obj : GraphObject(the_graph)
| . 3. M : traversal(the_graph)→ℕ
| . 4. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . |  M([inl(i) / s])≤M(s)
| . 5. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . |  ¬member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ M([inr(i) / s])<M(s)
| . 6. d : ℕ
| . 7. ∀d1:ℕ.
| . |  d1<d
| . |  ⇒
| . |  (∀s:traversal(the_graph), i:Vertices(the_graph).
| . |   M(s)≤d1
| . |   ⇒
| . |   dfs(the_obj;s;i) ∈ {s':traversal(the_graph)| M(s')≤d1 })
| . 8. s : traversal(the_graph)
| . 9. i : Vertices(the_graph)
| . 10. M(s)≤d
| . 11. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . ⊢  [inl(i) /
| . |   (the_obj.eacc((λs',j. dfs(the_obj;s';j)),[inr(i) / s],i))]
| . |  ∈ {s':traversal(the_graph)| M(s')≤d }
| . | by (InstHyp [i;s] 5) THEN (AssertBY (0<d) Auto)
| . |    THEN
| . |    (Inst
| . |     Thm* For any graph
| . |          ∀t:GraphObject(the_graph), T:Type.
| . |          t.eacc ∈ (T→V→T)→T→V→T
| . |     {gro_eacc_wf}
| . |     [the_graph;the_obj;{s':traversal(the_graph)| M(s')≤d-1 }])
| . |
| . 1. the_graph : Graph
| . 2. the_obj : GraphObject(the_graph)
| . 3. M : traversal(the_graph)→ℕ
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| . 4. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . |  M([inl(i) / s])≤M(s)
| . 5. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . |  ¬member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ M([inr(i) / s])<M(s)
| . 6. d : ℕ
| . 7. ∀d1:ℕ.
| . |  d1<d
| . |  ⇒
| . |  (∀s:traversal(the_graph), i:Vertices(the_graph).
| . |   M(s)≤d1
| . |   ⇒
| . |   dfs(the_obj;s;i) ∈ {s':traversal(the_graph)| M(s')≤d1 })
| . 8. s : traversal(the_graph)
| . 9. i : Vertices(the_graph)
| . 10. M(s)≤d
| . 11. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . 12. M([inr(i) / s])<M(s)
| . 13. 0<d
| . 14. the_obj.eacc
| . |   ∈ ({s':traversal(the_graph)| M(s')≤d-1 }→Vertices(the_graph)
| . |      →{s':traversal(the_graph)| M(s')≤d-1 })
| . |     →{s':traversal(the_graph)| M(s')≤d-1 }→Vertices(the_graph)
| . |     →{s':traversal(the_graph)| M(s')≤d-1 }
| . ⊢  [inl(i) /
| . |   (the_obj.eacc((λs',j. dfs(the_obj;s';j)),[inr(i) / s],i))]
| . |  ∈ {s':traversal(the_graph)| M(s')≤d }  by GenConclAtAddr [2;2]
| . |
| . |\
| . | .....wf .....
| . | 1. the_graph : Graph
| . | 2. the_obj : GraphObject(the_graph)
| . | 3. M : traversal(the_graph)→ℕ
| . | 4. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | |  M([inl(i) / s])≤M(s)
| . | 5. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | |  ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | |  ⇒
| . | |  M([inr(i) / s])<M(s)
| . | 6. d : ℕ
| . | 7. ∀d1:ℕ.
| . | |  d1<d
| . | |  ⇒
| . | |  (∀s:traversal(the_graph), i:Vertices(the_graph).
| . | |   M(s)≤d1
| . | |   ⇒
| . | |   dfs(the_obj;s;i) ∈ {s':traversal(the_graph)| M(s')≤d1 })
| . | 8. s : traversal(the_graph)
| . | 9. i : Vertices(the_graph)
| . | 10. M(s)≤d
| . | 11. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | 12. M([inr(i) / s])<M(s)
| . | 13. 0<d
| . | 14. the_obj.eacc
| . | |   ∈ ({s':traversal(the_graph)| M(s')≤d-1 }→
| . | |      Vertices(the_graph)→{s':traversal(the_graph)
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| . | |                            | M(s')≤d-1 })
| . | |     →{s':traversal(the_graph)| M(s')≤d-1 }→
| . | |     Vertices(the_graph)→{s':traversal(the_graph)| M(s')≤d-1 }
| . | ⊢  the_obj.eacc((λs',j. dfs(the_obj;s';j)),[inr(i) / s],i)
| . | |  ∈ {s':traversal(the_graph)| M(s')≤d-1 }  by Auto
| . | |
| . | |\
| . | | .....subterm .....
| . | | 1. the_graph : Graph
| . | | 2. the_obj : GraphObject(the_graph)
| . | | 3. M : traversal(the_graph)→ℕ
| . | | 4. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | |    M([inl(i) / s])≤M(s)
| . | | 5. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | |    ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | |    ⇒
| . | |    M([inr(i) / s])<M(s)
| . | | 6. d : ℕ
| . | | 7. ∀d1:ℕ.
| . | |    d1<d
| . | |    ⇒
| . | |    (∀s:traversal(the_graph), i:Vertices(the_graph).
| . | |     M(s)≤d1
| . | |     ⇒
| . | |     dfs(the_obj;s;i) ∈ {s':traversal(the_graph)| M(s')≤d1 })
| . | | 8. s : traversal(the_graph)
| . | | 9. i : Vertices(the_graph)
| . | | 10. M(s)≤d
| . | | 11. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | | 12. M([inr(i) / s])<M(s)
| . | | 13. 0<d
| . | | 14. the_obj.eacc
| . | |     ∈ ({s':traversal(the_graph)| M(s')≤d-1 }→
| . | |        Vertices(the_graph)→{s':traversal(the_graph)
| . | |                             | M(s')≤d-1 })
| . | |       →{s':traversal(the_graph)| M(s')≤d-1 }→
| . | |       Vertices(the_graph)→{s':traversal(the_graph)
| . | |                             | M(s')≤d-1 }
| . | | 15. s' : {s':traversal(the_graph)| M(s')≤d-1 }
| . | | 16. j : Vertices(the_graph)
| . | | ⊢  dfs(the_obj;s';j) ∈ {s':traversal(the_graph)| M(s')≤d-1 }
| . | |   by (Analyze -2) THEN (BackThru 7)
| . | |
| . | .\
| . | . .....subterm .....
| . | . 1. the_graph : Graph
| . | . 2. the_obj : GraphObject(the_graph)
| . | . 3. M : traversal(the_graph)→ℕ
| . | . 4. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | .    M([inl(i) / s])≤M(s)
| . | . 5. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . | .    ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | .    ⇒
| . | .    M([inr(i) / s])<M(s)
| . | . 6. d : ℕ
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| . | . 7. ∀d1:ℕ.
| . | .    d1<d
| . | .    ⇒
| . | .    (∀s:traversal(the_graph), i:Vertices(the_graph).
| . | .     M(s)≤d1
| . | .     ⇒
| . | .     dfs(the_obj;s;i) ∈ {s':traversal(the_graph)| M(s')≤d1 })
| . | . 8. s : traversal(the_graph)
| . | . 9. i : Vertices(the_graph)
| . | . 10. M(s)≤d
| . | . 11. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . | . 12. M([inr(i) / s])<M(s)
| . | . 13. 0<d
| . | . 14. the_obj.eacc
| . | .     ∈ ({s':traversal(the_graph)| M(s')≤d-1 }→
| . | .        Vertices(the_graph)→{s':traversal(the_graph)
| . | .                              | M(s')≤d-1 })
| . | .       →{s':traversal(the_graph)| M(s')≤d-1 }→
| . | .       Vertices(the_graph)→{s':traversal(the_graph)
| . | .                             | M(s')≤d-1 }
| . | . ⊢  [inr(i) / s] ∈ {s':traversal(the_graph)| M(s')≤d-1 }
| . | .   by Analyze
| . |
| . .\
| . . 1. the_graph : Graph
| . . 2. the_obj : GraphObject(the_graph)
| . . 3. M : traversal(the_graph)→ℕ
| . . 4. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . .    M([inl(i) / s])≤M(s)
| . . 5. ∀i:Vertices(the_graph), s:traversal(the_graph).
| . .    ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . .    ⇒
| . .    M([inr(i) / s])<M(s)
| . . 6. d : ℕ
| . . 7. ∀d1:ℕ.
| . .    d1<d
| . .    ⇒
| . .    (∀s:traversal(the_graph), i:Vertices(the_graph).
| . .     M(s)≤d1
| . .     ⇒
| . .     dfs(the_obj;s;i) ∈ {s':traversal(the_graph)| M(s')≤d1 })
| . . 8. s : traversal(the_graph)
| . . 9. i : Vertices(the_graph)
| . . 10. M(s)≤d
| . . 11. ¬member-paren(x,y.the_obj.eq(x,y);i;s)
| . . 12. M([inr(i) / s])<M(s)
| . . 13. 0<d
| . . 14. the_obj.eacc
| . .     ∈ ({s':traversal(the_graph)| M(s')≤d-1 }→
| . .        Vertices(the_graph)→{s':traversal(the_graph)
| . .                              | M(s')≤d-1 })
| . .       →{s':traversal(the_graph)| M(s')≤d-1 }→
| . .       Vertices(the_graph)→{s':traversal(the_graph)| M(s')≤d-1 }
| . . 15. z : {s':traversal(the_graph)| M(s')≤d-1 }
| . . 16. the_obj.eacc((λs',j. dfs(the_obj;s';j)),[inr(i) / s],i) = z
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| . . ⊢  [inl(i) / z] ∈ {s':traversal(the_graph)| M(s')≤d }
| . .   by (Analyze -2) THEN Analyze THEN (InstHyp [i;z] 4)
|
.\
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. M : traversal(the_graph)→ℕ
. 4. ∀i:Vertices(the_graph), s:traversal(the_graph).
.    M([inl(i) / s])≤M(s)
. 5. ∀i:Vertices(the_graph), s:traversal(the_graph).
.    ¬member-paren(x,y.the_obj.eq(x,y);i;s) ⇒ M([inr(i) / s])<M(s)
. 6. ∀d:ℕ, s:traversal(the_graph), i:Vertices(the_graph).
.    M(s)≤d ⇒ dfs(the_obj;s;i) ∈ {s':traversal(the_graph)| M(s')≤d }
. ⊢  ∀s:traversal(the_graph), i:Vertices(the_graph).
.    dfs(the_obj;s;i) ∈ traversal(the_graph)
.   by Auto THEN (InstHyp [M(s);s;i] -3) THEN (Analyze -1)
END
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