```
dfsl-properties  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), L:V List.
|  paren(V;dfsl(the_obj;L))
|  & no_repeats(V+V;dfsl(the_obj;L))
|  & dfsl-traversal(the_graph;L;dfsl(the_obj;L))
|  & (∀i:V.
|     (i ∈ L)
|     ⇒
|     (inr(i) ∈ dfsl(the_obj;L)) & (inl(i) ∈ dfsl(the_obj;L)))
| by UnivCD
|    THEN
|    (Inst Thm: dfsl_DASH_induction2
|     [the_graph;the_obj;λ2L,s. dfsl-traversal(the_graph;L;s)])
|
|\
| .....antecedent .....
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. Vertices(the_graph) List
| ⊢  dfsl-traversal(the_graph;nil;nil)
|   by BackThru
|      Thm* For any graph dfsl-traversal(the_graph;nil;nil)
|      {dfsl_DASH_traversal_DASH_nil}
|
|\
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. Vertices(the_graph) List
| 4. L1 : Vertices(the_graph) List
| 5. s : traversal(the_graph)
| 6. i : Vertices(the_graph)
| 7. paren(Vertices(the_graph);s)
| 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| 9. dfsl-traversal(the_graph;L1;s)
| ⊢  dfsl-traversal(the_graph;L1 @ [i];dfs(the_obj;s;i))
| | by (((Inst Thm: dfs_DASH_properties [the_graph;the_obj;s;i])
| |      THEN
| |      ExRepD
| |      THEN
| |      (Inst
| |       Thm* For any graph
| |            ∀the_obj:GraphObject(the_graph), s:Traversal, i:V.
| |            df-traversal(the_graph;s)
| |            ⇒
| |            (∀j:V.
| |             (inr(j) ∈ s) ⇒ ¬(inl(j) ∈ s) ⇒ j-the_graph->*i)
| |            ⇒
| |            df-traversal(the_graph;dfs(the_obj;s;i))
| |       {dfs_DASH_df_DASH_traversal}
| |       [the_graph;the_obj;s;i]))
| |     THENA
| |     (Auto THEN (Analyze 9) THEN (Try (Complete Auto))
| |      THEN
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| |      (Analyze -1)
| |      THEN
| |      (BackThru
| |       Thm* ∀s':(T+T) List.
| |            paren(T;s') ⇒ (∀i:T. (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
| |       {paren_balance2})))
| |    THEN
| |    (HypSubst -2 -1)
| |    THEN
| |    (HypSubst -2 0)
| |
| 1. the_graph : Graph
| 2. the_obj : GraphObject(the_graph)
| 3. Vertices(the_graph) List
| 4. L1 : Vertices(the_graph) List
| 5. s : traversal(the_graph)
| 6. i : Vertices(the_graph)
| 7. paren(Vertices(the_graph);s)
| 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| 9. dfsl-traversal(the_graph;L1;s)
| 10. s' : traversal(the_graph)
| 11. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
| 12. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
| |   ⇒
| |   (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
| 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
| 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
| 15. paren(Vertices(the_graph);s')
| 16. ∀j:Vertices(the_graph). (inr(j) ∈ s') ⇒ i-the_graph->*j
| 17. dfs(the_obj;s;i) = (s' @ s)
| 18. df-traversal(the_graph;s' @ s)
| ⊢  dfsl-traversal(the_graph;L1 @ [i];s' @ s)
| | by ((((Inst
| |        Thm* (∀x,y:T. Dec(x = y))
| |             ⇒
| |             (∀s:(T+T) List, i:T. Dec((inl(i) ∈ s)))
| |        {decidable__l_member_paren}
| |        [Vertices(the_graph);s;i])
| |       THENA
| |       (Auto
| |        THEN
| |        (Using ['the_obj',the_obj]
| |         (BackThru
| |          Thm* For any graph
| |               ∀the_obj:GraphObject(the_graph), x,y:V. Dec(x = y)
| |          {decidable__equal_gr_v}))))
| |      THEN
| |      (Analyze -1))
| |     THENL
| |     [Analyze 11
| |     ;((Analyze 12)
| |       THENA
| |       (Auto THEN (ParallelOp -1)
| |        THEN
| |        (BackThru
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| |         Thm* ∀s':(T+T) List.
| |              paren(T;s') ⇒ (∀i:T. (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
| |         {paren_balance2})))
| |      THEN
| |      ExRepD])
| |    THEN
| |    (HypSubst -1 0)
| |
| |\
| | .....antecedent .....
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. Vertices(the_graph) List
| | 4. L1 : Vertices(the_graph) List
| | 5. s : traversal(the_graph)
| | 6. i : Vertices(the_graph)
| | 7. paren(Vertices(the_graph);s)
| | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| | 9. dfsl-traversal(the_graph;L1;s)
| | 10. s' : traversal(the_graph)
| | 11. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
| |     ⇒
| |     (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
| | 12. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
| | 13. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
| | 14. paren(Vertices(the_graph);s')
| | 15. ∀j:Vertices(the_graph). (inr(j) ∈ s') ⇒ i-the_graph->*j
| | 16. dfs(the_obj;s;i) = (s' @ s)
| | 17. df-traversal(the_graph;s' @ s)
| | 18. (inl(i) ∈ s)
| | ⊢  (inr(i) ∈ s) ∨ (inl(i) ∈ s)  by ObviousFrom [-1]
| |
| |\
| | 1. the_graph : Graph
| | 2. the_obj : GraphObject(the_graph)
| | 3. Vertices(the_graph) List
| | 4. L1 : Vertices(the_graph) List
| | 5. s : traversal(the_graph)
| | 6. i : Vertices(the_graph)
| | 7. paren(Vertices(the_graph);s)
| | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| | 9. dfsl-traversal(the_graph;L1;s)
| | 10. s' : traversal(the_graph)
| | 11. ¬(inr(i) ∈ s) & ¬(inl(i) ∈ s)
| | |   ⇒
| | |   (∃s2:traversal(the_graph). s' = ([inl(i)] @ s2 @ [inr(i)]))
| | 12. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
| | 13. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
| | 14. paren(Vertices(the_graph);s')
| | 15. ∀j:Vertices(the_graph). (inr(j) ∈ s') ⇒ i-the_graph->*j
| | 16. dfs(the_obj;s;i) = (s' @ s)
| | 17. df-traversal(the_graph;s' @ s)
| | 18. (inl(i) ∈ s)
| | 19. s' = nil
| | ⊢  dfsl-traversal(the_graph;L1 @ [i];nil @ s)
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| | | by (AssertBY df-traversal(the_graph;s)
| | |     ((HypSubst -1 -3) THEN (Reduce -3)))
| | |    THEN
| | |    (Thin -2)
| | |    THEN
| | |    (RepeatFor 8 (Thin -3))
| | |    THEN
| | |    (Reduce 0)
| | |
| | 1. the_graph : Graph
| | 2. GraphObject(the_graph)
| | 3. Vertices(the_graph) List
| | 4. L1 : Vertices(the_graph) List
| | 5. s : traversal(the_graph)
| | 6. i : Vertices(the_graph)
| | 7. paren(Vertices(the_graph);s)
| | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| | 9. dfsl-traversal(the_graph;L1;s)
| | 10. (inl(i) ∈ s)
| | 11. df-traversal(the_graph;s)
| | ⊢  dfsl-traversal(the_graph;L1 @ [i];s)
| |   by (((BackThru
| |         Thm* For any graph
| |              ∀L1,L2:V List, s:Traversal.
| |              L1-->*L2
| |              ⇒
| |              dfsl-traversal(the_graph;L1;s)
| |              ⇒
| |              dfsl-traversal(the_graph;L1 @ L2;s)
| |         {dfsl_DASH_traversal_DASH_append})
| |        THEN
| |        (RWO
| |         Thm* For any graph ∀A:V List, x:V. A-->*[x] ⟺ A-->*x
| |         {list_DASH_list_DASH_connect_DASH_singleton}
| |         0))
| |       THEN
| |       (Analyze 9))
| |      THEN
| |      EasyHyp
| |
| .\
| . 1. the_graph : Graph
| . 2. the_obj : GraphObject(the_graph)
| . 3. Vertices(the_graph) List
| . 4. L1 : Vertices(the_graph) List
| . 5. s : traversal(the_graph)
| . 6. i : Vertices(the_graph)
| . 7. paren(Vertices(the_graph);s)
| . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . 9. dfsl-traversal(the_graph;L1;s)
| . 10. s' : traversal(the_graph)
| . 11. (inr(i) ∈ s) ∨ (inl(i) ∈ s) ⇒ s' = nil
| . 12. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
| . 13. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
| . 14. paren(Vertices(the_graph);s')
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| . 15. ∀j:Vertices(the_graph). (inr(j) ∈ s') ⇒ i-the_graph->*j
| . 16. dfs(the_obj;s;i) = (s' @ s)
| . 17. df-traversal(the_graph;s' @ s)
| . 18. ¬(inl(i) ∈ s)
| . 19. s2 : traversal(the_graph)
| . 20. s' = ([inl(i)] @ s2 @ [inr(i)])
| . ⊢  dfsl-traversal(the_graph;L1 @ [i];([inl(i)] @ s2 @ [inr(i)])
| . |  @ s)
| . | by let h ≡ 11 in
| . |    (Thin h)
| . |    THEN
| . |    (AssertBY
| . |     l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . |     ((MoveToHyp 0 h) THEN (SubstFor s' -1)
| . |      THEN
| . |      (RWW
| . |       Thm* ∀a,b,c:T List.
| . |            l_disjoint(T;a @ b;c)
| . |            ⟺
| . |            l_disjoint(T;a;c) & l_disjoint(T;b;c)
| . |       {l_disjoint_append}
| . |       -1)))
| . |    THEN
| . |    (Thin h)
| . |    THEN
| . |    (AssertBY paren(Vertices(the_graph);s2)
| . |     ((Inst Thm: paren_interval
| . |       [Vertices(the_graph);s';nil;s2;nil;i])
| . |      THEN
| . |      (All Reduce)))
| . |    THEN
| . |    (AssertBY
| . |     (no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . |      & ¬(inr(i) ∈ s2))
| . |     (((MoveToHyp 0 h) THEN (SubstFor s' -1)
| . |       THEN
| . |       (RWW
| . |        Thm* ∀l1,l2:T List.
| . |             no_repeats(T;l1 @ l2)
| . |             ⟺
| . |             l_disjoint(T;l1;l2)
| . |             & no_repeats(T;l1)
| . |             & no_repeats(T;l2)
| . |        {no_repeats_append_iff}
| . |        -1))
| . |      THEN
| . |      (RWO
| . |       Thm* ∀a,b:T List, t:T.
| . |            l_disjoint(T;b;[t / a])
| . |            ⟺
| . |            ¬(t ∈ b) & l_disjoint(T;b;a)
| . |       {l_disjoint_cons2}
| . |       -1)))
| . |    THEN
| . |    (Thin h)
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| . |    THEN
| . |    (Thin h)
| . |    THEN
| . |    (AssertBY
| . |     (∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j)
| . |     (Auto THEN BackThruSomeHyp THEN (SubstFor s' 0)))
| . |
| . |\
| . | .....aux .....
| . | 1. the_graph : Graph
| . | 2. the_obj : GraphObject(the_graph)
| . | 3. Vertices(the_graph) List
| . | 4. L1 : Vertices(the_graph) List
| . | 5. s : traversal(the_graph)
| . | 6. i : Vertices(the_graph)
| . | 7. paren(Vertices(the_graph);s)
| . | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . | 9. dfsl-traversal(the_graph;L1;s)
| . | 10. s' : traversal(the_graph)
| . | 11. ∀j:Vertices(the_graph). (inr(j) ∈ s') ⇒ i-the_graph->*j
| . | 12. dfs(the_obj;s;i) = (s' @ s)
| . | 13. df-traversal(the_graph;s' @ s)
| . | 14. ¬(inl(i) ∈ s)
| . | 15. s2 : traversal(the_graph)
| . | 16. s' = ([inl(i)] @ s2 @ [inr(i)])
| . | 17. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . | 18. paren(Vertices(the_graph);s2)
| . | 19. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . | 20. ¬(inr(i) ∈ s2)
| . | 21. j : Vertices(the_graph)
| . | 22. (inr(j) ∈ s2)
| . | ⊢  (inr(j) ∈ [inl(i)] @ s2 @ [inr(i)])  by ObviousFrom [-1]
| . |
| . .\
| . . 1. the_graph : Graph
| . . 2. the_obj : GraphObject(the_graph)
| . . 3. Vertices(the_graph) List
| . . 4. L1 : Vertices(the_graph) List
| . . 5. s : traversal(the_graph)
| . . 6. i : Vertices(the_graph)
| . . 7. paren(Vertices(the_graph);s)
| . . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . 9. dfsl-traversal(the_graph;L1;s)
| . . 10. s' : traversal(the_graph)
| . . 11. ∀j:Vertices(the_graph). (inr(j) ∈ s') ⇒ i-the_graph->*j
| . . 12. dfs(the_obj;s;i) = (s' @ s)
| . . 13. df-traversal(the_graph;s' @ s)
| . . 14. ¬(inl(i) ∈ s)
| . . 15. s2 : traversal(the_graph)
| . . 16. s' = ([inl(i)] @ s2 @ [inr(i)])
| . . 17. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . . 18. paren(Vertices(the_graph);s2)
| . . 19. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . |   & ¬(inr(i) ∈ s2)
| . . 20. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
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| . . ⊢  dfsl-traversal(the_graph;L1 @ [i];([inl(i)] @ s2 @ [inr(i)])
| . . |  @ s)
| . . | by let h ≡ 11 in
| . . |    (Thin h) THEN (Thin h) THEN (MoveToHyp 0 h)
| . . |    THEN
| . . |    (SubstFor s' -1)
| . . |    THEN
| . . |    (Thin h+2)
| . . |    THEN
| . . |    (Thin h-1)
| . . |
| . . 1. the_graph : Graph
| . . 2. GraphObject(the_graph)
| . . 3. Vertices(the_graph) List
| . . 4. L1 : Vertices(the_graph) List
| . . 5. s : traversal(the_graph)
| . . 6. i : Vertices(the_graph)
| . . 7. paren(Vertices(the_graph);s)
| . . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . 9. dfsl-traversal(the_graph;L1;s)
| . . 10. ¬(inl(i) ∈ s)
| . . 11. s2 : traversal(the_graph)
| . . 12. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . . 13. paren(Vertices(the_graph);s2)
| . . 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . |   & ¬(inr(i) ∈ s2)
| . . 15. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . 16. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . ⊢  dfsl-traversal(the_graph;L1 @ [i];([inl(i)] @ s2 @ [inr(i)])
| . . |  @ s)
| . . | by (AllHyps (λh.ParallelOp h)) THEN ExRepD THEN (Analyze 0)
| . . |    THEN
| . . |    (Analyze 0)
| . . |    THEN
| . . |    (Try Trivial)
| . . |
| . . |\
| . . | 1. the_graph : Graph
| . . | 2. GraphObject(the_graph)
| . . | 3. Vertices(the_graph) List
| . . | 4. L1 : Vertices(the_graph) List
| . . | 5. s : traversal(the_graph)
| . . | 6. i : Vertices(the_graph)
| . . | 7. paren(Vertices(the_graph);s)
| . . | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . | 9. df-traversal(the_graph;s)
| . . | 10. ∀i:Vertices(the_graph). (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . | 11. (∀i:Vertices(the_graph).
| . . | |    L1-the_graph->*i ⇒ ¬non-trivial-loop(the_graph;i))
| . . | |   ⇒
| . . | |   (∀L1@0,L2:Vertices(the_graph) List.
| . . | |    L1 = (L1@0 @ L2)
| . . | |    ⇒
| . . | |    (∃s1,s2:traversal(the_graph).
| . . | |     s = (s2 @ s1)
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| . . | |     & paren(Vertices(the_graph);s1)
| . . | |     & paren(Vertices(the_graph);s2)
| . . | |     & (∀j:Vertices(the_graph).
| . . | |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . | |        & ((inl(j) ∈ s2)
| . . | |           ⟺
| . . | |           L2-the_graph->*j & ¬L1@0-the_graph->*j))))
| . . | 12. ¬(inl(i) ∈ s)
| . . | 13. s2 : traversal(the_graph)
| . . | 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . . | 15. paren(Vertices(the_graph);s2)
| . . | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . | 17. ¬(inr(i) ∈ s2)
| . . | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . | 19. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . | ⊢  ∀i@0:Vertices(the_graph).
| . . | |  (inl(i@0) ∈ ([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . | |  ⇒
| . . | |  L1 @ [i]-the_graph->*i@0
| . . | | by ((RWW
| . . | |      Thm* ∀x:T, l1,l2:T List.
| . . | |           (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| . . | |      {member_append}
| . . | |      0)
| . . | |     THEN
| . . | |     (RW0
| . . | |      Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a    {member_singleton}
| . . | |      0)
| . . | |     THEN
| . . | |     (RW0
| . . | |      Thm* For any graph
| . . | |           ∀A,B:V List, i:V. A @ B-->*i ⟺ A-->*i ∨ B-->*i
| . . | |      {list_DASH_connect_DASH_append}
| . . | |      0))
| . . | |    THEN
| . . | |    SplitOrHyps
| . . | |
| . . | |\
| . . | | 1. the_graph : Graph
| . . | | 2. GraphObject(the_graph)
| . . | | 3. Vertices(the_graph) List
| . . | | 4. L1 : Vertices(the_graph) List
| . . | | 5. s : traversal(the_graph)
| . . | | 6. i : Vertices(the_graph)
| . . | | 7. paren(Vertices(the_graph);s)
| . . | | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . | | 9. df-traversal(the_graph;s)
| . . | | 10. ∀i:Vertices(the_graph). (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . | | 11. (∀i:Vertices(the_graph).
| . . | | |    L1-the_graph->*i ⇒ ¬non-trivial-loop(the_graph;i))
| . . | | |   ⇒
| . . | | |   (∀L1@0,L2:Vertices(the_graph) List.
| . . | | |    L1 = (L1@0 @ L2)
| . . | | |    ⇒
| . . | | |    (∃s1,s2:traversal(the_graph).
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| . . | | |     s = (s2 @ s1)
| . . | | |     & paren(Vertices(the_graph);s1)
| . . | | |     & paren(Vertices(the_graph);s2)
| . . | | |     & (∀j:Vertices(the_graph).
| . . | | |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . | | |        & ((inl(j) ∈ s2)
| . . | | |           ⟺
| . . | | |           L2-the_graph->*j & ¬L1@0-the_graph->*j))))
| . . | | 12. ¬(inl(i) ∈ s)
| . . | | 13. s2 : traversal(the_graph)
| . . | | 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . . | | 15. paren(Vertices(the_graph);s2)
| . . | | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . | | 17. ¬(inr(i) ∈ s2)
| . . | | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . | | 19. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . | | 20. i@0 : Vertices(the_graph)
| . . | | 21. inl(i@0) = inl(i)
| . . | | ⊢  L1-the_graph->*i@0 ∨ [i]-the_graph->*i@0
| . . | | | by Subst (i@0 = i) 0
| . . | | |
| . . | | |\
| . . | | | .....equality .....
| . . | | | 1. the_graph : Graph
| . . | | | 2. GraphObject(the_graph)
| . . | | | 3. Vertices(the_graph) List
| . . | | | 4. L1 : Vertices(the_graph) List
| . . | | | 5. s : traversal(the_graph)
| . . | | | 6. i : Vertices(the_graph)
| . . | | | 7. paren(Vertices(the_graph);s)
| . . | | | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . | | | 9. df-traversal(the_graph;s)
| . . | | | 10. ∀i:Vertices(the_graph).
| . . | | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . | | | 11. (∀i:Vertices(the_graph).
| . . | | |      L1-the_graph->*i ⇒ ¬non-trivial-loop(the_graph;i))
| . . | | |     ⇒
| . . | | |     (∀L1@0,L2:Vertices(the_graph) List.
| . . | | |      L1 = (L1@0 @ L2)
| . . | | |      ⇒
| . . | | |      (∃s1,s2:traversal(the_graph).
| . . | | |       s = (s2 @ s1)
| . . | | |       & paren(Vertices(the_graph);s1)
| . . | | |       & paren(Vertices(the_graph);s2)
| . . | | |       & (∀j:Vertices(the_graph).
| . . | | |          ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . | | |          & ((inl(j) ∈ s2)
| . . | | |             ⟺
| . . | | |             L2-the_graph->*j & ¬L1@0-the_graph->*j))))
| . . | | | 12. ¬(inl(i) ∈ s)
| . . | | | 13. s2 : traversal(the_graph)
| . . | | | 14. l_disjoint(Vertices(the_graph)+
| . . | | |     Vertices(the_graph);s2;s)
| . . | | | 15. paren(Vertices(the_graph);s2)
| . . | | | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
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| . . | | | 17. ¬(inr(i) ∈ s2)
| . . | | | 18. ∀j:Vertices(the_graph).
| . . | | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . | | | 19. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . | | | 20. i@0 : Vertices(the_graph)
| . . | | | 21. inl(i@0) = inl(i)
| . . | | | ⊢  i@0 = i  by ObviousFrom [-1]

| . . | | |
| . . | | .\
| . . | | . 1. the_graph : Graph
| . . | | . 2. GraphObject(the_graph)
| . . | | . 3. Vertices(the_graph) List
| . . | | . 4. L1 : Vertices(the_graph) List
| . . | | . 5. s : traversal(the_graph)
| . . | | . 6. i : Vertices(the_graph)
| . . | | . 7. paren(Vertices(the_graph);s)
| . . | | . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . | | . 9. df-traversal(the_graph;s)
| . . | | . 10. ∀i:Vertices(the_graph).
| . . | | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . | | . 11. (∀i:Vertices(the_graph).
| . . | | . |    L1-the_graph->*i ⇒ ¬non-trivial-loop(the_graph;i))
| . . | | . |   ⇒
| . . | | . |   (∀L1@0,L2:Vertices(the_graph) List.
| . . | | . |    L1 = (L1@0 @ L2)
| . . | | . |    ⇒
| . . | | . |    (∃s1,s2:traversal(the_graph).
| . . | | . |     s = (s2 @ s1)
| . . | | . |     & paren(Vertices(the_graph);s1)
| . . | | . |     & paren(Vertices(the_graph);s2)
| . . | | . |     & (∀j:Vertices(the_graph).
| . . | | . |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . | | . |        & ((inl(j) ∈ s2)
| . . | | . |           ⟺
| . . | | . |           L2-the_graph->*j & ¬L1@0-the_graph->*j))))
| . . | | . 12. ¬(inl(i) ∈ s)
| . . | | . 13. s2 : traversal(the_graph)
| . . | | . 14. l_disjoint(Vertices(the_graph)+
| . . | | . |   Vertices(the_graph);s2;s)
| . . | | . 15. paren(Vertices(the_graph);s2)
| . . | | . 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . | | . 17. ¬(inr(i) ∈ s2)
| . . | | . 18. ∀j:Vertices(the_graph).
| . . | | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . | | . 19. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . | | . 20. i@0 : Vertices(the_graph)
| . . | | . 21. inl(i@0) = inl(i)
| . . | | . ⊢  L1-the_graph->*i ∨ [i]-the_graph->*i
| . . | | . | by OrRight THEN (Unfold `list-connect` 0)
| . . | | . |    THEN
| . . | | . |    (Unfold `l_exists` 0)
| . . | | . |    THEN
| . . | | . |    (InstConcl [i])
| . . | | . |    THEN
| . . | | . |    (Try Connect)
```

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| . . | | . |
| . . | | . 1. the_graph : Graph
| . . | | . 2. GraphObject(the_graph)
| . . | | . 3. Vertices(the_graph) List
| . . | | . 4. L1 : Vertices(the_graph) List
| . . | | . 5. s : traversal(the_graph)
| . . | | . 6. i : Vertices(the_graph)
| . . | | . 7. paren(Vertices(the_graph);s)
| . . | | . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . | | . 9. df-traversal(the_graph;s)
| . . | | . 10. ∀i:Vertices(the_graph).
| . . | | .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . | | . 11. (∀i:Vertices(the_graph).
| . . | | .      L1-the_graph->*i ⇒ ¬non-trivial-loop(the_graph;i))
| . . | | .     ⇒
| . . | | .     (∀L1@0,L2:Vertices(the_graph) List.
| . . | | .      L1 = (L1@0 @ L2)
| . . | | .      ⇒
| . . | | .      (∃s1,s2:traversal(the_graph).
| . . | | .       s = (s2 @ s1)
| . . | | .       & paren(Vertices(the_graph);s1)
| . . | | .       & paren(Vertices(the_graph);s2)
| . . | | .       & (∀j:Vertices(the_graph).
| . . | | .          ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . | | .          & ((inl(j) ∈ s2)
| . . | | .             ⟺
| . . | | .             L2-the_graph->*j & ¬L1@0-the_graph->*j))))
| . . | | . 12. ¬(inl(i) ∈ s)
| . . | | . 13. s2 : traversal(the_graph)
| . . | | . 14. l_disjoint(Vertices(the_graph)+
| . . | | .     Vertices(the_graph);s2;s)
| . . | | . 15. paren(Vertices(the_graph);s2)
| . . | | . 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . | | . 17. ¬(inr(i) ∈ s2)
| . . | | . 18. ∀j:Vertices(the_graph).
| . . | | .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . | | . 19. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . | | . 20. i@0 : Vertices(the_graph)
| . . | | . 21. inl(i@0) = inl(i)
| . . | | . ⊢  (i ∈ [i])  by ObviousConcl
| . . | |
| . . | |\
| . . | | 1. the_graph : Graph
| . . | | 2. GraphObject(the_graph)
| . . | | 3. Vertices(the_graph) List
| . . | | 4. L1 : Vertices(the_graph) List
| . . | | 5. s : traversal(the_graph)
| . . | | 6. i : Vertices(the_graph)
| . . | | 7. paren(Vertices(the_graph);s)
| . . | | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . | | 9. df-traversal(the_graph;s)
| . . | | 10. ∀i:Vertices(the_graph). (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . | | 11. (∀i:Vertices(the_graph).
| . . | | |    L1-the_graph->*i ⇒ ¬non-trivial-loop(the_graph;i))
| . . | | |   ⇒
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| . . | | |   (∀L1@0,L2:Vertices(the_graph) List.
| . . | | |    L1 = (L1@0 @ L2)
| . . | | |    ⇒
| . . | | |    (∃s1,s2:traversal(the_graph).
| . . | | |     s = (s2 @ s1)
| . . | | |     & paren(Vertices(the_graph);s1)
| . . | | |     & paren(Vertices(the_graph);s2)
| . . | | |     & (∀j:Vertices(the_graph).
| . . | | |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . | | |        & ((inl(j) ∈ s2)
| . . | | |           ⟺
| . . | | |           L2-the_graph->*j & ¬L1@0-the_graph->*j))))
| . . | | 12. ¬(inl(i) ∈ s)
| . . | | 13. s2 : traversal(the_graph)
| . . | | 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . . | | 15. paren(Vertices(the_graph);s2)
| . . | | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . | | 17. ¬(inr(i) ∈ s2)
| . . | | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . | | 19. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . | | 20. i@0 : Vertices(the_graph)
| . . | | 21. (inl(i@0) ∈ s2)
| . . | | ⊢  L1-the_graph->*i@0 ∨ [i]-the_graph->*i@0
| . . | | | by InstHyp [i@0] -4
| . . | | |
| . . | | |\
| . . | | | .....antecedent .....
| . . | | | 1. the_graph : Graph
| . . | | | 2. GraphObject(the_graph)
| . . | | | 3. Vertices(the_graph) List
| . . | | | 4. L1 : Vertices(the_graph) List
| . . | | | 5. s : traversal(the_graph)
| . . | | | 6. i : Vertices(the_graph)
| . . | | | 7. paren(Vertices(the_graph);s)
| . . | | | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . | | | 9. df-traversal(the_graph;s)
| . . | | | 10. ∀i:Vertices(the_graph).
| . . | | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . | | | 11. (∀i:Vertices(the_graph).
| . . | | |      L1-the_graph->*i ⇒ ¬non-trivial-loop(the_graph;i))
| . . | | |     ⇒
| . . | | |     (∀L1@0,L2:Vertices(the_graph) List.
| . . | | |      L1 = (L1@0 @ L2)
| . . | | |      ⇒
| . . | | |      (∃s1,s2:traversal(the_graph).
| . . | | |       s = (s2 @ s1)
| . . | | |       & paren(Vertices(the_graph);s1)
| . . | | |       & paren(Vertices(the_graph);s2)
| . . | | |       & (∀j:Vertices(the_graph).
| . . | | |          ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . | | |          & ((inl(j) ∈ s2)
| . . | | |             ⟺
| . . | | |             L2-the_graph->*j & ¬L1@0-the_graph->*j))))
| . . | | | 12. ¬(inl(i) ∈ s)
| . . | | | 13. s2 : traversal(the_graph)
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| . . | | | 14. l_disjoint(Vertices(the_graph)+
| . . | | |     Vertices(the_graph);s2;s)
| . . | | | 15. paren(Vertices(the_graph);s2)
| . . | | | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . | | | 17. ¬(inr(i) ∈ s2)
| . . | | | 18. ∀j:Vertices(the_graph).
| . . | | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . | | | 19. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . | | | 20. i@0 : Vertices(the_graph)
| . . | | | 21. (inl(i@0) ∈ s2)
| . . | | | ⊢  (inr(i@0) ∈ s2)
| . . | | |   by BackThru
| . . | | |      Thm* ∀s':(T+T) List.
| . . | | |           paren(T;s')
| . . | | |           ⇒
| . . | | |           (∀i:T. (inl(i) ∈ s') ⇒ (inr(i) ∈ s'))
| . . | | |      {paren_balance1}
| . . | | |
| . . | | .\
| . . | | . 1. the_graph : Graph
| . . | | . 2. GraphObject(the_graph)
| . . | | . 3. Vertices(the_graph) List
| . . | | . 4. L1 : Vertices(the_graph) List
| . . | | . 5. s : traversal(the_graph)
| . . | | . 6. i : Vertices(the_graph)
| . . | | . 7. paren(Vertices(the_graph);s)
| . . | | . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . | | . 9. df-traversal(the_graph;s)
| . . | | . 10. ∀i:Vertices(the_graph).
| . . | | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . | | . 11. (∀i:Vertices(the_graph).
| . . | | . |    L1-the_graph->*i ⇒ ¬non-trivial-loop(the_graph;i))
| . . | | . |   ⇒
| . . | | . |   (∀L1@0,L2:Vertices(the_graph) List.
| . . | | . |    L1 = (L1@0 @ L2)
| . . | | . |    ⇒
| . . | | . |    (∃s1,s2:traversal(the_graph).
| . . | | . |     s = (s2 @ s1)
| . . | | . |     & paren(Vertices(the_graph);s1)
| . . | | . |     & paren(Vertices(the_graph);s2)
| . . | | . |     & (∀j:Vertices(the_graph).
| . . | | . |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . | | . |        & ((inl(j) ∈ s2)
| . . | | . |           ⟺
| . . | | . |           L2-the_graph->*j & ¬L1@0-the_graph->*j))))
| . . | | . 12. ¬(inl(i) ∈ s)
| . . | | . 13. s2 : traversal(the_graph)
| . . | | . 14. l_disjoint(Vertices(the_graph)+
| . . | | . |   Vertices(the_graph);s2;s)
| . . | | . 15. paren(Vertices(the_graph);s2)
| . . | | . 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . | | . 17. ¬(inr(i) ∈ s2)
| . . | | . 18. ∀j:Vertices(the_graph).
| . . | | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . | | . 19. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
```

```
| . . | | . 20. i@0 : Vertices(the_graph)
| . . | | . 21. (inl(i@0) ∈ s2)
| . . | | . 22. i-the_graph->*i@0
| . . | | . ⊢  L1-the_graph->*i@0 ∨ [i]-the_graph->*i@0
| . . | | . | by OrRight THEN (Unfold ‘list-connect‘ 0)
| . . | | . |    THEN
| . . | | . |    (Unfold ‘l_exists‘ 0)
| . . | | . |    THEN
| . . | | . |    (InstConcl [i])
| . . | | . |
| . . | | . 1. the_graph : Graph
| . . | | . 2. GraphObject(the_graph)
| . . | | . 3. Vertices(the_graph) List
| . . | | . 4. L1 : Vertices(the_graph) List
| . . | | . 5. s : traversal(the_graph)
| . . | | . 6. i : Vertices(the_graph)
| . . | | . 7. paren(Vertices(the_graph);s)
| . . | | . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . | | . 9. df-traversal(the_graph;s)
| . . | | . 10. ∀i:Vertices(the_graph).
| . . | | .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . | | . 11. (∀i:Vertices(the_graph).
| . . | | .      L1-the_graph->*i ⇒ ¬non-trivial-loop(the_graph;i))
| . . | | .     ⇒
| . . | | .     (∀L1@0,L2:Vertices(the_graph) List.
| . . | | .      L1 = (L1@0 @ L2)
| . . | | .      ⇒
| . . | | .      (∃s1,s2:traversal(the_graph).
| . . | | .       s = (s2 @ s1)
| . . | | .       & paren(Vertices(the_graph);s1)
| . . | | .       & paren(Vertices(the_graph);s2)
| . . | | .       & (∀j:Vertices(the_graph).
| . . | | .          ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . | | .          & ((inl(j) ∈ s2)
| . . | | .             ⟺
| . . | | .             L2-the_graph->*j & ¬L1@0-the_graph->*j))))
| . . | | . 12. ¬(inl(i) ∈ s)
| . . | | . 13. s2 : traversal(the_graph)
| . . | | . 14. l_disjoint(Vertices(the_graph)+
| . . | | .     Vertices(the_graph);s2;s)
| . . | | . 15. paren(Vertices(the_graph);s2)
| . . | | . 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . | | . 17. ¬(inr(i) ∈ s2)
| . . | | . 18. ∀j:Vertices(the_graph).
| . . | | .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . | | . 19. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . | | . 20. i@0 : Vertices(the_graph)
| . . | | . 21. (inl(i@0) ∈ s2)
| . . | | . 22. i-the_graph->*i@0
| . . | | . ⊢  (i ∈ [i])  by ObviousConcl
| . . | |
| . . | |\
| . . | | 1. the_graph : Graph
| . . | | 2. GraphObject(the_graph)
| . . | | 3. Vertices(the_graph) List
```

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| . . | | 4. L1 : Vertices(the_graph) List
| . . | | 5. s : traversal(the_graph)
| . . | | 6. i : Vertices(the_graph)
| . . | | 7. paren(Vertices(the_graph);s)
| . . | | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . | | 9. df-traversal(the_graph;s)
| . . | | 10. ∀i:Vertices(the_graph). (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . | | 11. (∀i:Vertices(the_graph).
| . . | |      L1-the_graph->*i ⇒ ¬non-trivial-loop(the_graph;i))
| . . | |     ⇒
| . . | |     (∀L1@0,L2:Vertices(the_graph) List.
| . . | |      L1 = (L1@0 @ L2)
| . . | |      ⇒
| . . | |      (∃s1,s2:traversal(the_graph).
| . . | |       s = (s2 @ s1)
| . . | |       & paren(Vertices(the_graph);s1)
| . . | |       & paren(Vertices(the_graph);s2)
| . . | |       & (∀j:Vertices(the_graph).
| . . | |          ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . | |          & ((inl(j) ∈ s2)
| . . | |             ⟺
| . . | |             L2-the_graph->*j & ¬L1@0-the_graph->*j))))
| . . | | 12. ¬(inl(i) ∈ s)
| . . | | 13. s2 : traversal(the_graph)
| . . | | 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . . | | 15. paren(Vertices(the_graph);s2)
| . . | | 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . | | 17. ¬(inr(i) ∈ s2)
| . . | | 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . | | 19. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . | | 20. i@0 : Vertices(the_graph)
| . . | | 21. inl(i@0) = inr(i)
| . . | | ⊢  L1-the_graph->*i@0 ∨ [i]-the_graph->*i@0
| . . | |   by ObviousContradiction [-1]
| . . | |
| . . | .\
| . . | . 1. the_graph : Graph
| . . | . 2. GraphObject(the_graph)
| . . | . 3. Vertices(the_graph) List
| . . | . 4. L1 : Vertices(the_graph) List
| . . | . 5. s : traversal(the_graph)
| . . | . 6. i : Vertices(the_graph)
| . . | . 7. paren(Vertices(the_graph);s)
| . . | . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . | . 9. df-traversal(the_graph;s)
| . . | . 10. ∀i:Vertices(the_graph). (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . | . 11. (∀i:Vertices(the_graph).
| . . | .      L1-the_graph->*i ⇒ ¬non-trivial-loop(the_graph;i))
| . . | .     ⇒
| . . | .     (∀L1@0,L2:Vertices(the_graph) List.
| . . | .      L1 = (L1@0 @ L2)
| . . | .      ⇒
| . . | .      (∃s1,s2:traversal(the_graph).
| . . | .       s = (s2 @ s1)
| . . | .       & paren(Vertices(the_graph);s1)
```

```
| . . | .       & paren(Vertices(the_graph);s2)
| . . | .       & (∀j:Vertices(the_graph).
| . . | .          ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . | .          & ((inl(j) ∈ s2)
| . . | .             ⟺
| . . | .             L2-the_graph->*j & ¬L1@0-the_graph->*j))))
| . . | . 12. ¬(inl(i) ∈ s)
| . . | . 13. s2 : traversal(the_graph)
| . . | . 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . . | . 15. paren(Vertices(the_graph);s2)
| . . | . 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . | . 17. ¬(inr(i) ∈ s2)
| . . | . 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . | . 19. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . | . 20. i@0 : Vertices(the_graph)
| . . | . 21. (inl(i@0) ∈ s)
| . . | . ⊢  L1-the_graph->*i@0 ∨ [i]-the_graph->*i@0
| . . | .   by OrLeft THEN EasyHyp
| . . |
| . . .\
| . . . 1. the_graph : Graph
| . . . 2. GraphObject(the_graph)
| . . . 3. Vertices(the_graph) List
| . . . 4. L1 : Vertices(the_graph) List
| . . . 5. s : traversal(the_graph)
| . . . 6. i : Vertices(the_graph)
| . . . 7. paren(Vertices(the_graph);s)
| . . . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . 9. df-traversal(the_graph;s)
| . . . 10. ∀i:Vertices(the_graph). (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . 11. (∀i:Vertices(the_graph).
| . . . |    L1-the_graph->*i ⇒ ¬non-trivial-loop(the_graph;i))
| . . . |   ⇒
| . . . |   (∀L1@0,L2:Vertices(the_graph) List.
| . . . |    L1 = (L1@0 @ L2)
| . . . |    ⇒
| . . . |    (∃s1,s2:traversal(the_graph).
| . . . |     s = (s2 @ s1)
| . . . |     & paren(Vertices(the_graph);s1)
| . . . |     & paren(Vertices(the_graph);s2)
| . . . |     & (∀j:Vertices(the_graph).
| . . . |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . |        & ((inl(j) ∈ s2)
| . . . |           ⟺
| . . . |           L2-the_graph->*j & ¬L1@0-the_graph->*j))))
| . . . 12. ¬(inl(i) ∈ s)
| . . . 13. s2 : traversal(the_graph)
| . . . 14. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . . . 15. paren(Vertices(the_graph);s2)
| . . . 16. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . . 17. ¬(inr(i) ∈ s2)
| . . . 18. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . 19. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . 20. ∀i@0:Vertices(the_graph).
| . . . |   L1 @ [i]-the_graph->*i@0
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| . . . |   ⇒
| . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . ⊢  ∀L1@0,L2:Vertices(the_graph) List.
| . . . |  (L1 @ [i]) = (L1@0 @ L2)
| . . . |  ⇒
| . . . |  (∃s1,s2@0:traversal(the_graph).
| . . . |   (([inl(i)] @ s2 @ [inr(i)]) @ s) = (s2@0 @ s1)
| . . . |   & paren(Vertices(the_graph);s1)
| . . . |   & paren(Vertices(the_graph);s2@0)
| . . . |   & (∀j:Vertices(the_graph).
| . . . |      ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . |      & ((inl(j) ∈ s2@0)
| . . . |         ⟺
| . . . |         L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | by (Analyze 11)
| . . . |    THENA
| . . . |    ((Auto THEN BackThruSomeHyp
| . . . |      THEN
| . . . |      (RWO
| . . . |       Thm* For any graph
| . . . |            ∀A,B:V List, i:V. A @ B-->*i ⟺ A-->*i ∨ B-->*i
| . . . |       {list_DASH_connect_DASH_append}

| . . . |       0))
| . . . |     THEN
| . . . |     OrLeft)
| . . . |
| . . . 1. the_graph : Graph
| . . . 2. GraphObject(the_graph)
| . . . 3. Vertices(the_graph) List
| . . . 4. L1 : Vertices(the_graph) List
| . . . 5. s : traversal(the_graph)
| . . . 6. i : Vertices(the_graph)
| . . . 7. paren(Vertices(the_graph);s)
| . . . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . 9. df-traversal(the_graph;s)
| . . . 10. ∀i:Vertices(the_graph). (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . 11. ¬(inl(i) ∈ s)
| . . . 12. s2 : traversal(the_graph)
| . . . 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . . . 14. paren(Vertices(the_graph);s2)
| . . . 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . . 16. ¬(inr(i) ∈ s2)
| . . . 17. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . 19. ∀i@0:Vertices(the_graph).
| . . . |   L1 @ [i]-the_graph->*i@0
| . . . |   ⇒
| . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . |   L1 = (L1@0 @ L2)
| . . . |   ⇒
| . . . |   (∃s1,s2:traversal(the_graph).
| . . . |    s = (s2 @ s1)
| . . . |    & paren(Vertices(the_graph);s1)
| . . . |    & paren(Vertices(the_graph);s2)
```

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| . . . |    & (∀j:Vertices(the_graph).
| . . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . |       & ((inl(j) ∈ s2)
| . . . |          ⟺
| . . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . ⊢  ∀L1@0,L2:Vertices(the_graph) List.
| . . . |  (L1 @ [i]) = (L1@0 @ L2)
| . . . |  ⇒
| . . . |  (∃s1,s2@0:traversal(the_graph).
| . . . |   (([inl(i)] @ s2 @ [inr(i)]) @ s) = (s2@0 @ s1)
| . . . |   & paren(Vertices(the_graph);s1)
| . . . |   & paren(Vertices(the_graph);s2@0)
| . . . |   & (∀j:Vertices(the_graph).
| . . . |      ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . |      & ((inl(j) ∈ s2@0)
| . . . |         ⟺
| . . . |         L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | by Auto THEN (EqualAppendsD -1) THEN (Analyze -1)
| . . . |
| . . . |\
| . . . | 1. the_graph : Graph
| . . . | 2. GraphObject(the_graph)
| . . . | 3. Vertices(the_graph) List
| . . . | 4. L1 : Vertices(the_graph) List
| . . . | 5. s : traversal(the_graph)
| . . . | 6. i : Vertices(the_graph)
| . . . | 7. paren(Vertices(the_graph);s)
| . . . | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . | 9. df-traversal(the_graph;s)
| . . . | 10. ∀i:Vertices(the_graph). (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | 11. ¬(inl(i) ∈ s)
| . . . | 12. s2 : traversal(the_graph)
| . . . | 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . . . | 14. paren(Vertices(the_graph);s2)
| . . . | 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . . | 16. ¬(inr(i) ∈ s2)
| . . . | 17. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | 19. ∀i@0:Vertices(the_graph).
| . . . | |   L1 @ [i]-the_graph->*i@0
| . . . | |   ⇒
| . . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | |   L1 = (L1@0 @ L2)
| . . . | |   ⇒
| . . . | |   (∃s1,s2:traversal(the_graph).
| . . . | |    s = (s2 @ s1)
| . . . | |    & paren(Vertices(the_graph);s1)
| . . . | |    & paren(Vertices(the_graph);s2)
| . . . | |    & (∀j:Vertices(the_graph).
| . . . | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | |       & ((inl(j) ∈ s2)
| . . . | |          ⟺
| . . . | |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | 21. L1@0 : Vertices(the_graph) List
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| . . . | 22. L2 : Vertices(the_graph) List
| . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | 24. e : Vertices(the_graph) List
| . . . | 25. L1 = (L1@0 @ e)
| . . . | 26. L2 = (e @ [i])
| . . . | ⊢  ∃s1,s2@0:traversal(the_graph).
| . . . | |   (([inl(i)] @ s2 @ [inr(i)]) @ s) = (s2@0 @ s1)
| . . . | |   & paren(Vertices(the_graph);s1)
| . . . | |   & paren(Vertices(the_graph);s2@0)
| . . . | |   & (∀j:Vertices(the_graph).
| . . . | |      ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | |      & ((inl(j) ∈ s2@0)
| . . . | |         ⟺
| . . . | |         L2-the_graph->*j & ¬L1@0-the_graph->*j))
| . . . | | by (InstHyp [L1@0;e] -7) THEN ExRepD
| . . . | |    THEN
| . . . | |    (InstConcl [s1;([inl(i)] @ s2 @ [inr(i)]) @ s3])
| . . . | |    THEN
| . . . | |    (Analyze 0)
| . . . | |
| . . . | |\
| . . . | | 1. the_graph : Graph
| . . . | | 2. GraphObject(the_graph)
| . . . | | 3. Vertices(the_graph) List
| . . . | | 4. L1 : Vertices(the_graph) List
| . . . | | 5. s : traversal(the_graph)
| . . . | | 6. i : Vertices(the_graph)
| . . . | | 7. paren(Vertices(the_graph);s)
| . . . | | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . | | 9. df-traversal(the_graph;s)
| . . . | | 10. ∀i:Vertices(the_graph).
| . . . | |      (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | | 11. ¬(inl(i) ∈ s)
| . . . | | 12. s2 : traversal(the_graph)
| . . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . | |     Vertices(the_graph);s2;s)
| . . . | | 14. paren(Vertices(the_graph);s2)
| . . . | | 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . . | | 16. ¬(inr(i) ∈ s2)
| . . . | | 17. ∀j:Vertices(the_graph).
| . . . | |      (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | | 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | | 19. ∀i@0:Vertices(the_graph).
| . . . | |     L1 @ [i]-the_graph->*i@0
| . . . | |     ⇒
| . . . | |     ¬non-trivial-loop(the_graph;i@0)
| . . . | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | |     L1 = (L1@0 @ L2)
| . . . | |     ⇒
| . . . | |     (∃s1,s2:traversal(the_graph).
| . . . | |      s = (s2 @ s1)
| . . . | |      & paren(Vertices(the_graph);s1)
| . . . | |      & paren(Vertices(the_graph);s2)
| . . . | |      & (∀j:Vertices(the_graph).
| . . . | |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
```

```
| . . . | |        & ((inl(j) ∈ s2)
| . . . | |           ⟺
| . . . | |           L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | | 21. L1@0 : Vertices(the_graph) List
| . . . | | 22. L2 : Vertices(the_graph) List
| . . . | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | | 24. e : Vertices(the_graph) List
| . . . | | 25. L1 = (L1@0 @ e)
| . . . | | 26. L2 = (e @ [i])
| . . . | | 27. s1 : traversal(the_graph)
| . . . | | 28. s3 : traversal(the_graph)
| . . . | | 29. s = (s3 @ s1)
| . . . | | 30. paren(Vertices(the_graph);s1)
| . . . | | 31. paren(Vertices(the_graph);s3)
| . . . | | 32. ∀j:Vertices(the_graph).
| . . . | |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | |     & ((inl(j) ∈ s3)
| . . . | |        ⟺
| . . . | |        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | | ⊢  (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | |    =
| . . . | |    ((([inl(i)] @ s2 @ [inr(i)]) @ s3) @ s1)
| . . . | |   by (RWW
| . . . | |       Thm* ∀a,b,c:Top List. ((a @ b) @ c) ~ (a @ b @ c)
| . . . | |       {append_assoc_sq}
| . . . | |       0)
| . . . | |      THEN
| . . . | |      (Repeat Analyze)
| . . . | |
| . . . | .\
| . . . | . 1. the_graph : Graph
| . . . | . 2. GraphObject(the_graph)
| . . . | . 3. Vertices(the_graph) List
| . . . | . 4. L1 : Vertices(the_graph) List
| . . . | . 5. s : traversal(the_graph)
| . . . | . 6. i : Vertices(the_graph)
| . . . | . 7. paren(Vertices(the_graph);s)
| . . . | . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . | . 9. df-traversal(the_graph;s)
| . . . | . 10. ∀i:Vertices(the_graph).
| . . . | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . 11. ¬(inl(i) ∈ s)
| . . . | . 12. s2 : traversal(the_graph)
| . . . | . 13. l_disjoint(Vertices(the_graph)+
| . . . | . |   Vertices(the_graph);s2;s)
| . . . | . 14. paren(Vertices(the_graph);s2)
| . . . | . 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . . | . 16. ¬(inr(i) ∈ s2)
| . . . | . 17. ∀j:Vertices(the_graph).
| . . . | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . 19. ∀i@0:Vertices(the_graph).
| . . . | . |   L1 @ [i]-the_graph->*i@0
| . . . | . |   ⇒
| . . . | . |   ¬non-trivial-loop(the_graph;i@0)
```

```
| . . . | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . |   L1 = (L1@0 @ L2)
| . . . | . |   ⇒
| . . . | . |   (∃s1,s2:traversal(the_graph).
| . . . | . |    s = (s2 @ s1)
| . . . | . |    & paren(Vertices(the_graph);s1)
| . . . | . |    & paren(Vertices(the_graph);s2)
| . . . | . |    & (∀j:Vertices(the_graph).
| . . . | . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . |       & ((inl(j) ∈ s2)
| . . . | . |          ⟺
| . . . | . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | . 21. L1@0 : Vertices(the_graph) List
| . . . | . 22. L2 : Vertices(the_graph) List
| . . . | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . 24. e : Vertices(the_graph) List
| . . . | . 25. L1 = (L1@0 @ e)
| . . . | . 26. L2 = (e @ [i])
| . . . | . 27. s1 : traversal(the_graph)
| . . . | . 28. s3 : traversal(the_graph)
| . . . | . 29. s = (s3 @ s1)
| . . . | . 30. paren(Vertices(the_graph);s1)
| . . . | . 31. paren(Vertices(the_graph);s3)
| . . . | . 32. ∀j:Vertices(the_graph).
| . . . | . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . |   & ((inl(j) ∈ s3)
| . . . | . |      ⟺
| . . . | . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . ⊢  paren(Vertices(the_graph);s1)
| . . . | . |  & paren(Vertices(the_graph);([inl(i)] @ s2 @ [inr(i)])
| . . . | . |    @ s3)
| . . . | . |  & (∀j:Vertices(the_graph).
| . . . | . |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . |     & ((inl(j) ∈ ([inl(i)] @ s2 @ [inr(i)]) @ s3)
| . . . | . |        ⟺
| . . . | . |        L2-the_graph->*j & ¬L1@0-the_graph->*j))
| . . . | . | by Analyze 0
| . . . | . |
| . . . | . |\
| . . . | . | 1. the_graph : Graph
| . . . | . | 2. GraphObject(the_graph)
| . . . | . | 3. Vertices(the_graph) List
| . . . | . | 4. L1 : Vertices(the_graph) List
| . . . | . | 5. s : traversal(the_graph)
| . . . | . | 6. i : Vertices(the_graph)
| . . . | . | 7. paren(Vertices(the_graph);s)
| . . . | . | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . | . | 9. df-traversal(the_graph;s)
| . . . | . | 10. ∀i:Vertices(the_graph).
| . . . | . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . | 11. ¬(inl(i) ∈ s)
| . . . | . | 12. s2 : traversal(the_graph)
| . . . | . | 13. l_disjoint(Vertices(the_graph)+
| . . . | . |     Vertices(the_graph);s2;s)
| . . . | . | 14. paren(Vertices(the_graph);s2)
```

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| . . . | . | 15. no_repeats(Vertices(the_graph)+
| . . . | . |     Vertices(the_graph);s2)
| . . . | . | 16. ¬(inr(i) ∈ s2)
| . . . | . | 17. ∀j:Vertices(the_graph).
| . . . | . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . | 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . | . |     @ s)
| . . . | . | 19. ∀i@0:Vertices(the_graph).
| . . . | . |     L1 @ [i]-the_graph->*i@0
| . . . | . |     ⇒
| . . . | . |     ¬non-trivial-loop(the_graph;i@0)
| . . . | . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . |     L1 = (L1@0 @ L2)
| . . . | . |     ⇒
| . . . | . |     (∃s1,s2:traversal(the_graph).
| . . . | . |      s = (s2 @ s1)
| . . . | . |      & paren(Vertices(the_graph);s1)
| . . . | . |      & paren(Vertices(the_graph);s2)
| . . . | . |      & (∀j:Vertices(the_graph).
| . . . | . |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . |         & ((inl(j) ∈ s2)
| . . . | . |            ⟺
| . . . | . |            L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | . | 21. L1@0 : Vertices(the_graph) List
| . . . | . | 22. L2 : Vertices(the_graph) List
| . . . | . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . | 24. e : Vertices(the_graph) List
| . . . | . | 25. L1 = (L1@0 @ e)
| . . . | . | 26. L2 = (e @ [i])
| . . . | . | 27. s1 : traversal(the_graph)
| . . . | . | 28. s3 : traversal(the_graph)
| . . . | . | 29. s = (s3 @ s1)
| . . . | . | 30. paren(Vertices(the_graph);s1)
| . . . | . | 31. paren(Vertices(the_graph);s3)
| . . . | . | 32. ∀j:Vertices(the_graph).
| . . . | . |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . |     & ((inl(j) ∈ s3)
| . . . | . |        ⟺
| . . . | . |        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . | ⊢  paren(Vertices(the_graph);s1)
| . . . | . |    & paren(Vertices(the_graph);([inl(i)]
| . . . | . |                                 @ s2
| . . . | . |                                 @ [inr(i)])
| . . . | . |      @ s3)
| . . . | . |   by ((Auto
| . . . | . |        THEN
| . . . | . |        (BackThru
| . . . | . |         Thm* ∀s',s3:(T+T) List.
| . . . | . |              paren(T;s')
| . . . | . |              ⇒
| . . . | . |              paren(T;s3) ⇒ paren(T;s3 @ s')
| . . . | . |         {append_paren})
| . . . | . |        THEN
| . . . | . |        (RecUnfold `paren` 0)
| . . . | . |        THEN
```

```
| . . . | . |        OrRight)
| . . . | . |       THEN
| . . . | . |       OrLeft)
| . . . | . |      THEN
| . . . | . |      (InstConcl [i;s2])
| . . . | . |
| . . . | . .\
| . . . | . . 1. the_graph : Graph
| . . . | . . 2. GraphObject(the_graph)
| . . . | . . 3. Vertices(the_graph) List
| . . . | . . 4. L1 : Vertices(the_graph) List
| . . . | . . 5. s : traversal(the_graph)
| . . . | . . 6. i : Vertices(the_graph)
| . . . | . . 7. paren(Vertices(the_graph);s)
| . . . | . . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . | . . 9. df-traversal(the_graph;s)
| . . . | . . 10. ∀i:Vertices(the_graph).
| . . . | . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . 11. ¬(inl(i) ∈ s)
| . . . | . . 12. s2 : traversal(the_graph)
| . . . | . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . |   Vertices(the_graph);s2;s)
| . . . | . . 14. paren(Vertices(the_graph);s2)
| . . . | . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . |   Vertices(the_graph);s2)
| . . . | . . 16. ¬(inr(i) ∈ s2)
| . . . | . . 17. ∀j:Vertices(the_graph).
| . . . | . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . | . . |   @ s)
| . . . | . . 19. ∀i@0:Vertices(the_graph).
| . . . | . . |   L1 @ [i]-the_graph->*i@0
| . . . | . . |   ⇒
| . . . | . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . |   L1 = (L1@0 @ L2)
| . . . | . . |   ⇒
| . . . | . . |   (∃s1,s2:traversal(the_graph).
| . . . | . . |    s = (s2 @ s1)
| . . . | . . |    & paren(Vertices(the_graph);s1)
| . . . | . . |    & paren(Vertices(the_graph);s2)
| . . . | . . |    & (∀j:Vertices(the_graph).
| . . . | . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . |       & ((inl(j) ∈ s2)
| . . . | . . |          ⟺
| . . . | . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | . . 21. L1@0 : Vertices(the_graph) List
| . . . | . . 22. L2 : Vertices(the_graph) List
| . . . | . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . 24. e : Vertices(the_graph) List
| . . . | . . 25. L1 = (L1@0 @ e)
| . . . | . . 26. L2 = (e @ [i])
| . . . | . . 27. s1 : traversal(the_graph)
| . . . | . . 28. s3 : traversal(the_graph)
| . . . | . . 29. s = (s3 @ s1)
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| . . . | . . 30. paren(Vertices(the_graph);s1)
| . . . | . . 31. paren(Vertices(the_graph);s3)
| . . . | . . 32. ∀j:Vertices(the_graph).
| . . . | . . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . |   & ((inl(j) ∈ s3)
| . . . | . . |      ⟺
| . . . | . . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . ⊢  ∀j:Vertices(the_graph).
| . . . | . . |  ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . |  & ((inl(j) ∈ ([inl(i)] @ s2 @ [inr(i)]) @ s3)
| . . . | . . |     ⟺
| . . . | . . |     L2-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | by ((ParallelOp -1) THEN (ParallelOp -1)
| . . . | . . |     THEN
| . . . | . . |     (RWW
| . . . | . . |      Thm* ∀x:T, l1,l2:T List.
| . . . | . . |           (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| . . . | . . |      {member_append}
| . . . | . . |      0))
| . . . | . . |    THEN
| . . . | . . |    (RWO
| . . . | . . |     Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a
| . . . | . . |     {member_singleton}
| . . . | . . |     0)
| . . . | . . |    THEN
| . . . | . . |    (SubstFor L2 0)
| . . . | . . |    THEN
| . . . | . . |    (RWO
| . . . | . . |     Thm* For any graph
| . . . | . . |          ∀A,B:V List, i:V.
| . . . | . . |          A @ B-->*i ⟺ A-->*i ∨ B-->*i
| . . . | . . |     {list_DASH_connect_DASH_append}
| . . . | . . |     0)
| . . . | . . |
| . . . | . . 1. the_graph : Graph
| . . . | . . 2. GraphObject(the_graph)
| . . . | . . 3. Vertices(the_graph) List

| . . . | . . 4. L1 : Vertices(the_graph) List
| . . . | . . 5. s : traversal(the_graph)
| . . . | . . 6. i : Vertices(the_graph)
| . . . | . . 7. paren(Vertices(the_graph);s)
| . . . | . . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . | . . 9. df-traversal(the_graph;s)
| . . . | . . 10. ∀i:Vertices(the_graph).
| . . . | . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . 11. ¬(inl(i) ∈ s)
| . . . | . . 12. s2 : traversal(the_graph)
| . . . | . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . |   Vertices(the_graph);s2;s)
| . . . | . . 14. paren(Vertices(the_graph);s2)
| . . . | . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . |   Vertices(the_graph);s2)
| . . . | . . 16. ¬(inr(i) ∈ s2)
| . . . | . . 17. ∀j:Vertices(the_graph).
| . . . | . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
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| . . . | . . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . | . . |   @ s)
| . . . | . . 19. ∀i@0:Vertices(the_graph).
| . . . | . . |   L1 @ [i]-the_graph->*i@0
| . . . | . . |   ⇒
| . . . | . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . |   L1 = (L1@0 @ L2)
| . . . | . . |   ⇒
| . . . | . . |   (∃s1,s2:traversal(the_graph).
| . . . | . . |    s = (s2 @ s1)
| . . . | . . |    & paren(Vertices(the_graph);s1)
| . . . | . . |    & paren(Vertices(the_graph);s2)
| . . . | . . |    & (∀j:Vertices(the_graph).
| . . . | . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . |       & ((inl(j) ∈ s2)
| . . . | . . |          ⟺
| . . . | . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | . . 21. L1@0 : Vertices(the_graph) List
| . . . | . . 22. L2 : Vertices(the_graph) List
| . . . | . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . 24. e : Vertices(the_graph) List
| . . . | . . 25. L1 = (L1@0 @ e)
| . . . | . . 26. L2 = (e @ [i])
| . . . | . . 27. s1 : traversal(the_graph)
| . . . | . . 28. s3 : traversal(the_graph)
| . . . | . . 29. s = (s3 @ s1)
| . . . | . . 30. paren(Vertices(the_graph);s1)
| . . . | . . 31. paren(Vertices(the_graph);s3)
| . . . | . . 32. ∀j:Vertices(the_graph).
| . . . | . . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . |   & ((inl(j) ∈ s3)
| . . . | . . |      ⟺
| . . . | . . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . 33. j : Vertices(the_graph)
| . . . | . . 34. (inl(j) ∈ s3)
| . . . | . . |   ⟺
| . . . | . . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . ⊢  inl(j) = inl(i) ∨ (inl(j) ∈ s2) ∨ inl(j) = inr(i)
| . . . | . . |  ∨ (inl(j) ∈ s3)
| . . . | . . |  ⟺
| . . . | . . |  e-the_graph->*j ∨ [i]-the_graph->*j
| . . . | . . |  & ¬L1@0-the_graph->*j  by RW (SweepDnC (HypC -1)) 0
| . . . | . . |
| . . . | . . 1. the_graph : Graph
| . . . | . . 2. GraphObject(the_graph)
| . . . | . . 3. Vertices(the_graph) List
| . . . | . . 4. L1 : Vertices(the_graph) List
| . . . | . . 5. s : traversal(the_graph)
| . . . | . . 6. i : Vertices(the_graph)
| . . . | . . 7. paren(Vertices(the_graph);s)
| . . . | . . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . | . . 9. df-traversal(the_graph;s)
| . . . | . . 10. ∀i:Vertices(the_graph).
| . . . | . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
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| . . . | . . 11. ¬(inl(i) ∈ s)
| . . . | . . 12. s2 : traversal(the_graph)
| . . . | . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . |   Vertices(the_graph);s2;s)
| . . . | . . 14. paren(Vertices(the_graph);s2)
| . . . | . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . |   Vertices(the_graph);s2)
| . . . | . . 16. ¬(inr(i) ∈ s2)
| . . . | . . 17. ∀j:Vertices(the_graph).
| . . . | . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . | . . |   @ s)
| . . . | . . 19. ∀i@0:Vertices(the_graph).
| . . . | . . |   L1 @ [i]-the_graph->*i@0
| . . . | . . |   ⇒
| . . . | . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . |   L1 = (L1@0 @ L2)
| . . . | . . |   ⇒
| . . . | . . |   (∃s1,s2:traversal(the_graph).
| . . . | . . |    s = (s2 @ s1)
| . . . | . . |    & paren(Vertices(the_graph);s1)
| . . . | . . |    & paren(Vertices(the_graph);s2)
| . . . | . . |    & (∀j:Vertices(the_graph).
| . . . | . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . |       & ((inl(j) ∈ s2)
| . . . | . . |          ⟺
| . . . | . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | . . 21. L1@0 : Vertices(the_graph) List
| . . . | . . 22. L2 : Vertices(the_graph) List
| . . . | . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . 24. e : Vertices(the_graph) List
| . . . | . . 25. L1 = (L1@0 @ e)
| . . . | . . 26. L2 = (e @ [i])
| . . . | . . 27. s1 : traversal(the_graph)
| . . . | . . 28. s3 : traversal(the_graph)
| . . . | . . 29. s = (s3 @ s1)
| . . . | . . 30. paren(Vertices(the_graph);s1)
| . . . | . . 31. paren(Vertices(the_graph);s3)
| . . . | . . 32. ∀j:Vertices(the_graph).
| . . . | . . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . |   & ((inl(j) ∈ s3)
| . . . | . . |      ⟺
| . . . | . . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . 33. j : Vertices(the_graph)
| . . . | . . 34. (inl(j) ∈ s3)
| . . . | . . |   ⟺
| . . . | . . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . ⊢  inl(j) = inl(i) ∨ (inl(j) ∈ s2) ∨ inl(j) = inr(i)
| . . . | . . |  ∨ e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . |  ⟺
| . . . | . . |  e-the_graph->*j ∨ [i]-the_graph->*j
| . . . | . . |  & ¬L1@0-the_graph->*j
| . . . | . . | by (Analyze 0) THEN (Analyze 0)
| . . . | . . |
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| . . . | . . |\
| . . . | . . | 1. the_graph : Graph
| . . . | . . | 2. GraphObject(the_graph)
| . . . | . . | 3. Vertices(the_graph) List
| . . . | . . | 4. L1 : Vertices(the_graph) List
| . . . | . . | 5. s : traversal(the_graph)
| . . . | . . | 6. i : Vertices(the_graph)
| . . . | . . | 7. paren(Vertices(the_graph);s)
| . . . | . . | 8. no_repeats(Vertices(the_graph)+
| . . . | . . | |  Vertices(the_graph);s)
| . . . | . . | 9. df-traversal(the_graph;s)
| . . . | . . | 10. ∀i:Vertices(the_graph).
| . . . | . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | 11. ¬(inl(i) ∈ s)
| . . . | . . | 12. s2 : traversal(the_graph)
| . . . | . . | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | |   Vertices(the_graph);s2;s)
| . . . | . . | 14. paren(Vertices(the_graph);s2)
| . . . | . . | 15. no_repeats(Vertices(the_graph)+
| . . . | . . | |   Vertices(the_graph);s2)
| . . . | . . | 16. ¬(inr(i) ∈ s2)
| . . . | . . | 17. ∀j:Vertices(the_graph).
| . . . | . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . | . . | |   @ s)
| . . . | . . | 19. ∀i@0:Vertices(the_graph).
| . . . | . . | |   L1 @ [i]-the_graph->*i@0
| . . . | . . | |   ⇒
| . . . | . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | |   L1 = (L1@0 @ L2)
| . . . | . . | |   ⇒
| . . . | . . | |   (∃s1,s2:traversal(the_graph).
| . . . | . . | |    s = (s2 @ s1)
| . . . | . . | |    & paren(Vertices(the_graph);s1)
| . . . | . . | |    & paren(Vertices(the_graph);s2)
| . . . | . . | |    & (∀j:Vertices(the_graph).
| . . . | . . | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | |       & ((inl(j) ∈ s2)
| . . . | . . | |          ⟺
| . . . | . . | |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | . . | 21. L1@0 : Vertices(the_graph) List
| . . . | . . | 22. L2 : Vertices(the_graph) List
| . . . | . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | 24. e : Vertices(the_graph) List
| . . . | . . | 25. L1 = (L1@0 @ e)
| . . . | . . | 26. L2 = (e @ [i])
| . . . | . . | 27. s1 : traversal(the_graph)
| . . . | . . | 28. s3 : traversal(the_graph)
| . . . | . . | 29. s = (s3 @ s1)
| . . . | . . | 30. paren(Vertices(the_graph);s1)
| . . . | . . | 31. paren(Vertices(the_graph);s3)
| . . . | . . | 32. ∀j:Vertices(the_graph).
| . . . | . . | |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | |   & ((inl(j) ∈ s3)
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| . . . | . . | |      ⟺
| . . . | . . | |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | 33. j : Vertices(the_graph)
| . . . | . . | 34. (inl(j) ∈ s3)
| . . . | . . | |   ⟺
| . . . | . . | |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | 35. inl(j) = inl(i) ∨ (inl(j) ∈ s2) ∨ inl(j) = inr(i)
| . . . | . . | |   ∨ e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | ⊢  e-the_graph->*j ∨ [i]-the_graph->*j
| . . . | . . | |  & ¬L1@0-the_graph->*j  by Analyze -1
| . . . | . . | |
| . . . | . . | |\
| . . . | . . | | 1. the_graph : Graph
| . . . | . . | | 2. GraphObject(the_graph)
| . . . | . . | | 3. Vertices(the_graph) List
| . . . | . . | | 4. L1 : Vertices(the_graph) List
| . . . | . . | | 5. s : traversal(the_graph)
| . . . | . . | | 6. i : Vertices(the_graph)
| . . . | . . | | 7. paren(Vertices(the_graph);s)
| . . . | . . | | 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | |  Vertices(the_graph);s)
| . . . | . . | | 9. df-traversal(the_graph;s)
| . . . | . . | | 10. ∀i:Vertices(the_graph).
| . . . | . . | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | 11. ¬(inl(i) ∈ s)
| . . . | . . | | 12. s2 : traversal(the_graph)
| . . . | . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | |   Vertices(the_graph);s2;s)
| . . . | . . | | 14. paren(Vertices(the_graph);s2)
| . . . | . . | | 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | |   Vertices(the_graph);s2)
| . . . | . . | | 16. ¬(inr(i) ∈ s2)
| . . . | . . | | 17. ∀j:Vertices(the_graph).
| . . . | . . | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | |                          @ s2
| . . . | . . | | |                          @ [inr(i)])
| . . . | . . | | |   @ s)
| . . . | . . | | 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | |   L1 @ [i]-the_graph->*i@0
| . . . | . . | | |   ⇒
| . . . | . . | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | |   L1 = (L1@0 @ L2)
| . . . | . . | | |   ⇒
| . . . | . . | | |   (∃s1,s2:traversal(the_graph).
| . . . | . . | | |    s = (s2 @ s1)
| . . . | . . | | |    & paren(Vertices(the_graph);s1)
| . . . | . . | | |    & paren(Vertices(the_graph);s2)
| . . . | . . | | |    & (∀j:Vertices(the_graph).
| . . . | . . | | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | |       & ((inl(j) ∈ s2)
| . . . | . . | | |          ⟺
| . . . | . . | | |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | . . | | 21. L1@0 : Vertices(the_graph) List
```

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| . . . | . . | | 22. L2 : Vertices(the_graph) List
| . . . | . . | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | 24. e : Vertices(the_graph) List
| . . . | . . | | 25. L1 = (L1@0 @ e)
| . . . | . . | | 26. L2 = (e @ [i])
| . . . | . . | | 27. s1 : traversal(the_graph)
| . . . | . . | | 28. s3 : traversal(the_graph)
| . . . | . . | | 29. s = (s3 @ s1)
| . . . | . . | | 30. paren(Vertices(the_graph);s1)
| . . . | . . | | 31. paren(Vertices(the_graph);s3)
| . . . | . . | | 32. ∀j:Vertices(the_graph).
| . . . | . . | | |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | |   & ((inl(j) ∈ s3)
| . . . | . . | | |      ⟺
| . . . | . . | | |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | 33. j : Vertices(the_graph)
| . . . | . . | | 34. (inl(j) ∈ s3)
| . . . | . . | | |   ⟺
| . . . | . . | | |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | 35. inl(j) = inl(i)
| . . . | . . | | |   ∨ (inl(j) ∈ s2)
| . . . | . . | | |   ∨ inl(j) = inr(i)
| . . . | . . | | ⊢  e-the_graph->*j ∨ [i]-the_graph->*j
| . . . | . . | | |  & ¬L1@0-the_graph->*j  by Analyze -1
| . . . | . . | | |
| . . . | . . | | |\
| . . . | . . | | | 1. the_graph : Graph
| . . . | . . | | | 2. GraphObject(the_graph)
| . . . | . . | | | 3. Vertices(the_graph) List
| . . . | . . | | | 4. L1 : Vertices(the_graph) List
| . . . | . . | | | 5. s : traversal(the_graph)
| . . . | . . | | | 6. i : Vertices(the_graph)
| . . . | . . | | | 7. paren(Vertices(the_graph);s)
| . . . | . . | | | 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | | |  Vertices(the_graph);s)
| . . . | . . | | | 9. df-traversal(the_graph;s)
| . . . | . . | | | 10. ∀i:Vertices(the_graph).
| . . . | . . | | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | | 11. ¬(inl(i) ∈ s)
| . . . | . . | | | 12. s2 : traversal(the_graph)
| . . . | . . | | | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | | |   Vertices(the_graph);s2;s)
| . . . | . . | | | 14. paren(Vertices(the_graph);s2)
| . . . | . . | | | 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | | |   Vertices(the_graph);s2)
| . . . | . . | | | 16. ¬(inr(i) ∈ s2)
| . . . | . . | | | 17. ∀j:Vertices(the_graph).
| . . . | . . | | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | | |                          @ s2
| . . . | . . | | | |                          @ [inr(i)])
| . . . | . . | | | |   @ s)
| . . . | . . | | | 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | | |   L1 @ [i]-the_graph->*i@0
| . . . | . . | | | |   ⇒
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| . . . | . . | | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | | |   L1 = (L1@0 @ L2)
| . . . | . . | | | |   ⇒
| . . . | . . | | | |   (∃s1,s2:traversal(the_graph).
| . . . | . . | | | |    s = (s2 @ s1)
| . . . | . . | | | |    & paren(Vertices(the_graph);s1)
| . . . | . . | | | |    & paren(Vertices(the_graph);s2)
| . . . | . . | | | |    & ...)
| . . . | . . | | | 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | | 22. L2 : Vertices(the_graph) List
| . . . | . . | | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | | 24. e : Vertices(the_graph) List
| . . . | . . | | | 25. L1 = (L1@0 @ e)
| . . . | . . | | | 26. L2 = (e @ [i])
| . . . | . . | | | 27. s1 : traversal(the_graph)
| . . . | . . | | | 28. s3 : traversal(the_graph)
| . . . | . . | | | 29. s = (s3 @ s1)
| . . . | . . | | | 30. paren(Vertices(the_graph);s1)
| . . . | . . | | | 31. paren(Vertices(the_graph);s3)
| . . . | . . | | | 32. ∀j:Vertices(the_graph).
| . . . | . . | | | |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | |   & ((inl(j) ∈ s3)
| . . . | . . | | | |      ⟺
| . . . | . . | | | |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | | 33. j : Vertices(the_graph)
| . . . | . . | | | 34. (inl(j) ∈ s3)
| . . . | . . | | | |   ⟺
| . . . | . . | | | |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | | 35. inl(j) = inl(i)
| . . . | . . | | | ⊢  e-the_graph->*j ∨ [i]-the_graph->*j
| . . . | . . | | | |  & ¬L1@0-the_graph->*j  by Subst (j = i) 0
| . . . | . . | | | |
| . . . | . . | | | |\
| . . . | . . | | | | 1. the_graph : Graph
| . . . | . . | | | | 2. GraphObject(the_graph)
| . . . | . . | | | | 3. Vertices(the_graph) List
| . . . | . . | | | | 4. L1 : Vertices(the_graph) List
| . . . | . . | | | | 5. s : traversal(the_graph)
| . . . | . . | | | | 6. i : Vertices(the_graph)
| . . . | . . | | | | 7. paren(Vertices(the_graph);s)
| . . . | . . | | | | 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | | |    Vertices(the_graph);s)
| . . . | . . | | | | 9. df-traversal(the_graph;s)
| . . . | . . | | | | 10. ∀i:Vertices(the_graph).
| . . . | . . | | | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | | | 11. ¬(inl(i) ∈ s)
| . . . | . . | | | | 12. s2 : traversal(the_graph)
| . . . | . . | | | | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | | |     Vertices(the_graph);s2;s)
| . . . | . . | | | | 14. paren(Vertices(the_graph);s2)
| . . . | . . | | | | 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | | |     Vertices(the_graph);s2)
| . . . | . . | | | | 16. ¬(inr(i) ∈ s2)
| . . . | . . | | | | 17. ∀j:Vertices(the_graph).
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| . . . | . . | | | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | | | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | | |                             @ s2
| . . . | . . | | | |                             @ [inr(i)])
| . . . | . . | | | |     @ s)
| . . . | . . | | | | 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | | |     L1 @ [i]-the_graph->*i@0
| . . . | . . | | | |     ⇒
| . . . | . . | | | |     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | | |     L1 = (L1@0 @ L2)
| . . . | . . | | | |     ⇒
| . . . | . . | | | |     (∃s1,s2:traversal(the_graph).
| . . . | . . | | | |      s = (s2 @ s1)
| . . . | . . | | | |      & paren(Vertices(the_graph);s1)
| . . . | . . | | | |      & paren(Vertices(the_graph);s2)
| . . . | . . | | | |      & (∀j:Vertices(the_graph).
| . . . | . . | | | |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | |         & ((inl(j) ∈ s2)
| . . . | . . | | | |            ⟺
| . . . | . . | | | |            L2-the_graph->*j
| . . . | . . | | | |            & ¬L1@0-the_graph->*j)))
| . . . | . . | | | | 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | | | 22. L2 : Vertices(the_graph) List
| . . . | . . | | | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | | | 24. e : Vertices(the_graph) List
| . . . | . . | | | | 25. L1 = (L1@0 @ e)
| . . . | . . | | | | 26. L2 = (e @ [i])
| . . . | . . | | | | 27. s1 : traversal(the_graph)
| . . . | . . | | | | 28. s3 : traversal(the_graph)
| . . . | . . | | | | 29. s = (s3 @ s1)
| . . . | . . | | | | 30. paren(Vertices(the_graph);s1)
| . . . | . . | | | | 31. paren(Vertices(the_graph);s3)
| . . . | . . | | | | 32. ∀j:Vertices(the_graph).
| . . . | . . | | | |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | |     & ((inl(j) ∈ s3)
| . . . | . . | | | |        ⟺
| . . . | . . | | | |        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | | | 33. j : Vertices(the_graph)
| . . . | . . | | | | 34. (inl(j) ∈ s3)
| . . . | . . | | | |     ⇒
| . . . | . . | | | |     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | | | 35. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . | | | |     & ¬L1@0-the_graph->*j)
| . . . | . . | | | | 36. inl(j) = inl(i)
| . . . | . . | | | | ⊢  j = i  by ObviousFrom [-1]
| . . . | . . | | | |
| . . . | . . | | | .\
| . . . | . . | | | . 1. the_graph : Graph
| . . . | . . | | | . 2. GraphObject(the_graph)
| . . . | . . | | | . 3. Vertices(the_graph) List
| . . . | . . | | | . 4. L1 : Vertices(the_graph) List

| . . . | . . | | | . 5. s : traversal(the_graph)
| . . . | . . | | | . 6. i : Vertices(the_graph)
| . . . | . . | | | . 7. paren(Vertices(the_graph);s)
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| . . . | . . | | | . 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | | . |  Vertices(the_graph);s)
| . . . | . . | | | . 9. df-traversal(the_graph;s)
| . . . | . . | | | . 10. ∀i:Vertices(the_graph).
| . . . | . . | | | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | | . 11. ¬(inl(i) ∈ s)
| . . . | . . | | | . 12. s2 : traversal(the_graph)
| . . . | . . | | | . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | | . |   Vertices(the_graph);s2;s)
| . . . | . . | | | . 14. paren(Vertices(the_graph);s2)
| . . . | . . | | | . 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | | . |   Vertices(the_graph);s2)
| . . . | . . | | | . 16. ¬(inr(i) ∈ s2)
| . . . | . . | | | . 17. ∀j:Vertices(the_graph).
| . . . | . . | | | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | | . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | | . |                         @ s2
| . . . | . . | | | . |                         @ [inr(i)])
| . . . | . . | | | . |   @ s)
| . . . | . . | | | . 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | | . |   L1 @ [i]-the_graph->*i@0
| . . . | . . | | | . |   ⇒
| . . . | . . | | | . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | | . |   L1 = (L1@0 @ L2)
| . . . | . . | | | . |   ⇒
| . . . | . . | | | . |   (∃s1,s2:traversal(the_graph).
| . . . | . . | | | . |    s = (s2 @ s1)
| . . . | . . | | | . |    & paren(Vertices(the_graph);s1)
| . . . | . . | | | . |    & paren(Vertices(the_graph);s2)
| . . . | . . | | | . |    & (∀j:Vertices(the_graph).
| . . . | . . | | | . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | . |       & ((inl(j) ∈ s2)
| . . . | . . | | | . |          ⟺
| . . . | . . | | | . |          L2-the_graph->*j
| . . . | . . | | | . |          & ¬L1@0-the_graph->*j)))
| . . . | . . | | | . 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | | . 22. L2 : Vertices(the_graph) List
| . . . | . . | | | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | | . 24. e : Vertices(the_graph) List
| . . . | . . | | | . 25. L1 = (L1@0 @ e)
| . . . | . . | | | . 26. L2 = (e @ [i])
| . . . | . . | | | . 27. s1 : traversal(the_graph)
| . . . | . . | | | . 28. s3 : traversal(the_graph)
| . . . | . . | | | . 29. s = (s3 @ s1)
| . . . | . . | | | . 30. paren(Vertices(the_graph);s1)
| . . . | . . | | | . 31. paren(Vertices(the_graph);s3)
| . . . | . . | | | . 32. ∀j:Vertices(the_graph).
| . . . | . . | | | . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | . |   & ((inl(j) ∈ s3)
| . . . | . . | | | . |      ⟺
| . . . | . . | | | . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | | . 33. j : Vertices(the_graph)
| . . . | . . | | | . 34. (inl(j) ∈ s3)
| . . . | . . | | | . |   ⟺
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| . . . | . . | | | . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | | . 35. inl(j) = inl(i)
| . . . | . . | | | . ⊢  e-the_graph->*i ∨ [i]-the_graph->*i
| . . . | . . | | | . |  & ¬L1@0-the_graph->*i  by Auto
| . . . | . . | | | . |
| . . . | . . | | | . |\
| . . . | . . | | | . | 1. the_graph : Graph
| . . . | . . | | | . | 2. GraphObject(the_graph)
| . . . | . . | | | . | 3. Vertices(the_graph) List
| . . . | . . | | | . | 4. L1 : Vertices(the_graph) List
| . . . | . . | | | . | 5. s : traversal(the_graph)
| . . . | . . | | | . | 6. i : Vertices(the_graph)
| . . . | . . | | | . | 7. paren(Vertices(the_graph);s)
| . . . | . . | | | . | 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | | . |    Vertices(the_graph);s)
| . . . | . . | | | . | 9. df-traversal(the_graph;s)
| . . . | . . | | | . | 10. ∀i:Vertices(the_graph).
| . . . | . . | | | . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | | . | 11. ¬(inl(i) ∈ s)
| . . . | . . | | | . | 12. s2 : traversal(the_graph)
| . . . | . . | | | . | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | | . |     Vertices(the_graph);s2;s)
| . . . | . . | | | . | 14. paren(Vertices(the_graph);s2)
| . . . | . . | | | . | 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | | . |     Vertices(the_graph);s2)
| . . . | . . | | | . | 16. ¬(inr(i) ∈ s2)
| . . . | . . | | | . | 17. ∀j:Vertices(the_graph).
| . . . | . . | | | . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | | . | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | | . |                              @ s2
| . . . | . . | | | . |                              @ [inr(i)])
| . . . | . . | | | . |     @ s)
| . . . | . . | | | . | 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | | . |     L1 @ [i]-the_graph->*i@0
| . . . | . . | | | . |     ⇒
| . . . | . . | | | . |     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | | . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | | . |     L1 = (L1@0 @ L2)
| . . . | . . | | | . |     ⇒
| . . . | . . | | | . |     (∃s1,s2:traversal(the_graph).
| . . . | . . | | | . |      s = (s2 @ s1)
| . . . | . . | | | . |      & paren(Vertices(the_graph);s1)

| . . . | . . | | | . |      & paren(Vertices(the_graph);s2)
| . . . | . . | | | . |      & (∀j:Vertices(the_graph).
| . . . | . . | | | . |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | . |         & ((inl(j) ∈ s2)
| . . . | . . | | | . |            ⟺
| . . . | . . | | | . |            L2-the_graph->*j
| . . . | . . | | | . |            & ¬L1@0-the_graph->*j)))
| . . . | . . | | | . | 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | | . | 22. L2 : Vertices(the_graph) List
| . . . | . . | | | . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | | . | 24. e : Vertices(the_graph) List
| . . . | . . | | | . | 25. L1 = (L1@0 @ e)
| . . . | . . | | | . | 26. L2 = (e @ [i])
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| . . . | . . | | | . | 27. s1 : traversal(the_graph)
| . . . | . . | | | . | 28. s3 : traversal(the_graph)
| . . . | . . | | | . | 29. s = (s3 @ s1)
| . . . | . . | | | . | 30. paren(Vertices(the_graph);s1)
| . . . | . . | | | . | 31. paren(Vertices(the_graph);s3)
| . . . | . . | | | . | 32. ∀j:Vertices(the_graph).
| . . . | . . | | | . |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | . |     & ((inl(j) ∈ s3)
| . . . | . . | | | . |        ⟺
| . . . | . . | | | . |        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | | . | 33. j : Vertices(the_graph)
| . . . | . . | | | . | 34. (inl(j) ∈ s3)
| . . . | . . | | | . |     ⇒
| . . . | . . | | | . |     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | | . | 35. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . | | | . |     & ¬L1@0-the_graph->*j)
| . . . | . . | | | . | 36. inl(j) = inl(i)
| . . . | . . | | | . | ⊢  e-the_graph->*i ∨ [i]-the_graph->*i
| . . . | . . | | | . |   by ((RWO
| . . . | . . | | | . |        Thm* For any graph
| . . . | . . | | | . |             ∀i,j:V.
| . . . | . . | | | . |             [i]-->*j ⟺ i-the_graph->*j
| . . . | . . | | | . |        {list_DASH_connect_DASH_singleton}
| . . . | . . | | | . |        0)
| . . . | . . | | | . |       THEN
| . . . | . . | | | . |       OrRight)
| . . . | . . | | | . |      THEN
| . . . | . . | | | . |      Connect
| . . . | . . | | | . |
| . . . | . . | | | . .\
| . . . | . . | | | . . 1. the_graph : Graph
| . . . | . . | | | . . 2. GraphObject(the_graph)
| . . . | . . | | | . . 3. Vertices(the_graph) List
| . . . | . . | | | . . 4. L1 : Vertices(the_graph) List
| . . . | . . | | | . . 5. s : traversal(the_graph)
| . . . | . . | | | . . 6. i : Vertices(the_graph)
| . . . | . . | | | . . 7. paren(Vertices(the_graph);s)
| . . . | . . | | | . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | | . . |  Vertices(the_graph);s)
| . . . | . . | | | . . 9. df-traversal(the_graph;s)
| . . . | . . | | | . . 10. ∀i:Vertices(the_graph).
| . . . | . . | | | . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | | . . 11. ¬(inl(i) ∈ s)
| . . . | . . | | | . . 12. s2 : traversal(the_graph)
| . . . | . . | | | . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | | . . |   Vertices(the_graph);s2;s)
| . . . | . . | | | . . 14. paren(Vertices(the_graph);s2)
| . . . | . . | | | . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | | . . |   Vertices(the_graph);s2)
| . . . | . . | | | . . 16. ¬(inr(i) ∈ s2)
| . . . | . . | | | . . 17. ∀j:Vertices(the_graph).
| . . . | . . | | | . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | | . . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | | . . |                         @ s2
| . . . | . . | | | . . |                         @ [inr(i)])
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| . . . | . . | | | . . |   @ s)
| . . . | . . | | | . . 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | | . . |   L1 @ [i]-the_graph->*i@0
| . . . | . . | | | . . |   ⇒
| . . . | . . | | | . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | | . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | | . . |   L1 = (L1@0 @ L2)
| . . . | . . | | | . . |   ⇒
| . . . | . . | | | . . |   (∃s1,s2:traversal(the_graph).
| . . . | . . | | | . . |    s = (s2 @ s1)
| . . . | . . | | | . . |    & paren(Vertices(the_graph);s1)
| . . . | . . | | | . . |    & paren(Vertices(the_graph);s2)
| . . . | . . | | | . . |    & (∀j:Vertices(the_graph).
| . . . | . . | | | . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | . . |       & ((inl(j) ∈ s2)
| . . . | . . | | | . . |          ⟺
| . . . | . . | | | . . |          L2-the_graph->*j
| . . . | . . | | | . . |          & ¬L1@0-the_graph->*j)))
| . . . | . . | | | . . 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | | . . 22. L2 : Vertices(the_graph) List
| . . . | . . | | | . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | | . . 24. e : Vertices(the_graph) List
| . . . | . . | | | . . 25. L1 = (L1@0 @ e)
| . . . | . . | | | . . 26. L2 = (e @ [i])
| . . . | . . | | | . . 27. s1 : traversal(the_graph)
| . . . | . . | | | . . 28. s3 : traversal(the_graph)
| . . . | . . | | | . . 29. s = (s3 @ s1)
| . . . | . . | | | . . 30. paren(Vertices(the_graph);s1)
| . . . | . . | | | . . 31. paren(Vertices(the_graph);s3)
| . . . | . . | | | . . 32. ∀j:Vertices(the_graph).
| . . . | . . | | | . . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | . . |   & ((inl(j) ∈ s3)

| . . . | . . | | | . . |      ⟺
| . . . | . . | | | . . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | | . . 33. j : Vertices(the_graph)
| . . . | . . | | | . . 34. (inl(j) ∈ s3)
| . . . | . . | | | . . |   ⇒
| . . . | . . | | | . . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | | . . 35. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . | | | . . |   & ¬L1@0-the_graph->*j)
| . . . | . . | | | . . 36. inl(j) = inl(i)
| . . . | . . | | | . . ⊢  ¬L1@0-the_graph->*i  by ParallelOp 11
| . . . | . . | | | . . |
| . . . | . . | | | . . 1. the_graph : Graph
| . . . | . . | | | . . 2. GraphObject(the_graph)
| . . . | . . | | | . . 3. Vertices(the_graph) List
| . . . | . . | | | . . 4. L1 : Vertices(the_graph) List
| . . . | . . | | | . . 5. s : traversal(the_graph)
| . . . | . . | | | . . 6. i : Vertices(the_graph)
| . . . | . . | | | . . 7. paren(Vertices(the_graph);s)
| . . . | . . | | | . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | | . . |  Vertices(the_graph);s)
| . . . | . . | | | . . 9. df-traversal(the_graph;s)
| . . . | . . | | | . . 10. ∀i:Vertices(the_graph).
| . . . | . . | | | . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
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| . . . | . . | | | . . 11. s2 : traversal(the_graph)
| . . . | . . | | | . . 12. l_disjoint(Vertices(the_graph)+
| . . . | . . | | | . . |   Vertices(the_graph);s2;s)
| . . . | . . | | | . . 13. paren(Vertices(the_graph);s2)
| . . . | . . | | | . . 14. no_repeats(Vertices(the_graph)+
| . . . | . . | | | . . |   Vertices(the_graph);s2)
| . . . | . . | | | . . 15. ¬(inr(i) ∈ s2)
| . . . | . . | | | . . 16. ∀j:Vertices(the_graph).
| . . . | . . | | | . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | | . . 17. df-traversal(the_graph;([inl(i)]
| . . . | . . | | | . . |                           @ s2
| . . . | . . | | | . . |                           @ [inr(i)])
| . . . | . . | | | . . |   @ s)
| . . . | . . | | | . . 18. ∀i@0:Vertices(the_graph).
| . . . | . . | | | . . |   L1 @ [i]-the_graph->*i@0
| . . . | . . | | | . . |   ⇒
| . . . | . . | | | . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | | . . 19. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | | . . |   L1 = (L1@0 @ L2)
| . . . | . . | | | . . |   ⇒
| . . . | . . | | | . . |   (∃s1,s2:traversal(the_graph).
| . . . | . . | | | . . |    s = (s2 @ s1)
| . . . | . . | | | . . |    & paren(Vertices(the_graph);s1)
| . . . | . . | | | . . |    & paren(Vertices(the_graph);s2)
| . . . | . . | | | . . |    & (∀j:Vertices(the_graph).
| . . . | . . | | | . . |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | . . |       & ((inl(j) ∈ s2)
| . . . | . . | | | . . |          ⟺
| . . . | . . | | | . . |          L2-the_graph->*j
| . . . | . . | | | . . |          & ¬L1@0-the_graph->*j)))
| . . . | . . | | | . . 20. L1@0 : Vertices(the_graph) List
| . . . | . . | | | . . 21. L2 : Vertices(the_graph) List
| . . . | . . | | | . . 22. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | | . . 23. e : Vertices(the_graph) List
| . . . | . . | | | . . 24. L1 = (L1@0 @ e)
| . . . | . . | | | . . 25. L2 = (e @ [i])
| . . . | . . | | | . . 26. s1 : traversal(the_graph)
| . . . | . . | | | . . 27. s3 : traversal(the_graph)
| . . . | . . | | | . . 28. s = (s3 @ s1)
| . . . | . . | | | . . 29. paren(Vertices(the_graph);s1)
| . . . | . . | | | . . 30. paren(Vertices(the_graph);s3)
| . . . | . . | | | . . 31. ∀j:Vertices(the_graph).
| . . . | . . | | | . . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | . . |   & ((inl(j) ∈ s3)
| . . . | . . | | | . . |      ⟺
| . . . | . . | | | . . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | | . . 32. j : Vertices(the_graph)
| . . . | . . | | | . . 33. (inl(j) ∈ s3)
| . . . | . . | | | . . |   ⇒
| . . . | . . | | | . . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | | . . 34. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . | | | . . |   & ¬L1@0-the_graph->*j)
| . . . | . . | | | . . 35. inl(j) = inl(i)
| . . . | . . | | | . . 36. L1@0-the_graph->*i
| . . . | . . | | | . . ⊢  (inl(i) ∈ s)
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| . . . | . . | | | . . | by ((SubstFor s 0)
| . . . | . . | | | . . |     THEN
| . . . | . . | | | . . |     (RWO
| . . . | . . | | | . . |      Thm* ∀x:T, l1,l2:T List.
| . . . | . . | | | . . |           (x ∈ l1 @ l2)
| . . . | . . | | | . . |           ⟺
| . . . | . . | | | . . |           (x ∈ l1) ∨ (x ∈ l2)
| . . . | . . | | | . . |      {member_append}
| . . . | . . | | | . . |      0))
| . . . | . . | | | . . |    THEN
| . . . | . . | | | . . |    OrRight
| . . . | . . | | | . . |
| . . . | . . | | | . . 1. the_graph : Graph
| . . . | . . | | | . . 2. GraphObject(the_graph)
| . . . | . . | | | . . 3. Vertices(the_graph) List
| . . . | . . | | | . . 4. L1 : Vertices(the_graph) List
| . . . | . . | | | . . 5. s : traversal(the_graph)
| . . . | . . | | | . . 6. i : Vertices(the_graph)
| . . . | . . | | | . . 7. paren(Vertices(the_graph);s)
| . . . | . . | | | . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | | . . |  Vertices(the_graph);s)
| . . . | . . | | | . . 9. df-traversal(the_graph;s)
| . . . | . . | | | . . 10. ∀i:Vertices(the_graph).
| . . . | . . | | | . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | | . . 11. s2 : traversal(the_graph)
| . . . | . . | | | . . 12. l_disjoint(Vertices(the_graph)+
| . . . | . . | | | . . |   Vertices(the_graph);s2;s)
| . . . | . . | | | . . 13. paren(Vertices(the_graph);s2)
| . . . | . . | | | . . 14. no_repeats(Vertices(the_graph)+
| . . . | . . | | | . . |   Vertices(the_graph);s2)
| . . . | . . | | | . . 15. ¬(inr(i) ∈ s2)
| . . . | . . | | | . . 16. ∀j:Vertices(the_graph).
| . . . | . . | | | . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | | . . 17. df-traversal(the_graph;([inl(i)]
| . . . | . . | | | . . |                           @ s2
| . . . | . . | | | . . |                           @ [inr(i)])
| . . . | . . | | | . . |   @ s)
| . . . | . . | | | . . 18. ∀i@0:Vertices(the_graph).
| . . . | . . | | | . . |   L1 @ [i]-the_graph->*i@0
| . . . | . . | | | . . |   ⇒
| . . . | . . | | | . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | | . . 19. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | | . . |   L1 = (L1@0 @ L2)
| . . . | . . | | | . . |   ⇒
| . . . | . . | | | . . |   (∃s1,s2:traversal(the_graph).
| . . . | . . | | | . . |    s = (s2 @ s1)
| . . . | . . | | | . . |    & paren(Vertices(the_graph);s1)
| . . . | . . | | | . . |    & paren(Vertices(the_graph);s2)
| . . . | . . | | | . . |    & (∀j:Vertices(the_graph).
| . . . | . . | | | . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | . . |       & ((inl(j) ∈ s2)
| . . . | . . | | | . . |          ⟺
| . . . | . . | | | . . |          L2-the_graph->*j
| . . . | . . | | | . . |          & ¬L1@0-the_graph->*j)))
| . . . | . . | | | . . 20. L1@0 : Vertices(the_graph) List
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| . . . | . . | | | . . 21. L2 : Vertices(the_graph) List
| . . . | . . | | | . . 22. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | | . . 23. e : Vertices(the_graph) List
| . . . | . . | | | . . 24. L1 = (L1@0 @ e)
| . . . | . . | | | . . 25. L2 = (e @ [i])
| . . . | . . | | | . . 26. s1 : traversal(the_graph)
| . . . | . . | | | . . 27. s3 : traversal(the_graph)
| . . . | . . | | | . . 28. s = (s3 @ s1)
| . . . | . . | | | . . 29. paren(Vertices(the_graph);s1)
| . . . | . . | | | . . 30. paren(Vertices(the_graph);s3)
| . . . | . . | | | . . 31. ∀j:Vertices(the_graph).
| . . . | . . | | | . . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | . . |   & ((inl(j) ∈ s3)
| . . . | . . | | | . . |      ⟺
| . . . | . . | | | . . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | | . . 32. j : Vertices(the_graph)
| . . . | . . | | | . . 33. (inl(j) ∈ s3)
| . . . | . . | | | . . |   ⇒
| . . . | . . | | | . . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | | . . 34. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . | | | . . |   & ¬L1@0-the_graph->*j)
| . . . | . . | | | . . 35. inl(j) = inl(i)
| . . . | . . | | | . . 36. L1@0-the_graph->*i
| . . . | . . | | | . . ⊢  (inl(i) ∈ s1)  by InstHyp [i] -6
| . . . | . . | | | . . |
| . . . | . . | | | . . 1. the_graph : Graph
| . . . | . . | | | . . 2. GraphObject(the_graph)
| . . . | . . | | | . . 3. Vertices(the_graph) List
| . . . | . . | | | . . 4. L1 : Vertices(the_graph) List
| . . . | . . | | | . . 5. s : traversal(the_graph)
| . . . | . . | | | . . 6. i : Vertices(the_graph)
| . . . | . . | | | . . 7. paren(Vertices(the_graph);s)
| . . . | . . | | | . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | | . .    Vertices(the_graph);s)
| . . . | . . | | | . . 9. df-traversal(the_graph;s)
| . . . | . . | | | . . 10. ∀i:Vertices(the_graph).
| . . . | . . | | | . .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | | . . 11. s2 : traversal(the_graph)
| . . . | . . | | | . . 12. l_disjoint(Vertices(the_graph)+
| . . . | . . | | | . .     Vertices(the_graph);s2;s)
| . . . | . . | | | . . 13. paren(Vertices(the_graph);s2)
| . . . | . . | | | . . 14. no_repeats(Vertices(the_graph)+
| . . . | . . | | | . .     Vertices(the_graph);s2)
| . . . | . . | | | . . 15. ¬(inr(i) ∈ s2)
| . . . | . . | | | . . 16. ∀j:Vertices(the_graph).
| . . . | . . | | | . .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | | . . 17. df-traversal(the_graph;([inl(i)]
| . . . | . . | | | . .                              @ s2
| . . . | . . | | | . .                              @ [inr(i)])
| . . . | . . | | | . .     @ s)
| . . . | . . | | | . . 18. ∀i@0:Vertices(the_graph).
| . . . | . . | | | . .     L1 @ [i]-the_graph->*i@0
| . . . | . . | | | . .     ⇒
| . . . | . . | | | . .     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | | . . 19. ∀L1@0,L2:Vertices(the_graph) List.
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| . . . | . . | | | . .     L1 = (L1@0 @ L2)
| . . . | . . | | | . .     ⇒
| . . . | . . | | | . .     (∃s1,s2:traversal(the_graph).
| . . . | . . | | | . .      s = (s2 @ s1)
| . . . | . . | | | . .      & paren(Vertices(the_graph);s1)
| . . . | . . | | | . .      & paren(Vertices(the_graph);s2)
| . . . | . . | | | . .      & (∀j:Vertices(the_graph).
| . . . | . . | | | . .         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | . .         & ((inl(j) ∈ s2)
| . . . | . . | | | . .            ⟺
| . . . | . . | | | . .            L2-the_graph->*j
| . . . | . . | | | . .            & ¬L1@0-the_graph->*j)))
| . . . | . . | | | . . 20. L1@0 : Vertices(the_graph) List
| . . . | . . | | | . . 21. L2 : Vertices(the_graph) List
| . . . | . . | | | . . 22. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | | . . 23. e : Vertices(the_graph) List
| . . . | . . | | | . . 24. L1 = (L1@0 @ e)
| . . . | . . | | | . . 25. L2 = (e @ [i])
| . . . | . . | | | . . 26. s1 : traversal(the_graph)
| . . . | . . | | | . . 27. s3 : traversal(the_graph)
| . . . | . . | | | . . 28. s = (s3 @ s1)
| . . . | . . | | | . . 29. paren(Vertices(the_graph);s1)
| . . . | . . | | | . . 30. paren(Vertices(the_graph);s3)
| . . . | . . | | | . . 31. ∀j:Vertices(the_graph).
| . . . | . . | | | . .     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | | . .     & ((inl(j) ∈ s3)
| . . . | . . | | | . .        ⟺
| . . . | . . | | | . .        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | | . . 32. j : Vertices(the_graph)
| . . . | . . | | | . . 33. (inl(j) ∈ s3)
| . . . | . . | | | . .     ⇒
| . . . | . . | | | . .     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | | . . 34. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . | | | . .     & ¬L1@0-the_graph->*j)
| . . . | . . | | | . . 35. inl(j) = inl(i)
| . . . | . . | | | . . 36. L1@0-the_graph->*i
| . . . | . . | | | . . 37. ((inl(i) ∈ s1) ⟺ L1@0-the_graph->*i)
| . . . | . . | | | . .     & ((inl(i) ∈ s3)
| . . . | . . | | | . .        ⟺
| . . . | . . | | | . .        e-the_graph->*i & ¬L1@0-the_graph->*i)
| . . . | . . | | | . . ⊢  (inl(i) ∈ s1)
| . . . | . . | | | . .   by (Analyze -1) THEN (Thin -1)
| . . . | . . | | | . .      THEN
| . . . | . . | | | . .      (Analyze -1)
| . . . | . . | | | . .      THEN
| . . . | . . | | | . .      (Analyze -1)
| . . . | . . | | |
| . . . | . . | | | .\
| . . . | . . | | . 1. the_graph : Graph
| . . . | . . | | . 2. GraphObject(the_graph)
| . . . | . . | | . 3. Vertices(the_graph) List
| . . . | . . | | . 4. L1 : Vertices(the_graph) List
| . . . | . . | | . 5. s : traversal(the_graph)
| . . . | . . | | . 6. i : Vertices(the_graph)
| . . . | . . | | . 7. paren(Vertices(the_graph);s)
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| . . . | . . | | . 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | . |  Vertices(the_graph);s)
| . . . | . . | | . 9. df-traversal(the_graph;s)
| . . . | . . | | . 10. ∀i:Vertices(the_graph).
| . . . | . . | | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | . 11. ¬(inl(i) ∈ s)
| . . . | . . | | . 12. s2 : traversal(the_graph)
| . . . | . . | | . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | . |   Vertices(the_graph);s2;s)
| . . . | . . | | . 14. paren(Vertices(the_graph);s2)
| . . . | . . | | . 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | . |   Vertices(the_graph);s2)
| . . . | . . | | . 16. ¬(inr(i) ∈ s2)
| . . . | . . | | . 17. ∀j:Vertices(the_graph).
| . . . | . . | | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | . |                         @ s2
| . . . | . . | | . |                         @ [inr(i)])
| . . . | . . | | . |   @ s)
| . . . | . . | | . 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | . |   L1 @ [i]-the_graph->*i@0
| . . . | . . | | . |   ⇒
| . . . | . . | | . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | . |   L1 = (L1@0 @ L2)
| . . . | . . | | . |   ⇒
| . . . | . . | | . |   (∃s1,s2:traversal(the_graph).
| . . . | . . | | . |    s = (s2 @ s1)
| . . . | . . | | . |    & paren(Vertices(the_graph);s1)
| . . . | . . | | . |    & paren(Vertices(the_graph);s2)
| . . . | . . | | . |    & ...)
| . . . | . . | | . 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | . 22. L2 : Vertices(the_graph) List
| . . . | . . | | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | . 24. e : Vertices(the_graph) List
| . . . | . . | | . 25. L1 = (L1@0 @ e)
| . . . | . . | | . 26. L2 = (e @ [i])
| . . . | . . | | . 27. s1 : traversal(the_graph)
| . . . | . . | | . 28. s3 : traversal(the_graph)
| . . . | . . | | . 29. s = (s3 @ s1)
| . . . | . . | | . 30. paren(Vertices(the_graph);s1)
| . . . | . . | | . 31. paren(Vertices(the_graph);s3)
| . . . | . . | | . 32. ∀j:Vertices(the_graph).
| . . . | . . | | . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | . |   & ((inl(j) ∈ s3)
| . . . | . . | | . |      ⟺
| . . . | . . | | . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | . 33. j : Vertices(the_graph)
| . . . | . . | | . 34. (inl(j) ∈ s3)
| . . . | . . | | . |   ⟺
| . . . | . . | | . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | . 35. (inl(j) ∈ s2) ∨ inl(j) = inr(i)
| . . . | . . | | . ⊢  e-the_graph->*j ∨ [i]-the_graph->*j
| . . . | . . | | . |  & ¬L1@0-the_graph->*j
| . . . | . . | | . | by (RWO
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| . . . | . . | | . |     Thm* For any graph
| . . . | . . | | . |          ∀i,j:V. [i]-->*j ⟺ i-the_graph->*j
| . . . | . . | | . |     {list_DASH_connect_DASH_singleton}
| . . . | . . | | . |     0)
| . . . | . . | | . |    THEN
| . . . | . . | | . |    (Analyze -1)
| . . . | . . | | . |
| . . . | . . | | . |\
| . . . | . . | | . | 1. the_graph : Graph
| . . . | . . | | . | 2. GraphObject(the_graph)
| . . . | . . | | . | 3. Vertices(the_graph) List
| . . . | . . | | . | 4. L1 : Vertices(the_graph) List
| . . . | . . | | . | 5. s : traversal(the_graph)
| . . . | . . | | . | 6. i : Vertices(the_graph)
| . . . | . . | | . | 7. paren(Vertices(the_graph);s)
| . . . | . . | | . | 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | . | |  Vertices(the_graph);s)
| . . . | . . | | . | 9. df-traversal(the_graph;s)
| . . . | . . | | . | 10. ∀i:Vertices(the_graph).
| . . . | . . | | . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | . | 11. ¬(inl(i) ∈ s)
| . . . | . . | | . | 12. s2 : traversal(the_graph)
| . . . | . . | | . | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | . | |   Vertices(the_graph);s2;s)
| . . . | . . | | . | 14. paren(Vertices(the_graph);s2)
| . . . | . . | | . | 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | . | |   Vertices(the_graph);s2)
| . . . | . . | | . | 16. ¬(inr(i) ∈ s2)
| . . . | . . | | . | 17. ∀j:Vertices(the_graph).
| . . . | . . | | . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | . | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | . | |                         @ s2
| . . . | . . | | . | |                         @ [inr(i)])
| . . . | . . | | . | |   @ s)
| . . . | . . | | . | 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | . | |   L1 @ [i]-the_graph->*i@0
| . . . | . . | | . | |   ⇒
| . . . | . . | | . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | . | |   L1 = (L1@0 @ L2)
| . . . | . . | | . | |   ⇒
| . . . | . . | | . | |   (∃s1,s2:traversal(the_graph).
| . . . | . . | | . | |    s = (s2 @ s1)
| . . . | . . | | . | |    & paren(Vertices(the_graph);s1)
| . . . | . . | | . | |    & paren(Vertices(the_graph);s2)
| . . . | . . | | . | |    & (∀j:Vertices(the_graph).
| . . . | . . | | . | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | . | |       & ((inl(j) ∈ s2)
| . . . | . . | | . | |          ⟺
| . . . | . . | | . | |          L2-the_graph->*j
| . . . | . . | | . | |          & ¬L1@0-the_graph->*j)))
| . . . | . . | | . | 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | . | 22. L2 : Vertices(the_graph) List
| . . . | . . | | . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | . | 24. e : Vertices(the_graph) List
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| . . . | . . | | . | 25. L1 = (L1@0 @ e)
| . . . | . . | | . | 26. L2 = (e @ [i])
| . . . | . . | | . | 27. s1 : traversal(the_graph)
| . . . | . . | | . | 28. s3 : traversal(the_graph)
| . . . | . . | | . | 29. s = (s3 @ s1)
| . . . | . . | | . | 30. paren(Vertices(the_graph);s1)
| . . . | . . | | . | 31. paren(Vertices(the_graph);s3)
| . . . | . . | | . | 32. ∀j:Vertices(the_graph).
| . . . | . . | | . | |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | . | |   & ((inl(j) ∈ s3)
| . . . | . . | | . | |      ⟺
| . . . | . . | | . | |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | . | 33. j : Vertices(the_graph)
| . . . | . . | | . | 34. (inl(j) ∈ s3)
| . . . | . . | | . | |   ⟺
| . . . | . . | | . | |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | . | 35. (inl(j) ∈ s2)
| . . . | . . | | . | ⊢  e-the_graph->*j ∨ i-the_graph->*j
| . . . | . . | | . | |  & ¬L1@0-the_graph->*j  by Auto
| . . . | . . | | . | |
| . . . | . . | | . | |\
| . . . | . . | | . | | 1. the_graph : Graph
| . . . | . . | | . | | 2. GraphObject(the_graph)
| . . . | . . | | . | | 3. Vertices(the_graph) List
| . . . | . . | | . | | 4. L1 : Vertices(the_graph) List
| . . . | . . | | . | | 5. s : traversal(the_graph)
| . . . | . . | | . | | 6. i : Vertices(the_graph)
| . . . | . . | | . | | 7. paren(Vertices(the_graph);s)
| . . . | . . | | . | | 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | . | |    Vertices(the_graph);s)
| . . . | . . | | . | | 9. df-traversal(the_graph;s)
| . . . | . . | | . | | 10. ∀i:Vertices(the_graph).
| . . . | . . | | . | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | . | | 11. ¬(inl(i) ∈ s)
| . . . | . . | | . | | 12. s2 : traversal(the_graph)
| . . . | . . | | . | | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | . | |     Vertices(the_graph);s2;s)
| . . . | . . | | . | | 14. paren(Vertices(the_graph);s2)
| . . . | . . | | . | | 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | . | |     Vertices(the_graph);s2)
| . . . | . . | | . | | 16. ¬(inr(i) ∈ s2)
| . . . | . . | | . | | 17. ∀j:Vertices(the_graph).
| . . . | . . | | . | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | . | | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | . | |                             @ s2
| . . . | . . | | . | |                             @ [inr(i)])
| . . . | . . | | . | |     @ s)
| . . . | . . | | . | | 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | . | |     L1 @ [i]-the_graph->*i@0
| . . . | . . | | . | |     ⇒
| . . . | . . | | . | |     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | . | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | . | |     L1 = (L1@0 @ L2)
| . . . | . . | | . | |     ⇒
| . . . | . . | | . | |     (∃s1,s2:traversal(the_graph).
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| . . . | . . | | . | |      s = (s2 @ s1)
| . . . | . . | | . | |      & paren(Vertices(the_graph);s1)
| . . . | . . | | . | |      & paren(Vertices(the_graph);s2)
| . . . | . . | | . | |      & (∀j:Vertices(the_graph).
| . . . | . . | | . | |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | . | |         & ((inl(j) ∈ s2)
| . . . | . . | | . | |            ⟺
| . . . | . . | | . | |            L2-the_graph->*j
| . . . | . . | | . | |            & ¬L1@0-the_graph->*j)))
| . . . | . . | | . | | 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | . | | 22. L2 : Vertices(the_graph) List
| . . . | . . | | . | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | . | | 24. e : Vertices(the_graph) List
| . . . | . . | | . | | 25. L1 = (L1@0 @ e)
| . . . | . . | | . | | 26. L2 = (e @ [i])
| . . . | . . | | . | | 27. s1 : traversal(the_graph)
| . . . | . . | | . | | 28. s3 : traversal(the_graph)
| . . . | . . | | . | | 29. s = (s3 @ s1)
| . . . | . . | | . | | 30. paren(Vertices(the_graph);s1)
| . . . | . . | | . | | 31. paren(Vertices(the_graph);s3)
| . . . | . . | | . | | 32. ∀j:Vertices(the_graph).
| . . . | . . | | . | |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | . | |     & ((inl(j) ∈ s3)
| . . . | . . | | . | |        ⟺
| . . . | . . | | . | |        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | . | | 33. j : Vertices(the_graph)
| . . . | . . | | . | | 34. (inl(j) ∈ s3)
| . . . | . . | | . | |     ⇒
| . . . | . . | | . | |     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | . | | 35. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . | | . | |     & ¬L1@0-the_graph->*j)
| . . . | . . | | . | | 36. (inl(j) ∈ s2)
| . . . | . . | | . | | ⊢  e-the_graph->*j ∨ i-the_graph->*j
| . . . | . . | | . | |   by (OrRight THEN BackThruSomeHyp)
| . . . | . . | | . | |      THEN
| . . . | . . | | . | |      (BackThru
| . . . | . . | | . | |       Thm* ∀s':(T+T) List.
| . . . | . . | | . | |            paren(T;s')
| . . . | . . | | . | |            ⇒
| . . . | . . | | . | |            (∀i:T.
| . . . | . . | | . | |             (inl(i) ∈ s') ⇒ (inr(i) ∈ s'))
| . . . | . . | | . | |       {paren_balance1})
| . . . | . . | | . | |
| . . . | . . | | . | .\
| . . . | . . | | . | . 1. the_graph : Graph
| . . . | . . | | . | . 2. GraphObject(the_graph)
| . . . | . . | | . | . 3. Vertices(the_graph) List
| . . . | . . | | . | . 4. L1 : Vertices(the_graph) List
| . . . | . . | | . | . 5. s : traversal(the_graph)
| . . . | . . | | . | . 6. i : Vertices(the_graph)
| . . . | . . | | . | . 7. paren(Vertices(the_graph);s)
| . . . | . . | | . | . 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | . | . |  Vertices(the_graph);s)
| . . . | . . | | . | . 9. df-traversal(the_graph;s)
| . . . | . . | | . | . 10. ∀i:Vertices(the_graph).
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| . . . | . . | | . | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | . | . 11. ¬(inl(i) ∈ s)
| . . . | . . | | . | . 12. s2 : traversal(the_graph)
| . . . | . . | | . | . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | . | . |   Vertices(the_graph);s2;s)
| . . . | . . | | . | . 14. paren(Vertices(the_graph);s2)
| . . . | . . | | . | . 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | . | . |   Vertices(the_graph);s2)
| . . . | . . | | . | . 16. ¬(inr(i) ∈ s2)
| . . . | . . | | . | . 17. ∀j:Vertices(the_graph).
| . . . | . . | | . | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | . | . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | . | . |                         @ s2
| . . . | . . | | . | . |                         @ [inr(i)])
| . . . | . . | | . | . |   @ s)
| . . . | . . | | . | . 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | . | . |   L1 @ [i]-the_graph->*i@0
| . . . | . . | | . | . |   ⇒
| . . . | . . | | . | . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | . | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | . | . |   L1 = (L1@0 @ L2)
| . . . | . . | | . | . |   ⇒
| . . . | . . | | . | . |   (∃s1,s2:traversal(the_graph).
| . . . | . . | | . | . |    s = (s2 @ s1)
| . . . | . . | | . | . |    & paren(Vertices(the_graph);s1)
| . . . | . . | | . | . |    & paren(Vertices(the_graph);s2)
| . . . | . . | | . | . |    & (∀j:Vertices(the_graph).
| . . . | . . | | . | . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | . | . |       & ((inl(j) ∈ s2)
| . . . | . . | | . | . |          ⟺
| . . . | . . | | . | . |          L2-the_graph->*j
| . . . | . . | | . | . |          & ¬L1@0-the_graph->*j)))
| . . . | . . | | . | . 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | . | . 22. L2 : Vertices(the_graph) List
| . . . | . . | | . | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | . | . 24. e : Vertices(the_graph) List
| . . . | . . | | . | . 25. L1 = (L1@0 @ e)

| . . . | . . | | . | . 26. L2 = (e @ [i])
| . . . | . . | | . | . 27. s1 : traversal(the_graph)
| . . . | . . | | . | . 28. s3 : traversal(the_graph)
| . . . | . . | | . | . 29. s = (s3 @ s1)
| . . . | . . | | . | . 30. paren(Vertices(the_graph);s1)
| . . . | . . | | . | . 31. paren(Vertices(the_graph);s3)
| . . . | . . | | . | . 32. ∀j:Vertices(the_graph).
| . . . | . . | | . | . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | . | . |   & ((inl(j) ∈ s3)
| . . . | . . | | . | . |      ⟺
| . . . | . . | | . | . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | . | . 33. j : Vertices(the_graph)
| . . . | . . | | . | . 34. (inl(j) ∈ s3)
| . . . | . . | | . | . |   ⇒
| . . . | . . | | . | . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | . | . 35. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . | | . | . |   & ¬L1@0-the_graph->*j)
| . . . | . . | | . | . 36. (inl(j) ∈ s2)
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| . . . | . . | | . | . ⊢  ¬L1@0-the_graph->*j
| . . . | . . | | . | . | by (Analyze 0) THEN (Assert (inl(j) ∈ s1))
| . . . | . . | | . | . |
| . . . | . . | | . | . |\
| . . . | . . | | . | . | .....assertion .....
| . . . | . . | | . | . | 1. the_graph : Graph
| . . . | . . | | . | . | 2. GraphObject(the_graph)
| . . . | . . | | . | . | 3. Vertices(the_graph) List
| . . . | . . | | . | . | 4. L1 : Vertices(the_graph) List
| . . . | . . | | . | . | 5. s : traversal(the_graph)
| . . . | . . | | . | . | 6. i : Vertices(the_graph)
| . . . | . . | | . | . | 7. paren(Vertices(the_graph);s)
| . . . | . . | | . | . | 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | . | . |    Vertices(the_graph);s)
| . . . | . . | | . | . | 9. df-traversal(the_graph;s)
| . . . | . . | | . | . | 10. ∀i:Vertices(the_graph).
| . . . | . . | | . | . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | . | . | 11. ¬(inl(i) ∈ s)
| . . . | . . | | . | . | 12. s2 : traversal(the_graph)
| . . . | . . | | . | . | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | . | . |     Vertices(the_graph);s2;s)
| . . . | . . | | . | . | 14. paren(Vertices(the_graph);s2)
| . . . | . . | | . | . | 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | . | . |     Vertices(the_graph);s2)
| . . . | . . | | . | . | 16. ¬(inr(i) ∈ s2)
| . . . | . . | | . | . | 17. ∀j:Vertices(the_graph).
| . . . | . . | | . | . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | . | . | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | . | . |                             @ s2
| . . . | . . | | . | . |                             @ [inr(i)])
| . . . | . . | | . | . |     @ s)
| . . . | . . | | . | . | 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | . | . |     L1 @ [i]-the_graph->*i@0
| . . . | . . | | . | . |     ⇒
| . . . | . . | | . | . |     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | . | . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | . | . |     L1 = (L1@0 @ L2)
| . . . | . . | | . | . |     ⇒
| . . . | . . | | . | . |     (∃s1,s2:traversal(the_graph).
| . . . | . . | | . | . |      s = (s2 @ s1)
| . . . | . . | | . | . |      & paren(Vertices(the_graph);s1)
| . . . | . . | | . | . |      & paren(Vertices(the_graph);s2)
| . . . | . . | | . | . |      & (∀j:Vertices(the_graph).
| . . . | . . | | . | . |         ...
| . . . | . . | | . | . |         & ((inl(j) ∈ s2)
| . . . | . . | | . | . |            ⟺
| . . . | . . | | . | . |            L2-the_graph->*j
| . . . | . . | | . | . |            & ¬L1@0-the_graph->*j)))
| . . . | . . | | . | . | 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | . | . | 22. L2 : Vertices(the_graph) List
| . . . | . . | | . | . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | . | . | 24. e : Vertices(the_graph) List
| . . . | . . | | . | . | 25. L1 = (L1@0 @ e)
| . . . | . . | | . | . | 26. L2 = (e @ [i])
| . . . | . . | | . | . | 27. s1 : traversal(the_graph)
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| . . . | . . | | . | . | 28. s3 : traversal(the_graph)
| . . . | . . | | . | . | 29. s = (s3 @ s1)
| . . . | . . | | . | . | 30. paren(Vertices(the_graph);s1)
| . . . | . . | | . | . | 31. paren(Vertices(the_graph);s3)
| . . . | . . | | . | . | 32. ∀j:Vertices(the_graph).
| . . . | . . | | . | . |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | . | . |     & ...
| . . . | . . | | . | . | 33. j : Vertices(the_graph)
| . . . | . . | | . | . | 34. (inl(j) ∈ s3)
| . . . | . . | | . | . |     ⇒
| . . . | . . | | . | . |     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | . | . | 35. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . | | . | . |     & ¬L1@0-the_graph->*j)
| . . . | . . | | . | . | 36. (inl(j) ∈ s2)
| . . . | . . | | . | . | 37. L1@0-the_graph->*j
| . . . | . . | | . | . | ⊢  (inl(j) ∈ s1)
| . . . | . . | | . | . |   by (InstHyp [j] -6) THEN (Analyze -1)
| . . . | . . | | . | . |      THEN
| . . . | . . | | . | . |      (Analyze -2)
| . . . | . . | | . | . |      THEN
| . . . | . . | | . | . |      (Analyze -2)
| . . . | . . | | . | . |
| . . . | . . | | . | . .\
| . . . | . . | | . | . . 1. the_graph : Graph
| . . . | . . | | . | . . 2. GraphObject(the_graph)
| . . . | . . | | . | . . 3. Vertices(the_graph) List
| . . . | . . | | . | . . 4. L1 : Vertices(the_graph) List
| . . . | . . | | . | . . 5. s : traversal(the_graph)
| . . . | . . | | . | . . 6. i : Vertices(the_graph)
| . . . | . . | | . | . . 7. paren(Vertices(the_graph);s)
| . . . | . . | | . | . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | . | . . |  Vertices(the_graph);s)
| . . . | . . | | . | . . 9. df-traversal(the_graph;s)
| . . . | . . | | . | . . 10. ∀i:Vertices(the_graph).
| . . . | . . | | . | . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | . | . . 11. ¬(inl(i) ∈ s)
| . . . | . . | | . | . . 12. s2 : traversal(the_graph)
| . . . | . . | | . | . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | . | . . |   Vertices(the_graph);s2;s)
| . . . | . . | | . | . . 14. paren(Vertices(the_graph);s2)
| . . . | . . | | . | . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | . | . . |   Vertices(the_graph);s2)
| . . . | . . | | . | . . 16. ¬(inr(i) ∈ s2)
| . . . | . . | | . | . . 17. ∀j:Vertices(the_graph).
| . . . | . . | | . | . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | . | . . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | . | . . |                         @ s2
| . . . | . . | | . | . . |                         @ [inr(i)])
| . . . | . . | | . | . . |   @ s)
| . . . | . . | | . | . . 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | . | . . |   L1 @ [i]-the_graph->*i@0
| . . . | . . | | . | . . |   ⇒
| . . . | . . | | . | . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | . | . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | . | . . |   L1 = (L1@0 @ L2)
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| . . . | . . | | . | . . |   ⇒
| . . . | . . | | . | . . |   (∃s1,s2:traversal(the_graph).
| . . . | . . | | . | . . |    s = (s2 @ s1)
| . . . | . . | | . | . . |    & paren(Vertices(the_graph);s1)
| . . . | . . | | . | . . |    & paren(Vertices(the_graph);s2)
| . . . | . . | | . | . . |    & (∀j:Vertices(the_graph).
| . . . | . . | | . | . . |       ...
| . . . | . . | | . | . . |       & ((inl(j) ∈ s2)
| . . . | . . | | . | . . |          ⟺
| . . . | . . | | . | . . |          L2-the_graph->*j
| . . . | . . | | . | . . |          & ¬L1@0-the_graph->*j)))
| . . . | . . | | . | . . 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | . | . . 22. L2 : Vertices(the_graph) List
| . . . | . . | | . | . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | . | . . 24. e : Vertices(the_graph) List
| . . . | . . | | . | . . 25. L1 = (L1@0 @ e)
| . . . | . . | | . | . . 26. L2 = (e @ [i])
| . . . | . . | | . | . . 27. s1 : traversal(the_graph)
| . . . | . . | | . | . . 28. s3 : traversal(the_graph)
| . . . | . . | | . | . . 29. s = (s3 @ s1)
| . . . | . . | | . | . . 30. paren(Vertices(the_graph);s1)
| . . . | . . | | . | . . 31. paren(Vertices(the_graph);s3)
| . . . | . . | | . | . . 32. ∀j:Vertices(the_graph).
| . . . | . . | | . | . . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | . | . . |   & ...
| . . . | . . | | . | . . 33. j : Vertices(the_graph)
| . . . | . . | | . | . . 34. (inl(j) ∈ s3)
| . . . | . . | | . | . . |   ⇒
| . . . | . . | | . | . . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | . | . . 35. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . | | . | . . |   & ¬L1@0-the_graph->*j)
| . . . | . . | | . | . . 36. (inl(j) ∈ s2)
| . . . | . . | | . | . . 37. L1@0-the_graph->*j
| . . . | . . | | . | . . 38. (inl(j) ∈ s1)
| . . . | . . | | . | . . ⊢  False
| . . . | . . | | . | . . | by (Unfold `l_disjoint` 13)
| . . . | . . | | . | . . |    THEN
| . . . | . . | | . | . . |    (InstHyp [inl(j)] 13)
| . . . | . . | | . | . . |    THEN
| . . . | . . | | . | . . |    (Analyze -1)
| . . . | . . | | . | . . |    THEN
| . . . | . . | | . | . . |    (SubstFor s 0)
| . . . | . . | | . | . . |
| . . . | . . | | . | . . 1. the_graph : Graph
| . . . | . . | | . | . . 2. GraphObject(the_graph)
| . . . | . . | | . | . . 3. Vertices(the_graph) List
| . . . | . . | | . | . . 4. L1 : Vertices(the_graph) List
| . . . | . . | | . | . . 5. s : traversal(the_graph)
| . . . | . . | | . | . . 6. i : Vertices(the_graph)
| . . . | . . | | . | . . 7. paren(Vertices(the_graph);s)
| . . . | . . | | . | . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | . | . .    Vertices(the_graph);s)
| . . . | . . | | . | . . 9. df-traversal(the_graph;s)
| . . . | . . | | . | . . 10. ∀i:Vertices(the_graph).
| . . . | . . | | . | . .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
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| . . . | . . | | . | . . 11. ¬(inl(i) ∈ s)
| . . . | . . | | . | . . 12. s2 : traversal(the_graph)
| . . . | . . | | . | . . 13. ∀x:Vertices(the_graph)+
| . . . | . . | | . | . .        Vertices(the_graph).
| . . . | . . | | . | . .     ¬((x ∈ s2) & (x ∈ s))
| . . . | . . | | . | . . 14. paren(Vertices(the_graph);s2)
| . . . | . . | | . | . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | . | . .     Vertices(the_graph);s2)
| . . . | . . | | . | . . 16. ¬(inr(i) ∈ s2)
| . . . | . . | | . | . . 17. ∀j:Vertices(the_graph).
| . . . | . . | | . | . .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | . | . . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | . | . .                             @ s2
| . . . | . . | | . | . .                             @ [inr(i)])
| . . . | . . | | . | . .     @ s)
| . . . | . . | | . | . . 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | . | . .     L1 @ [i]-the_graph->*i@0
| . . . | . . | | . | . .     ⇒
| . . . | . . | | . | . .     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | . | . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | . | . .     L1 = (L1@0 @ L2)
| . . . | . . | | . | . .     ⇒
| . . . | . . | | . | . .     (∃s1,s2:traversal(the_graph).
| . . . | . . | | . | . .      s = (s2 @ s1)
| . . . | . . | | . | . .      & paren(Vertices(the_graph);s1)
| . . . | . . | | . | . .      & paren(Vertices(the_graph);s2)
| . . . | . . | | . | . .      & (∀j:Vertices(the_graph).
| . . . | . . | | . | . .          ...
| . . . | . . | | . | . .          & ((inl(j) ∈ s2)
| . . . | . . | | . | . .             ⟺
| . . . | . . | | . | . .             L2-the_graph->*j
| . . . | . . | | . | . .             & ¬L1@0-the_graph->*j)))
| . . . | . . | | . | . . 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | . | . . 22. L2 : Vertices(the_graph) List
| . . . | . . | | . | . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | . | . . 24. e : Vertices(the_graph) List
| . . . | . . | | . | . . 25. L1 = (L1@0 @ e)
| . . . | . . | | . | . . 26. L2 = (e @ [i])
| . . . | . . | | . | . . 27. s1 : traversal(the_graph)
| . . . | . . | | . | . . 28. s3 : traversal(the_graph)
| . . . | . . | | . | . . 29. s = (s3 @ s1)
| . . . | . . | | . | . . 30. paren(Vertices(the_graph);s1)
| . . . | . . | | . | . . 31. paren(Vertices(the_graph);s3)
| . . . | . . | | . | . . 32. ∀j:Vertices(the_graph).
| . . . | . . | | . | . .     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | . | . .     & ...
| . . . | . . | | . | . . 33. j : Vertices(the_graph)
| . . . | . . | | . | . . 34. (inl(j) ∈ s3)
| . . . | . . | | . | . .     ⇒
| . . . | . . | | . | . .     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | . | . . 35. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . | | . | . .     & ¬L1@0-the_graph->*j)
| . . . | . . | | . | . . 36. (inl(j) ∈ s2)
| . . . | . . | | . | . . 37. L1@0-the_graph->*j
| . . . | . . | | . | . . 38. (inl(j) ∈ s1)
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| . . . | . . | | . | . . ⊢  (inl(j) ∈ s3 @ s1)  by ObviousFrom [-1]
| . . . | . . | | . |
| . . . | . . | | . .\
| . . . | . . | | . . 1. the_graph : Graph
| . . . | . . | | . . 2. GraphObject(the_graph)
| . . . | . . | | . . 3. Vertices(the_graph) List
| . . . | . . | | . . 4. L1 : Vertices(the_graph) List
| . . . | . . | | . . 5. s : traversal(the_graph)
| . . . | . . | | . . 6. i : Vertices(the_graph)
| . . . | . . | | . . 7. paren(Vertices(the_graph);s)
| . . . | . . | | . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . | | . .    Vertices(the_graph);s)
| . . . | . . | | . . 9. df-traversal(the_graph;s)
| . . . | . . | | . . 10. ∀i:Vertices(the_graph).
| . . . | . . | | . .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | | . . 11. ¬(inl(i) ∈ s)
| . . . | . . | | . . 12. s2 : traversal(the_graph)
| . . . | . . | | . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | | . .     Vertices(the_graph);s2;s)
| . . . | . . | | . . 14. paren(Vertices(the_graph);s2)
| . . . | . . | | . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . | | . .     Vertices(the_graph);s2)
| . . . | . . | | . . 16. ¬(inr(i) ∈ s2)
| . . . | . . | | . . 17. ∀j:Vertices(the_graph).
| . . . | . . | | . .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | | . . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | | . .                             @ s2
| . . . | . . | | . .                             @ [inr(i)])
| . . . | . . | | . .     @ s)
| . . . | . . | | . . 19. ∀i@0:Vertices(the_graph).
| . . . | . . | | . .     L1 @ [i]-the_graph->*i@0
| . . . | . . | | . .     ⇒
| . . . | . . | | . .     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | | . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | | . .     L1 = (L1@0 @ L2)
| . . . | . . | | . .     ⇒
| . . . | . . | | . .     (∃s1,s2:traversal(the_graph).
| . . . | . . | | . .      s = (s2 @ s1)
| . . . | . . | | . .      & paren(Vertices(the_graph);s1)
| . . . | . . | | . .      & paren(Vertices(the_graph);s2)
| . . . | . . | | . .      & (∀j:Vertices(the_graph).
| . . . | . . | | . .         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | . .         & ((inl(j) ∈ s2)
| . . . | . . | | . .            ⟺
| . . . | . . | | . .            L2-the_graph->*j
| . . . | . . | | . .            & ¬L1@0-the_graph->*j)))
| . . . | . . | | . . 21. L1@0 : Vertices(the_graph) List
| . . . | . . | | . . 22. L2 : Vertices(the_graph) List
| . . . | . . | | . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | | . . 24. e : Vertices(the_graph) List
| . . . | . . | | . . 25. L1 = (L1@0 @ e)
| . . . | . . | | . . 26. L2 = (e @ [i])
| . . . | . . | | . . 27. s1 : traversal(the_graph)
| . . . | . . | | . . 28. s3 : traversal(the_graph)
| . . . | . . | | . . 29. s = (s3 @ s1)
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| . . . | . . | | . . 30. paren(Vertices(the_graph);s1)
| . . . | . . | | . . 31. paren(Vertices(the_graph);s3)
| . . . | . . | | . . 32. ∀j:Vertices(the_graph).
| . . . | . . | | . .     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | | . .     & ((inl(j) ∈ s3)
| . . . | . . | | . .        ⟺
| . . . | . . | | . .        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | | . . 33. j : Vertices(the_graph)
| . . . | . . | | . . 34. (inl(j) ∈ s3)
| . . . | . . | | . .     ⟺
| . . . | . . | | . .     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | | . . 35. inl(j) = inr(i)
| . . . | . . | | . . ⊢  e-the_graph->*j ∨ i-the_graph->*j
| . . . | . . | | . .    & ¬L1@0-the_graph->*j
| . . . | . . | | . .   by ObviousContradiction [-1]
| . . . | . . | |
| . . . | . . | .\
| . . . | . . | . 1. the_graph : Graph
| . . . | . . | . 2. GraphObject(the_graph)
| . . . | . . | . 3. Vertices(the_graph) List
| . . . | . . | . 4. L1 : Vertices(the_graph) List
| . . . | . . | . 5. s : traversal(the_graph)
| . . . | . . | . 6. i : Vertices(the_graph)
| . . . | . . | . 7. paren(Vertices(the_graph);s)
| . . . | . . | . 8. no_repeats(Vertices(the_graph)+
| . . . | . . | . |  Vertices(the_graph);s)
| . . . | . . | . 9. df-traversal(the_graph;s)
| . . . | . . | . 10. ∀i:Vertices(the_graph).
| . . . | . . | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | . 11. ¬(inl(i) ∈ s)
| . . . | . . | . 12. s2 : traversal(the_graph)
| . . . | . . | . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | . |   Vertices(the_graph);s2;s)
| . . . | . . | . 14. paren(Vertices(the_graph);s2)
| . . . | . . | . 15. no_repeats(Vertices(the_graph)+
| . . . | . . | . |   Vertices(the_graph);s2)
| . . . | . . | . 16. ¬(inr(i) ∈ s2)
| . . . | . . | . 17. ∀j:Vertices(the_graph).
| . . . | . . | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | . |                           @ s2
| . . . | . . | . |                           @ [inr(i)])
| . . . | . . | . |   @ s)
| . . . | . . | . 19. ∀i@0:Vertices(the_graph).
| . . . | . . | . |   L1 @ [i]-the_graph->*i@0
| . . . | . . | . |   ⇒
| . . . | . . | . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | . |   L1 = (L1@0 @ L2)
| . . . | . . | . |   ⇒
| . . . | . . | . |   (∃s1,s2:traversal(the_graph).
| . . . | . . | . |    s = (s2 @ s1)
| . . . | . . | . |    & paren(Vertices(the_graph);s1)
| . . . | . . | . |    & paren(Vertices(the_graph);s2)
| . . . | . . | . |    & (∀j:Vertices(the_graph).
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| . . . | . . | . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | . |       & ((inl(j) ∈ s2)
| . . . | . . | . |          ⟺
| . . . | . . | . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | . . | . 21. L1@0 : Vertices(the_graph) List
| . . . | . . | . 22. L2 : Vertices(the_graph) List
| . . . | . . | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | . 24. e : Vertices(the_graph) List
| . . . | . . | . 25. L1 = (L1@0 @ e)
| . . . | . . | . 26. L2 = (e @ [i])
| . . . | . . | . 27. s1 : traversal(the_graph)
| . . . | . . | . 28. s3 : traversal(the_graph)
| . . . | . . | . 29. s = (s3 @ s1)
| . . . | . . | . 30. paren(Vertices(the_graph);s1)
| . . . | . . | . 31. paren(Vertices(the_graph);s3)
| . . . | . . | . 32. ∀j:Vertices(the_graph).
| . . . | . . | . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | . |   & ((inl(j) ∈ s3)
| . . . | . . | . |      ⟺
| . . . | . . | . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | . 33. j : Vertices(the_graph)
| . . . | . . | . 34. (inl(j) ∈ s3)
| . . . | . . | . |   ⟺
| . . . | . . | . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | . 35. e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | . ⊢  e-the_graph->*j ∨ [i]-the_graph->*j
| . . . | . . | . |  & ¬L1@0-the_graph->*j  by Auto
| . . . | . . | . |
| . . . | . . | . 1. the_graph : Graph
| . . . | . . | . 2. GraphObject(the_graph)
| . . . | . . | . 3. Vertices(the_graph) List
| . . . | . . | . 4. L1 : Vertices(the_graph) List
| . . . | . . | . 5. s : traversal(the_graph)
| . . . | . . | . 6. i : Vertices(the_graph)
| . . . | . . | . 7. paren(Vertices(the_graph);s)
| . . . | . . | . 8. no_repeats(Vertices(the_graph)+
| . . . | . . | .    Vertices(the_graph);s)
| . . . | . . | . 9. df-traversal(the_graph;s)
| . . . | . . | . 10. ∀i:Vertices(the_graph).
| . . . | . . | .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . | . 11. ¬(inl(i) ∈ s)
| . . . | . . | . 12. s2 : traversal(the_graph)
| . . . | . . | . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . | .     Vertices(the_graph);s2;s)
| . . . | . . | . 14. paren(Vertices(the_graph);s2)
| . . . | . . | . 15. no_repeats(Vertices(the_graph)+
| . . . | . . | .     Vertices(the_graph);s2)
| . . . | . . | . 16. ¬(inr(i) ∈ s2)
| . . . | . . | . 17. ∀j:Vertices(the_graph).
| . . . | . . | .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . | . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . | .                             @ s2

| . . . | . . | .                             @ [inr(i)])
| . . . | . . | .     @ s)
| . . . | . . | . 19. ∀i@0:Vertices(the_graph).
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| . . . | . . | .     L1 @ [i]-the_graph->*i@0
| . . . | . . | .     ⇒
| . . . | . . | .     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . | .     L1 = (L1@0 @ L2)
| . . . | . . | .     ⇒
| . . . | . . | .     (∃s1,s2:traversal(the_graph).
| . . . | . . | .      s = (s2 @ s1)
| . . . | . . | .      & paren(Vertices(the_graph);s1)
| . . . | . . | .      & paren(Vertices(the_graph);s2)
| . . . | . . | .      & (∀j:Vertices(the_graph).
| . . . | . . | .         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | .         & ((inl(j) ∈ s2)
| . . . | . . | .            ⟺
| . . . | . . | .            L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | . . | . 21. L1@0 : Vertices(the_graph) List
| . . . | . . | . 22. L2 : Vertices(the_graph) List
| . . . | . . | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . | . 24. e : Vertices(the_graph) List
| . . . | . . | . 25. L1 = (L1@0 @ e)
| . . . | . . | . 26. L2 = (e @ [i])
| . . . | . . | . 27. s1 : traversal(the_graph)
| . . . | . . | . 28. s3 : traversal(the_graph)
| . . . | . . | . 29. s = (s3 @ s1)
| . . . | . . | . 30. paren(Vertices(the_graph);s1)
| . . . | . . | . 31. paren(Vertices(the_graph);s3)
| . . . | . . | . 32. ∀j:Vertices(the_graph).
| . . . | . . | .     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . | .     & ((inl(j) ∈ s3)
| . . . | . . | .        ⟺
| . . . | . . | .        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . | . 33. j : Vertices(the_graph)
| . . . | . . | . 34. (inl(j) ∈ s3)
| . . . | . . | .     ⇒
| . . . | . . | .     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . | . 35. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . | .     & ¬L1@0-the_graph->*j)
| . . . | . . | . 36. e-the_graph->*j
| . . . | . . | . 37. ¬L1@0-the_graph->*j
| . . . | . . | . ⊢  e-the_graph->*j ∨ [i]-the_graph->*j
| . . . | . . | .   by ObviousFrom [-2]
| . . . | . . |
| . . . | . . .\
| . . . | . . . 1. the_graph : Graph
| . . . | . . . 2. GraphObject(the_graph)
| . . . | . . . 3. Vertices(the_graph) List
| . . . | . . . 4. L1 : Vertices(the_graph) List
| . . . | . . . 5. s : traversal(the_graph)
| . . . | . . . 6. i : Vertices(the_graph)
| . . . | . . . 7. paren(Vertices(the_graph);s)
| . . . | . . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . . |  Vertices(the_graph);s)
| . . . | . . . 9. df-traversal(the_graph;s)
| . . . | . . . 10. ∀i:Vertices(the_graph).
| . . . | . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
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| . . . | . . . 11. ¬(inl(i) ∈ s)
| . . . | . . . 12. s2 : traversal(the_graph)
| . . . | . . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . |   Vertices(the_graph);s2;s)
| . . . | . . . 14. paren(Vertices(the_graph);s2)
| . . . | . . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . . |   Vertices(the_graph);s2)
| . . . | . . . 16. ¬(inr(i) ∈ s2)
| . . . | . . . 17. ∀j:Vertices(the_graph).
| . . . | . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . | . . . |   @ s)
| . . . | . . . 19. ∀i@0:Vertices(the_graph).
| . . . | . . . |   L1 @ [i]-the_graph->*i@0
| . . . | . . . |   ⇒
| . . . | . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . |   L1 = (L1@0 @ L2)
| . . . | . . . |   ⇒
| . . . | . . . |   (∃s1,s2:traversal(the_graph).
| . . . | . . . |    s = (s2 @ s1)
| . . . | . . . |    & paren(Vertices(the_graph);s1)
| . . . | . . . |    & paren(Vertices(the_graph);s2)
| . . . | . . . |    & (∀j:Vertices(the_graph).
| . . . | . . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . |       & ((inl(j) ∈ s2)
| . . . | . . . |          ⟺
| . . . | . . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | . . . 21. L1@0 : Vertices(the_graph) List
| . . . | . . . 22. L2 : Vertices(the_graph) List
| . . . | . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . 24. e : Vertices(the_graph) List
| . . . | . . . 25. L1 = (L1@0 @ e)
| . . . | . . . 26. L2 = (e @ [i])
| . . . | . . . 27. s1 : traversal(the_graph)
| . . . | . . . 28. s3 : traversal(the_graph)
| . . . | . . . 29. s = (s3 @ s1)
| . . . | . . . 30. paren(Vertices(the_graph);s1)
| . . . | . . . 31. paren(Vertices(the_graph);s3)
| . . . | . . . 32. ∀j:Vertices(the_graph).
| . . . | . . . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . |   & ((inl(j) ∈ s3)
| . . . | . . . |      ⟺
| . . . | . . . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . 33. j : Vertices(the_graph)
| . . . | . . . 34. (inl(j) ∈ s3)
| . . . | . . . |   ⟺
| . . . | . . . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . 35. e-the_graph->*j ∨ [i]-the_graph->*j
| . . . | . . . |   & ¬L1@0-the_graph->*j
| . . . | . . . ⊢  inl(j) = inl(i) ∨ (inl(j) ∈ s2) ∨ inl(j) = inr(i)
| . . . | . . . |  ∨ e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . | by (Analyze -1) THEN (Analyze -2)
| . . . | . . . |
| . . . | . . . |\
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| . . . | . . . | 1. the_graph : Graph
| . . . | . . . | 2. GraphObject(the_graph)
| . . . | . . . | 3. Vertices(the_graph) List
| . . . | . . . | 4. L1 : Vertices(the_graph) List
| . . . | . . . | 5. s : traversal(the_graph)
| . . . | . . . | 6. i : Vertices(the_graph)
| . . . | . . . | 7. paren(Vertices(the_graph);s)
| . . . | . . . | 8. no_repeats(Vertices(the_graph)+
| . . . | . . . |    Vertices(the_graph);s)
| . . . | . . . | 9. df-traversal(the_graph;s)
| . . . | . . . | 10. ∀i:Vertices(the_graph).
| . . . | . . . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . | 11. ¬(inl(i) ∈ s)
| . . . | . . . | 12. s2 : traversal(the_graph)
| . . . | . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . |     Vertices(the_graph);s2;s)
| . . . | . . . | 14. paren(Vertices(the_graph);s2)
| . . . | . . . | 15. no_repeats(Vertices(the_graph)+
| . . . | . . . |     Vertices(the_graph);s2)
| . . . | . . . | 16. ¬(inr(i) ∈ s2)
| . . . | . . . | 17. ∀j:Vertices(the_graph).
| . . . | . . . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . . |                              @ s2
| . . . | . . . |                              @ [inr(i)])
| . . . | . . . |     @ s)
| . . . | . . . | 19. ∀i@0:Vertices(the_graph).
| . . . | . . . |     L1 @ [i]-the_graph->*i@0
| . . . | . . . |     ⇒
| . . . | . . . |     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . |     L1 = (L1@0 @ L2)
| . . . | . . . |     ⇒
| . . . | . . . |     (∃s1,s2:traversal(the_graph).
| . . . | . . . |      s = (s2 @ s1)
| . . . | . . . |      & paren(Vertices(the_graph);s1)
| . . . | . . . |      & paren(Vertices(the_graph);s2)
| . . . | . . . |      & (∀j:Vertices(the_graph).
| . . . | . . . |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . |         & ((inl(j) ∈ s2)
| . . . | . . . |            ⟺
| . . . | . . . |            L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | . . . | 21. L1@0 : Vertices(the_graph) List
| . . . | . . . | 22. L2 : Vertices(the_graph) List
| . . . | . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . | 24. e : Vertices(the_graph) List
| . . . | . . . | 25. L1 = (L1@0 @ e)
| . . . | . . . | 26. L2 = (e @ [i])
| . . . | . . . | 27. s1 : traversal(the_graph)
| . . . | . . . | 28. s3 : traversal(the_graph)
| . . . | . . . | 29. s = (s3 @ s1)
| . . . | . . . | 30. paren(Vertices(the_graph);s1)
| . . . | . . . | 31. paren(Vertices(the_graph);s3)
| . . . | . . . | 32. ∀j:Vertices(the_graph).
| . . . | . . . |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
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| . . . | . . . |     & ((inl(j) ∈ s3)
| . . . | . . . |        ⟺
| . . . | . . . |        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . | 33. j : Vertices(the_graph)
| . . . | . . . | 34. (inl(j) ∈ s3)
| . . . | . . . |     ⟺
| . . . | . . . |     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . | 35. e-the_graph->*j
| . . . | . . . | 36. ¬L1@0-the_graph->*j
| . . . | . . . | ⊢  inl(j) = inl(i) ∨ (inl(j) ∈ s2) ∨ inl(j) = inr(i)
| . . . | . . . |    ∨ e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . |   by OrRight
| . . . | . . . |
| . . . | . . . .\
| . . . | . . . . 1. the_graph : Graph
| . . . | . . . . 2. GraphObject(the_graph)
| . . . | . . . . 3. Vertices(the_graph) List
| . . . | . . . . 4. L1 : Vertices(the_graph) List
| . . . | . . . . 5. s : traversal(the_graph)
| . . . | . . . . 6. i : Vertices(the_graph)
| . . . | . . . . 7. paren(Vertices(the_graph);s)
| . . . | . . . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . |  Vertices(the_graph);s)
| . . . | . . . . 9. df-traversal(the_graph;s)
| . . . | . . . . 10. ∀i:Vertices(the_graph).
| . . . | . . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . 11. ¬(inl(i) ∈ s)
| . . . | . . . . 12. s2 : traversal(the_graph)
| . . . | . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . |   Vertices(the_graph);s2;s)
| . . . | . . . . 14. paren(Vertices(the_graph);s2)
| . . . | . . . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . |   Vertices(the_graph);s2)
| . . . | . . . . 16. ¬(inr(i) ∈ s2)
| . . . | . . . . 17. ∀j:Vertices(the_graph).
| . . . | . . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . . . |                           @ s2
| . . . | . . . . |                           @ [inr(i)])
| . . . | . . . . |   @ s)
| . . . | . . . . 19. ∀i@0:Vertices(the_graph).
| . . . | . . . . |   L1 @ [i]-the_graph->*i@0
| . . . | . . . . |   ⇒
| . . . | . . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . |   L1 = (L1@0 @ L2)
| . . . | . . . . |   ⇒
| . . . | . . . . |   (∃s1,s2:traversal(the_graph).
| . . . | . . . . |    s = (s2 @ s1)
| . . . | . . . . |    & paren(Vertices(the_graph);s1)
| . . . | . . . . |    & paren(Vertices(the_graph);s2)
| . . . | . . . . |    & (∀j:Vertices(the_graph).
| . . . | . . . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . |       & ((inl(j) ∈ s2)
| . . . | . . . . |          ⟺
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| . . . | . . . . |         L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . | . . . . 21. L1@0 : Vertices(the_graph) List
| . . . | . . . . 22. L2 : Vertices(the_graph) List
| . . . | . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . 24. e : Vertices(the_graph) List
| . . . | . . . . 25. L1 = (L1@0 @ e)
| . . . | . . . . 26. L2 = (e @ [i])
| . . . | . . . . 27. s1 : traversal(the_graph)
| . . . | . . . . 28. s3 : traversal(the_graph)
| . . . | . . . . 29. s = (s3 @ s1)
| . . . | . . . . 30. paren(Vertices(the_graph);s1)
| . . . | . . . . 31. paren(Vertices(the_graph);s3)
| . . . | . . . . 32. ∀j:Vertices(the_graph).
| . . . | . . . . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . |   & ((inl(j) ∈ s3)
| . . . | . . . . |      ⟺
| . . . | . . . . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . 33. j : Vertices(the_graph)
| . . . | . . . . 34. (inl(j) ∈ s3)
| . . . | . . . . |   ⟺
| . . . | . . . . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . 35. [i]-the_graph->*j
| . . . | . . . . 36. ¬L1@0-the_graph->*j
| . . . | . . . . ⊢  inl(j) = inl(i) ∨ (inl(j) ∈ s2) ∨ inl(j) = inr(i)
| . . . | . . . . |  ∨ e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | by Assert
| . . . | . . . . |    ((inl(j) ∈ s2) ∨ (inl(j) ∈ s)
| . . . | . . . . |     ∨ inl(j) = inl(i))
| . . . | . . . . |
| . . . | . . . . |\
| . . . | . . . . | .....assertion .....
| . . . | . . . . | 1. the_graph : Graph
| . . . | . . . . | 2. GraphObject(the_graph)
| . . . | . . . . | 3. Vertices(the_graph) List
| . . . | . . . . | 4. L1 : Vertices(the_graph) List
| . . . | . . . . | 5. s : traversal(the_graph)
| . . . | . . . . | 6. i : Vertices(the_graph)
| . . . | . . . . | 7. paren(Vertices(the_graph);s)
| . . . | . . . . | 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . | |  Vertices(the_graph);s)
| . . . | . . . . | 9. df-traversal(the_graph;s)
| . . . | . . . . | 10. ∀i:Vertices(the_graph).
| . . . | . . . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . | 11. ¬(inl(i) ∈ s)
| . . . | . . . . | 12. s2 : traversal(the_graph)
| . . . | . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . | |   Vertices(the_graph);s2;s)
| . . . | . . . . | 14. paren(Vertices(the_graph);s2)
| . . . | . . . . | 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . | |   Vertices(the_graph);s2)
| . . . | . . . . | 16. ¬(inr(i) ∈ s2)
| . . . | . . . . | 17. ∀j:Vertices(the_graph).
| . . . | . . . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . . . | |                            @ s2
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| . . . | . . . . | |                            @ [inr(i)])
| . . . | . . . . | |   @ s)
| . . . | . . . . | 19. ∀i@0:Vertices(the_graph).
| . . . | . . . . | |   L1 @ [i]-the_graph->*i@0
| . . . | . . . . | |   ⇒
| . . . | . . . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . | |   L1 = (L1@0 @ L2)
| . . . | . . . . | |   ⇒
| . . . | . . . . | |   (∃s1,s2:traversal(the_graph).
| . . . | . . . . | |    s = (s2 @ s1)
| . . . | . . . . | |    & paren(Vertices(the_graph);s1)
| . . . | . . . . | |    & paren(Vertices(the_graph);s2)
| . . . | . . . . | |    & ...)
| . . . | . . . . | 21. L1@0 : Vertices(the_graph) List
| . . . | . . . . | 22. L2 : Vertices(the_graph) List
| . . . | . . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . | 24. e : Vertices(the_graph) List
| . . . | . . . . | 25. L1 = (L1@0 @ e)
| . . . | . . . . | 26. L2 = (e @ [i])
| . . . | . . . . | 27. s1 : traversal(the_graph)
| . . . | . . . . | 28. s3 : traversal(the_graph)
| . . . | . . . . | 29. s = (s3 @ s1)
| . . . | . . . . | 30. paren(Vertices(the_graph);s1)
| . . . | . . . . | 31. paren(Vertices(the_graph);s3)
| . . . | . . . . | 32. ∀j:Vertices(the_graph).
| . . . | . . . . | |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | |   & ((inl(j) ∈ s3)
| . . . | . . . . | |      ⟺
| . . . | . . . . | |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . | 33. j : Vertices(the_graph)
| . . . | . . . . | 34. (inl(j) ∈ s3)
| . . . | . . . . | |   ⟺
| . . . | . . . . | |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | 35. [i]-the_graph->*j
| . . . | . . . . | 36. ¬L1@0-the_graph->*j
| . . . | . . . . | ⊢  (inl(j) ∈ s2) ∨ (inl(j) ∈ s) ∨ inl(j) = inl(i)
| . . . | . . . . | | by RWO
| . . . | . . . . | |    Thm* For any graph
| . . . | . . . . | |         ∀i,j:V. [i]-->*j ⟺ i-the_graph->*j
| . . . | . . . . | |    {list_DASH_connect_DASH_singleton}
| . . . | . . . . | |    -2
| . . . | . . . . | |
| . . . | . . . . | 1. the_graph : Graph
| . . . | . . . . | 2. GraphObject(the_graph)
| . . . | . . . . | 3. Vertices(the_graph) List
| . . . | . . . . | 4. L1 : Vertices(the_graph) List
| . . . | . . . . | 5. s : traversal(the_graph)
| . . . | . . . . | 6. i : Vertices(the_graph)
| . . . | . . . . | 7. paren(Vertices(the_graph);s)
| . . . | . . . . | 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . | |  Vertices(the_graph);s)
| . . . | . . . . | 9. df-traversal(the_graph;s)
| . . . | . . . . | 10. ∀i:Vertices(the_graph).
| . . . | . . . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
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| . . . | . . . . | 11. ¬(inl(i) ∈ s)
| . . . | . . . . | 12. s2 : traversal(the_graph)
| . . . | . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . | |   Vertices(the_graph);s2;s)
| . . . | . . . . | 14. paren(Vertices(the_graph);s2)
| . . . | . . . . | 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . | |   Vertices(the_graph);s2)
| . . . | . . . . | 16. ¬(inr(i) ∈ s2)
| . . . | . . . . | 17. ∀j:Vertices(the_graph).
| . . . | . . . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . . . | |                         @ s2
| . . . | . . . . | |                         @ [inr(i)])
| . . . | . . . . | |   @ s)
| . . . | . . . . | 19. ∀i@0:Vertices(the_graph).
| . . . | . . . . | |   L1 @ [i]-the_graph->*i@0
| . . . | . . . . | |   ⇒
| . . . | . . . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . | |   L1 = (L1@0 @ L2)
| . . . | . . . . | |   ⇒
| . . . | . . . . | |   (∃s1,s2:traversal(the_graph).
| . . . | . . . . | |    s = (s2 @ s1)
| . . . | . . . . | |    & paren(Vertices(the_graph);s1)
| . . . | . . . . | |    & paren(Vertices(the_graph);s2)
| . . . | . . . . | |    & ...)
| . . . | . . . . | 21. L1@0 : Vertices(the_graph) List
| . . . | . . . . | 22. L2 : Vertices(the_graph) List
| . . . | . . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . | 24. e : Vertices(the_graph) List
| . . . | . . . . | 25. L1 = (L1@0 @ e)
| . . . | . . . . | 26. L2 = (e @ [i])
| . . . | . . . . | 27. s1 : traversal(the_graph)
| . . . | . . . . | 28. s3 : traversal(the_graph)
| . . . | . . . . | 29. s = (s3 @ s1)
| . . . | . . . . | 30. paren(Vertices(the_graph);s1)
| . . . | . . . . | 31. paren(Vertices(the_graph);s3)
| . . . | . . . . | 32. ∀j:Vertices(the_graph).
| . . . | . . . . | |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | |   & ((inl(j) ∈ s3)
| . . . | . . . . | |      ⟺
| . . . | . . . . | |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . | 33. j : Vertices(the_graph)
| . . . | . . . . | 34. (inl(j) ∈ s3)
| . . . | . . . . | |   ⟺
| . . . | . . . . | |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | 35. i-the_graph->*j
| . . . | . . . . | 36. ¬L1@0-the_graph->*j
| . . . | . . . . | ⊢  (inl(j) ∈ s2) ∨ (inl(j) ∈ s) ∨ inl(j) = inl(i)
| . . . | . . . . | | by let h ≡ 18 in
| . . . | . . . . | |    (Unfold ‘df-traversal‘ h) THEN (Analyze h)
| . . . | . . . . | |    THEN
| . . . | . . . . | |    (InstHyp [i;nil;(s2 @ [inr(i)]) @ s;j] h+1)
| . . . | . . . . | |
| . . . | . . . . | |\
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| . . . | . . . . | | 1. the_graph : Graph
| . . . | . . . . | | 2. GraphObject(the_graph)
| . . . | . . . . | | 3. Vertices(the_graph) List
| . . . | . . . . | | 4. L1 : Vertices(the_graph) List
| . . . | . . . . | | 5. s : traversal(the_graph)
| . . . | . . . . | | 6. i : Vertices(the_graph)
| . . . | . . . . | | 7. paren(Vertices(the_graph);s)
| . . . | . . . . | | 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . | | |  Vertices(the_graph);s)
| . . . | . . . . | | 9. df-traversal(the_graph;s)
| . . . | . . . . | | 10. ∀i:Vertices(the_graph).
| . . . | . . . . | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . | | 11. ¬(inl(i) ∈ s)
| . . . | . . . . | | 12. s2 : traversal(the_graph)
| . . . | . . . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . | | |   Vertices(the_graph);s2;s)
| . . . | . . . . | | 14. paren(Vertices(the_graph);s2)
| . . . | . . . . | | 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . | | |   Vertices(the_graph);s2)
| . . . | . . . . | | 16. ¬(inr(i) ∈ s2)
| . . . | . . . . | | 17. ∀j:Vertices(the_graph).
| . . . | . . . . | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . | | 18. ∀i@0:Vertices(the_graph)
| . . . | . . . . | | |   , s1,s2@0:traversal(the_graph).
| . . . | . . . . | | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | | |   =
| . . . | . . . . | | |   (s1 @ [inr(i@0)] @ s2@0)
| . . . | . . . . | | |   ⇒
| . . . | . . . . | | |   (∀j:Vertices(the_graph).
| . . . | . . . . | | |    (inr(j) ∈ s2@0)
| . . . | . . . . | | |    ⇒
| . . . | . . . . | | |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . | . . . . | | 19. ∀i@0:Vertices(the_graph)
| . . . | . . . . | | |   , s1,s2@0:traversal(the_graph).
| . . . | . . . . | | |   (∀j:Vertices(the_graph).
| . . . | . . . . | | |    i@0-the_graph->*j
| . . . | . . . . | | |    ⇒
| . . . | . . . . | | |    ¬non-trivial-loop(the_graph;j))
| . . . | . . . . | | |   ⇒
| . . . | . . . . | | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | | |   =
| . . . | . . . . | | |   (s1 @ [inl(i@0)] @ s2@0)
| . . . | . . . . | | |   ⇒
| . . . | . . . . | | |   (∀j:Vertices(the_graph).
| . . . | . . . . | | |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . | . . . . | | 20. ∀i@0:Vertices(the_graph).
| . . . | . . . . | | |   L1 @ [i]-the_graph->*i@0
| . . . | . . . . | | |   ⇒
| . . . | . . . . | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . | | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . | | |   L1 = (L1@0 @ L2)
| . . . | . . . . | | |   ⇒
| . . . | . . . . | | |   (∃s1,s2:traversal(the_graph).
| . . . | . . . . | | |    s = (s2 @ s1)
| . . . | . . . . | | |    & paren(Vertices(the_graph);s1)
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| . . . | . . . . | | |    & paren(Vertices(the_graph);s2)
| . . . | . . . . | | |    & (∀j:Vertices(the_graph).
| . . . | . . . . | | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | | |       & ((inl(j) ∈ s2)
| . . . | . . . . | | |          ⟺
| . . . | . . . . | | |          L2-the_graph->*j
| . . . | . . . . | | |          & ¬L1@0-the_graph->*j)))
| . . . | . . . . | | 22. L1@0 : Vertices(the_graph) List
| . . . | . . . . | | 23. L2 : Vertices(the_graph) List
| . . . | . . . . | | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . | | 25. e : Vertices(the_graph) List
| . . . | . . . . | | 26. L1 = (L1@0 @ e)
| . . . | . . . . | | 27. L2 = (e @ [i])
| . . . | . . . . | | 28. s1 : traversal(the_graph)
| . . . | . . . . | | 29. s3 : traversal(the_graph)
| . . . | . . . . | | 30. s = (s3 @ s1)
| . . . | . . . . | | 31. paren(Vertices(the_graph);s1)
| . . . | . . . . | | 32. paren(Vertices(the_graph);s3)
| . . . | . . . . | | 33. ∀j:Vertices(the_graph).
| . . . | . . . . | | |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | | |   & ((inl(j) ∈ s3)
| . . . | . . . . | | |      ⟺
| . . . | . . . . | | |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . | | 34. j : Vertices(the_graph)
| . . . | . . . . | | 35. (inl(j) ∈ s3)
| . . . | . . . . | | |   ⇒
| . . . | . . . . | | |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | | 36. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . . . | | |   & ¬L1@0-the_graph->*j)
| . . . | . . . . | | 37. i-the_graph->*j
| . . . | . . . . | | 38. ¬L1@0-the_graph->*j
| . . . | . . . . | | 39. j1 : Vertices(the_graph)
| . . . | . . . . | | 40. i-the_graph->*j1
| . . . | . . . . | | ⊢  ¬non-trivial-loop(the_graph;j1)
| . . . | . . . . | | | by (BackThruSomeHyp
| . . . | . . . . | | |     THEN
| . . . | . . . . | | |     (RWO
| . . . | . . . . | | |      Thm* For any graph
| . . . | . . . . | | |           ∀A,B:V List, i:V.
| . . . | . . . . | | |           A @ B-->*i ⟺ A-->*i ∨ B-->*i
| . . . | . . . . | | |      {list_DASH_connect_DASH_append}
| . . . | . . . . | | |      0))
| . . . | . . . . | | |    THEN
| . . . | . . . . | | |    (RWO
| . . . | . . . . | | |     Thm* For any graph
| . . . | . . . . | | |          ∀i,j:V. [i]-->*j ⟺ i-the_graph->*j
| . . . | . . . . | | |     {list_DASH_connect_DASH_singleton}
| . . . | . . . . | | |     0)
| . . . | . . . . | | |
| . . . | . . . . | | 1. the_graph : Graph
| . . . | . . . . | | 2. GraphObject(the_graph)
| . . . | . . . . | | 3. Vertices(the_graph) List
| . . . | . . . . | | 4. L1 : Vertices(the_graph) List
| . . . | . . . . | | 5. s : traversal(the_graph)
| . . . | . . . . | | 6. i : Vertices(the_graph)
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| . . . | . . . . | | 7. paren(Vertices(the_graph);s)
| . . . | . . . . | | 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . | |    Vertices(the_graph);s)
| . . . | . . . . | | 9. df-traversal(the_graph;s)
| . . . | . . . . | | 10. ∀i:Vertices(the_graph).
| . . . | . . . . | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . | | 11. ¬(inl(i) ∈ s)
| . . . | . . . . | | 12. s2 : traversal(the_graph)
| . . . | . . . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . | |     Vertices(the_graph);s2;s)
| . . . | . . . . | | 14. paren(Vertices(the_graph);s2)
| . . . | . . . . | | 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . | |     Vertices(the_graph);s2)
| . . . | . . . . | | 16. ¬(inr(i) ∈ s2)
| . . . | . . . . | | 17. ∀j:Vertices(the_graph).
| . . . | . . . . | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . | | 18. ∀i@0:Vertices(the_graph)
| . . . | . . . . | |     , s1,s2@0:traversal(the_graph).
| . . . | . . . . | |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | |     =
| . . . | . . . . | |     (s1 @ [inr(i@0)] @ s2@0)
| . . . | . . . . | |     ⇒
| . . . | . . . . | |     (∀j:Vertices(the_graph).
| . . . | . . . . | |      (inr(j) ∈ s2@0)
| . . . | . . . . | |      ⇒
| . . . | . . . . | |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . | . . . . | | 19. ∀i@0:Vertices(the_graph)
| . . . | . . . . | |     , s1,s2@0:traversal(the_graph).
| . . . | . . . . | |     (∀j:Vertices(the_graph).
| . . . | . . . . | |      i@0-the_graph->*j
| . . . | . . . . | |      ⇒
| . . . | . . . . | |      ¬non-trivial-loop(the_graph;j))
| . . . | . . . . | |     ⇒
| . . . | . . . . | |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | |     =
| . . . | . . . . | |     (s1 @ [inl(i@0)] @ s2@0)
| . . . | . . . . | |     ⇒
| . . . | . . . . | |     (∀j:Vertices(the_graph).
| . . . | . . . . | |      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . | . . . . | | 20. ∀i@0:Vertices(the_graph).
| . . . | . . . . | |     L1 @ [i]-the_graph->*i@0
| . . . | . . . . | |     ⇒
| . . . | . . . . | |     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . | | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . | |     L1 = (L1@0 @ L2)
| . . . | . . . . | |     ⇒
| . . . | . . . . | |     (∃s1,s2:traversal(the_graph).
| . . . | . . . . | |      s = (s2 @ s1)
| . . . | . . . . | |      & paren(Vertices(the_graph);s1)
| . . . | . . . . | |      & paren(Vertices(the_graph);s2)
| . . . | . . . . | |      & (∀j:Vertices(the_graph).
| . . . | . . . . | |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | |         & ((inl(j) ∈ s2)
| . . . | . . . . | |            ⟺
| . . . | . . . . | |            L2-the_graph->*j
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| . . . | . . . . | |           & ¬L1@0-the_graph->*j)))
| . . . | . . . . | | 22. L1@0 : Vertices(the_graph) List
| . . . | . . . . | | 23. L2 : Vertices(the_graph) List
| . . . | . . . . | | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . | | 25. e : Vertices(the_graph) List
| . . . | . . . . | | 26. L1 = (L1@0 @ e)
| . . . | . . . . | | 27. L2 = (e @ [i])
| . . . | . . . . | | 28. s1 : traversal(the_graph)
| . . . | . . . . | | 29. s3 : traversal(the_graph)
| . . . | . . . . | | 30. s = (s3 @ s1)
| . . . | . . . . | | 31. paren(Vertices(the_graph);s1)
| . . . | . . . . | | 32. paren(Vertices(the_graph);s3)
| . . . | . . . . | | 33. ∀j:Vertices(the_graph).
| . . . | . . . . | |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | |     & ((inl(j) ∈ s3)
| . . . | . . . . | |        ⟺
| . . . | . . . . | |        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . | | 34. j : Vertices(the_graph)
| . . . | . . . . | | 35. (inl(j) ∈ s3)
| . . . | . . . . | |     ⇒
| . . . | . . . . | |     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | | 36. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . . . | |     & ¬L1@0-the_graph->*j)
| . . . | . . . . | | 37. i-the_graph->*j
| . . . | . . . . | | 38. ¬L1@0-the_graph->*j
| . . . | . . . . | | 39. j1 : Vertices(the_graph)
| . . . | . . . . | | 40. i-the_graph->*j1
| . . . | . . . . | | ⊢  L1-the_graph->*j1 ∨ i-the_graph->*j1
| . . . | . . . . | |   by ObviousFrom [-1]
| . . . | . . . . | |
| . . . | . . . . | |\
| . . . | . . . . | | 1. the_graph : Graph
| . . . | . . . . | | 2. GraphObject(the_graph)
| . . . | . . . . | | 3. Vertices(the_graph) List
| . . . | . . . . | | 4. L1 : Vertices(the_graph) List
| . . . | . . . . | | 5. s : traversal(the_graph)
| . . . | . . . . | | 6. i : Vertices(the_graph)
| . . . | . . . . | | 7. paren(Vertices(the_graph);s)
| . . . | . . . . | | 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . | |    Vertices(the_graph);s)
| . . . | . . . . | | 9. df-traversal(the_graph;s)
| . . . | . . . . | | 10. ∀i:Vertices(the_graph).
| . . . | . . . . | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . | | 11. ¬(inl(i) ∈ s)
| . . . | . . . . | | 12. s2 : traversal(the_graph)
| . . . | . . . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . | |     Vertices(the_graph);s2;s)

| . . . | . . . . | | 14. paren(Vertices(the_graph);s2)
| . . . | . . . . | | 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . | |     Vertices(the_graph);s2)
| . . . | . . . . | | 16. ¬(inr(i) ∈ s2)
| . . . | . . . . | | 17. ∀j:Vertices(the_graph).
| . . . | . . . . | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . | | 18. ∀i@0:Vertices(the_graph)
| . . . | . . . . | |     , s1,s2@0:traversal(the_graph).
```

```
| . . . | . . . . | |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | |     =
| . . . | . . . . | |     (s1 @ [inr(i@0)] @ s2@0)
| . . . | . . . . | |     ⇒
| . . . | . . . . | |     (∀j:Vertices(the_graph).
| . . . | . . . . | |      (inr(j) ∈ s2@0)
| . . . | . . . . | |      ⇒
| . . . | . . . . | |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . | . . . . | | 19. ∀i@0:Vertices(the_graph)
| . . . | . . . . | |     , s1,s2@0:traversal(the_graph).
| . . . | . . . . | |     (∀j:Vertices(the_graph).
| . . . | . . . . | |      i@0-the_graph->*j
| . . . | . . . . | |      ⇒
| . . . | . . . . | |      ¬non-trivial-loop(the_graph;j))
| . . . | . . . . | |     ⇒
| . . . | . . . . | |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | |     =
| . . . | . . . . | |     (s1 @ [inl(i@0)] @ s2@0)
| . . . | . . . . | |     ⇒
| . . . | . . . . | |     (∀j:Vertices(the_graph).
| . . . | . . . . | |      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . | . . . . | | 20. ∀i@0:Vertices(the_graph).
| . . . | . . . . | |     L1 @ [i]-the_graph->*i@0
| . . . | . . . . | |     ⇒
| . . . | . . . . | |     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . | | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . | |     L1 = (L1@0 @ L2)
| . . . | . . . . | |     ⇒
| . . . | . . . . | |     (∃s1,s2:traversal(the_graph).
| . . . | . . . . | |      s = (s2 @ s1)
| . . . | . . . . | |      & paren(Vertices(the_graph);s1)
| . . . | . . . . | |      & paren(Vertices(the_graph);s2)
| . . . | . . . . | |      & (∀j:Vertices(the_graph).
| . . . | . . . . | |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | |         & ((inl(j) ∈ s2)
| . . . | . . . . | |            ⟺
| . . . | . . . . | |            L2-the_graph->*j
| . . . | . . . . | |            & ¬L1@0-the_graph->*j)))
| . . . | . . . . | | 22. L1@0 : Vertices(the_graph) List
| . . . | . . . . | | 23. L2 : Vertices(the_graph) List
| . . . | . . . . | | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . | | 25. e : Vertices(the_graph) List
| . . . | . . . . | | 26. L1 = (L1@0 @ e)
| . . . | . . . . | | 27. L2 = (e @ [i])
| . . . | . . . . | | 28. s1 : traversal(the_graph)
| . . . | . . . . | | 29. s3 : traversal(the_graph)
| . . . | . . . . | | 30. s = (s3 @ s1)
| . . . | . . . . | | 31. paren(Vertices(the_graph);s1)
| . . . | . . . . | | 32. paren(Vertices(the_graph);s3)
| . . . | . . . . | | 33. ∀j:Vertices(the_graph).
| . . . | . . . . | |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | |     & ((inl(j) ∈ s3)
| . . . | . . . . | |        ⟺
| . . . | . . . . | |        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . | | 34. j : Vertices(the_graph)
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| . . . | . . . . | | 35. (inl(j) ∈ s3)
| . . . | . . . . | |     ⇒
| . . . | . . . . | |     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | | 36. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . . . | |     & ¬L1@0-the_graph->*j)
| . . . | . . . . | | 37. i-the_graph->*j
| . . . | . . . . | | 38. ¬L1@0-the_graph->*j
| . . . | . . . . | | ⊢  (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | |    =
| . . . | . . . . | |    (nil @ [inl(i)] @ (s2 @ [inr(i)]) @ s)
| . . . | . . . . | |   by Reduce 0
| . . . | . . . . | |
| . . . | . . . . | .\
| . . . | . . . . | . 1. the_graph : Graph
| . . . | . . . . | . 2. GraphObject(the_graph)
| . . . | . . . . | . 3. Vertices(the_graph) List
| . . . | . . . . | . 4. L1 : Vertices(the_graph) List
| . . . | . . . . | . 5. s : traversal(the_graph)
| . . . | . . . . | . 6. i : Vertices(the_graph)
| . . . | . . . . | . 7. paren(Vertices(the_graph);s)
| . . . | . . . . | . 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . |  Vertices(the_graph);s)
| . . . | . . . . | . 9. df-traversal(the_graph;s)
| . . . | . . . . | . 10. ∀i:Vertices(the_graph).
| . . . | . . . . | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . | . 11. ¬(inl(i) ∈ s)
| . . . | . . . . | . 12. s2 : traversal(the_graph)
| . . . | . . . . | . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . | . |   Vertices(the_graph);s2;s)
| . . . | . . . . | . 14. paren(Vertices(the_graph);s2)
| . . . | . . . . | . 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . |   Vertices(the_graph);s2)
| . . . | . . . . | . 16. ¬(inr(i) ∈ s2)
| . . . | . . . . | . 17. ∀j:Vertices(the_graph).
| . . . | . . . . | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . | . 18. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . |   , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . |   =
| . . . | . . . . | . |   (s1 @ [inr(i@0)] @ s2@0)
| . . . | . . . . | . |   ⇒
| . . . | . . . . | . |   (∀j:Vertices(the_graph).
| . . . | . . . . | . |    (inr(j) ∈ s2@0)
| . . . | . . . . | . |    ⇒
| . . . | . . . . | . |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . | . . . . | . 19. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . |   , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . |   (∀j:Vertices(the_graph).
| . . . | . . . . | . |    i@0-the_graph->*j
| . . . | . . . . | . |    ⇒
| . . . | . . . . | . |    ¬non-trivial-loop(the_graph;j))
| . . . | . . . . | . |   ⇒
| . . . | . . . . | . |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . |   =
| . . . | . . . . | . |   (s1 @ [inl(i@0)] @ s2@0)
```

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| . . . | . . . . | . |   ⇒
| . . . | . . . . | . |   (∀j:Vertices(the_graph).
| . . . | . . . . | . |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . | . . . . | . 20. ∀i@0:Vertices(the_graph).
| . . . | . . . . | . |   L1 @ [i]-the_graph->*i@0
| . . . | . . . . | . |   ⇒
| . . . | . . . . | . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . | . 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . | . |   L1 = (L1@0 @ L2)
| . . . | . . . . | . |   ⇒
| . . . | . . . . | . |   (∃s1,s2:traversal(the_graph).
| . . . | . . . . | . |    s = (s2 @ s1)
| . . . | . . . . | . |    & paren(Vertices(the_graph);s1)
| . . . | . . . . | . |    & paren(Vertices(the_graph);s2)
| . . . | . . . . | . |    & (∀j:Vertices(the_graph).
| . . . | . . . . | . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | . |       & ((inl(j) ∈ s2)
| . . . | . . . . | . |          ⟺
| . . . | . . . . | . |          L2-the_graph->*j
| . . . | . . . . | . |          & ¬L1@0-the_graph->*j)))
| . . . | . . . . | . 22. L1@0 : Vertices(the_graph) List
| . . . | . . . . | . 23. L2 : Vertices(the_graph) List
| . . . | . . . . | . 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . | . 25. e : Vertices(the_graph) List
| . . . | . . . . | . 26. L1 = (L1@0 @ e)
| . . . | . . . . | . 27. L2 = (e @ [i])
| . . . | . . . . | . 28. s1 : traversal(the_graph)
| . . . | . . . . | . 29. s3 : traversal(the_graph)
| . . . | . . . . | . 30. s = (s3 @ s1)
| . . . | . . . . | . 31. paren(Vertices(the_graph);s1)
| . . . | . . . . | . 32. paren(Vertices(the_graph);s3)
| . . . | . . . . | . 33. ∀j:Vertices(the_graph).
| . . . | . . . . | . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | . |   & ((inl(j) ∈ s3)
| . . . | . . . . | . |      ⟺
| . . . | . . . . | . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . | . 34. j : Vertices(the_graph)
| . . . | . . . . | . 35. (inl(j) ∈ s3)
| . . . | . . . . | . |   ⇒
| . . . | . . . . | . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | . 36. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . . . | . |   & ¬L1@0-the_graph->*j)
| . . . | . . . . | . 37. i-the_graph->*j
| . . . | . . . . | . 38. ¬L1@0-the_graph->*j
| . . . | . . . . | . 39. (inr(j) ∈ (s2 @ [inr(i)]) @ s)
| . . . | . . . . | . ⊢  (inl(j) ∈ s2) ∨ (inl(j) ∈ s)
| . . . | . . . . | . |  ∨ inl(j) = inl(i)
| . . . | . . . . | . | by ((RWW
| . . . | . . . . | . |      Thm* ∀x:T, l1,l2:T List.
| . . . | . . . . | . |           (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| . . . | . . . . | . |      {member_append}
| . . . | . . . . | . |      -1)
| . . . | . . . . | . |     THEN
| . . . | . . . . | . |     (RWO
| . . . | . . . . | . |      Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a
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| . . . | . . . . | . |      {member_singleton}
| . . . | . . . . | . |      -1))
| . . . | . . . . | . |    THEN
| . . . | . . . . | . |    (Analyze -1)
| . . . | . . . . | . |
| . . . | . . . . | . |\
| . . . | . . . . | . | 1. the_graph : Graph
| . . . | . . . . | . | 2. GraphObject(the_graph)
| . . . | . . . . | . | 3. Vertices(the_graph) List
| . . . | . . . . | . | 4. L1 : Vertices(the_graph) List
| . . . | . . . . | . | 5. s : traversal(the_graph)
| . . . | . . . . | . | 6. i : Vertices(the_graph)
| . . . | . . . . | . | 7. paren(Vertices(the_graph);s)
| . . . | . . . . | . | 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . | |  Vertices(the_graph);s)
| . . . | . . . . | . | 9. df-traversal(the_graph;s)
| . . . | . . . . | . | 10. ∀i:Vertices(the_graph).
| . . . | . . . . | . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . | . | 11. ¬(inl(i) ∈ s)
| . . . | . . . . | . | 12. s2 : traversal(the_graph)
| . . . | . . . . | . | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . | . | |   Vertices(the_graph);s2;s)
| . . . | . . . . | . | 14. paren(Vertices(the_graph);s2)
| . . . | . . . . | . | 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . | |   Vertices(the_graph);s2)
| . . . | . . . . | . | 16. ¬(inr(i) ∈ s2)
| . . . | . . . . | . | 17. ∀j:Vertices(the_graph).
| . . . | . . . . | . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . | . | 18. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . | |   , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . | |   =
| . . . | . . . . | . | |   (s1 @ [inr(i@0)] @ s2@0)
| . . . | . . . . | . | |   ⇒
| . . . | . . . . | . | |   (∀j:Vertices(the_graph).
| . . . | . . . . | . | |    (inr(j) ∈ s2@0)
| . . . | . . . . | . | |    ⇒
| . . . | . . . . | . | |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . | . . . . | . | 19. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . | |   , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . | |   (∀j:Vertices(the_graph).
| . . . | . . . . | . | |    i@0-the_graph->*j
| . . . | . . . . | . | |    ⇒
| . . . | . . . . | . | |    ¬non-trivial-loop(the_graph;j))
| . . . | . . . . | . | |   ⇒
| . . . | . . . . | . | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . | |   =
| . . . | . . . . | . | |   (s1 @ [inl(i@0)] @ s2@0)
| . . . | . . . . | . | |   ⇒
| . . . | . . . . | . | |   (∀j:Vertices(the_graph).
| . . . | . . . . | . | |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . | . . . . | . | 20. ∀i@0:Vertices(the_graph).
| . . . | . . . . | . | |   L1 @ [i]-the_graph->*i@0
| . . . | . . . . | . | |   ⇒
| . . . | . . . . | . | |   ¬non-trivial-loop(the_graph;i@0)
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| . . . | . . . . | . | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . | . | |   L1 = (L1@0 @ L2)
| . . . | . . . . | . | |   ⇒
| . . . | . . . . | . | |   (∃s1,s2:traversal(the_graph).
| . . . | . . . . | . | |    s = (s2 @ s1)
| . . . | . . . . | . | |    & paren(Vertices(the_graph);s1)
| . . . | . . . . | . | |    & paren(Vertices(the_graph);s2)
| . . . | . . . . | . | |    & (∀j:Vertices(the_graph).
| . . . | . . . . | . | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | . | |       & ((inl(j) ∈ s2)
| . . . | . . . . | . | |          ⟺
| . . . | . . . . | . | |          L2-the_graph->*j
| . . . | . . . . | . | |          & ¬L1@0-the_graph->*j)))
| . . . | . . . . | . | 22. L1@0 : Vertices(the_graph) List
| . . . | . . . . | . | 23. L2 : Vertices(the_graph) List
| . . . | . . . . | . | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . | . | 25. e : Vertices(the_graph) List
| . . . | . . . . | . | 26. L1 = (L1@0 @ e)
| . . . | . . . . | . | 27. L2 = (e @ [i])
| . . . | . . . . | . | 28. s1 : traversal(the_graph)
| . . . | . . . . | . | 29. s3 : traversal(the_graph)
| . . . | . . . . | . | 30. s = (s3 @ s1)
| . . . | . . . . | . | 31. paren(Vertices(the_graph);s1)
| . . . | . . . . | . | 32. paren(Vertices(the_graph);s3)
| . . . | . . . . | . | 33. ∀j:Vertices(the_graph).
| . . . | . . . . | . | |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | . | |   & ((inl(j) ∈ s3)
| . . . | . . . . | . | |      ⟺
| . . . | . . . . | . | |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . | . | 34. j : Vertices(the_graph)
| . . . | . . . . | . | 35. (inl(j) ∈ s3)
| . . . | . . . . | . | |   ⇒
| . . . | . . . . | . | |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | . | 36. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . . . | . | |   & ¬L1@0-the_graph->*j)
| . . . | . . . . | . | 37. i-the_graph->*j
| . . . | . . . . | . | 38. ¬L1@0-the_graph->*j
| . . . | . . . . | . | 39. (inr(j) ∈ s2) ∨ inr(j) = inr(i)
| . . . | . . . . | . | ⊢  (inl(j) ∈ s2) ∨ (inl(j) ∈ s)
| . . . | . . . . | . | |  ∨ inl(j) = inl(i)  by Analyze -1
| . . . | . . . . | . | |
| . . . | . . . . | . | |\
| . . . | . . . . | . | | 1. the_graph : Graph
| . . . | . . . . | . | | 2. GraphObject(the_graph)
| . . . | . . . . | . | | 3. Vertices(the_graph) List
| . . . | . . . . | . | | 4. L1 : Vertices(the_graph) List
| . . . | . . . . | . | | 5. s : traversal(the_graph)
| . . . | . . . . | . | | 6. i : Vertices(the_graph)
| . . . | . . . . | . | | 7. paren(Vertices(the_graph);s)
| . . . | . . . . | . | | 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . | | |  Vertices(the_graph);s)
| . . . | . . . . | . | | 9. df-traversal(the_graph;s)
| . . . | . . . . | . | | 10. ∀i:Vertices(the_graph).
| . . . | . . . . | . | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . | . | | 11. ¬(inl(i) ∈ s)
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| . . . | . . . . | . | | 12. s2 : traversal(the_graph)
| . . . | . . . . | . | | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . | . | | |   Vertices(the_graph);s2;s)
| . . . | . . . . | . | | 14. paren(Vertices(the_graph);s2)
| . . . | . . . . | . | | 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . | | |   Vertices(the_graph);s2)
| . . . | . . . . | . | | 16. ¬(inr(i) ∈ s2)
| . . . | . . . . | . | | 17. ∀j:Vertices(the_graph).
| . . . | . . . . | . | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . | . | | 18. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . | | |   , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . | | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . | | |   =
| . . . | . . . . | . | | |   (s1 @ [inr(i@0)] @ s2@0)
| . . . | . . . . | . | | |   ⇒
| . . . | . . . . | . | | |   (∀j:Vertices(the_graph).
| . . . | . . . . | . | | |    (inr(j) ∈ s2@0)
| . . . | . . . . | . | | |    ⇒
| . . . | . . . . | . | | |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . | . . . . | . | | 19. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . | | |   , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . | | |   (∀j:Vertices(the_graph).
| . . . | . . . . | . | | |    i@0-the_graph->*j
| . . . | . . . . | . | | |    ⇒
| . . . | . . . . | . | | |    ¬non-trivial-loop(the_graph;j))
| . . . | . . . . | . | | |   ⇒
| . . . | . . . . | . | | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . | | |   =
| . . . | . . . . | . | | |   (s1 @ [inl(i@0)] @ s2@0)
| . . . | . . . . | . | | |   ⇒
| . . . | . . . . | . | | |   (∀j:Vertices(the_graph).
| . . . | . . . . | . | | |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . | . . . . | . | | 20. ∀i@0:Vertices(the_graph).
| . . . | . . . . | . | | |   L1 @ [i]-the_graph->*i@0
| . . . | . . . . | . | | |   ⇒
| . . . | . . . . | . | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . | . | | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . | . | | |   L1 = (L1@0 @ L2)
| . . . | . . . . | . | | |   ⇒
| . . . | . . . . | . | | |   (∃s1,s2:traversal(the_graph).
| . . . | . . . . | . | | |    s = (s2 @ s1)
| . . . | . . . . | . | | |    & paren(Vertices(the_graph);s1)
| . . . | . . . . | . | | |    & paren(Vertices(the_graph);s2)
| . . . | . . . . | . | | |    & (∀j:Vertices(the_graph).
| . . . | . . . . | . | | |        ...
| . . . | . . . . | . | | |       & ((inl(j) ∈ s2)
| . . . | . . . . | . | | |          ⟺
| . . . | . . . . | . | | |          L2-the_graph->*j
| . . . | . . . . | . | | |          & ¬L1@0-the_graph->*j)))
| . . . | . . . . | . | | 22. L1@0 : Vertices(the_graph) List
| . . . | . . . . | . | | 23. L2 : Vertices(the_graph) List
| . . . | . . . . | . | | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . | . | | 25. e : Vertices(the_graph) List
| . . . | . . . . | . | | 26. L1 = (L1@0 @ e)
| . . . | . . . . | . | | 27. L2 = (e @ [i])
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| . . . | . . . . | . | | 28. s1 : traversal(the_graph)
| . . . | . . . . | . | | 29. s3 : traversal(the_graph)
| . . . | . . . . | . | | 30. s = (s3 @ s1)
| . . . | . . . . | . | | 31. paren(Vertices(the_graph);s1)
| . . . | . . . . | . | | 32. paren(Vertices(the_graph);s3)
| . . . | . . . . | . | | 33. ∀j:Vertices(the_graph).
| . . . | . . . . | . | | |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | . | | |   & ...
| . . . | . . . . | . | | 34. j : Vertices(the_graph)
| . . . | . . . . | . | | 35. (inl(j) ∈ s3)
| . . . | . . . . | . | | |   ⇒
| . . . | . . . . | . | | |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | . | | 36. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . . . | . | | |   & ¬L1@0-the_graph->*j)
| . . . | . . . . | . | | 37. i-the_graph->*j
| . . . | . . . . | . | | 38. ¬L1@0-the_graph->*j
| . . . | . . . . | . | | 39. (inr(j) ∈ s2)
| . . . | . . . . | . | | ⊢  (inl(j) ∈ s2) ∨ (inl(j) ∈ s)
| . . . | . . . . | . | | |  ∨ inl(j) = inl(i)
| . . . | . . . . | . | | | by FwdThru
| . . . | . . . . | . | | |    Thm* ∀s':(T+T) List.
| . . . | . . . . | . | | |         paren(T;s')
| . . . | . . . . | . | | |         ⇒
| . . . | . . . . | . | | |         (∀i:T.
| . . . | . . . . | . | | |          (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
| . . . | . . . . | . | | |    {paren_balance2}
| . . . | . . . . | . | | |    [14;-1]
| . . . | . . . . | . | | |
| . . . | . . . . | . | | 1. the_graph : Graph
| . . . | . . . . | . | | 2. GraphObject(the_graph)
| . . . | . . . . | . | | 3. Vertices(the_graph) List
| . . . | . . . . | . | | 4. L1 : Vertices(the_graph) List
| . . . | . . . . | . | | 5. s : traversal(the_graph)
| . . . | . . . . | . | | 6. i : Vertices(the_graph)
| . . . | . . . . | . | | 7. paren(Vertices(the_graph);s)
| . . . | . . . . | . | | 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . | |    Vertices(the_graph);s)
| . . . | . . . . | . | | 9. df-traversal(the_graph;s)
| . . . | . . . . | . | | 10. ∀i:Vertices(the_graph).
| . . . | . . . . | . | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . | . | | 11. ¬(inl(i) ∈ s)
| . . . | . . . . | . | | 12. s2 : traversal(the_graph)
| . . . | . . . . | . | | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . | . | |     Vertices(the_graph);s2;s)
| . . . | . . . . | . | | 14. paren(Vertices(the_graph);s2)
| . . . | . . . . | . | | 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . | |     Vertices(the_graph);s2)
| . . . | . . . . | . | | 16. ¬(inr(i) ∈ s2)
| . . . | . . . . | . | | 17. ∀j:Vertices(the_graph).
| . . . | . . . . | . | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . | . | | 18. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . | |     , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . | |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . | |     =
| . . . | . . . . | . | |     (s1 @ [inr(i@0)] @ s2@0)
```

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| . . . | . . . . | . | |     ⇒
| . . . | . . . . | . | |     (∀j:Vertices(the_graph).
| . . . | . . . . | . | |      (inr(j) ∈ s2@0)
| . . . | . . . . | . | |      ⇒
| . . . | . . . . | . | |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . | . . . . | . | | 19. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . | |     , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . | |     (∀j:Vertices(the_graph).
| . . . | . . . . | . | |      i@0-the_graph->*j
| . . . | . . . . | . | |      ⇒
| . . . | . . . . | . | |      ¬non-trivial-loop(the_graph;j))
| . . . | . . . . | . | |     ⇒
| . . . | . . . . | . | |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . | |     =
| . . . | . . . . | . | |     (s1 @ [inl(i@0)] @ s2@0)
| . . . | . . . . | . | |     ⇒
| . . . | . . . . | . | |     (∀j:Vertices(the_graph).
| . . . | . . . . | . | |      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . | . . . . | . | | 20. ∀i@0:Vertices(the_graph).
| . . . | . . . . | . | |     L1 @ [i]-the_graph->*i@0
| . . . | . . . . | . | |     ⇒
| . . . | . . . . | . | |     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . | . | | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . | . | |     L1 = (L1@0 @ L2)
| . . . | . . . . | . | |     ⇒
| . . . | . . . . | . | |     (∃s1,s2:traversal(the_graph).
| . . . | . . . . | . | |      s = (s2 @ s1)
| . . . | . . . . | . | |      & paren(Vertices(the_graph);s1)
| . . . | . . . . | . | |      & paren(Vertices(the_graph);s2)
| . . . | . . . . | . | |      & (∀j:Vertices(the_graph).
| . . . | . . . . | . | |          ...
| . . . | . . . . | . | |          & ((inl(j) ∈ s2)
| . . . | . . . . | . | |             ⟺
| . . . | . . . . | . | |             L2-the_graph->*j
| . . . | . . . . | . | |             & ¬L1@0-the_graph->*j)))
| . . . | . . . . | . | | 22. L1@0 : Vertices(the_graph) List
| . . . | . . . . | . | | 23. L2 : Vertices(the_graph) List
| . . . | . . . . | . | | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . | . | | 25. e : Vertices(the_graph) List
| . . . | . . . . | . | | 26. L1 = (L1@0 @ e)
| . . . | . . . . | . | | 27. L2 = (e @ [i])
| . . . | . . . . | . | | 28. s1 : traversal(the_graph)
| . . . | . . . . | . | | 29. s3 : traversal(the_graph)
| . . . | . . . . | . | | 30. s = (s3 @ s1)
| . . . | . . . . | . | | 31. paren(Vertices(the_graph);s1)
| . . . | . . . . | . | | 32. paren(Vertices(the_graph);s3)
| . . . | . . . . | . | | 33. ∀j:Vertices(the_graph).
| . . . | . . . . | . | |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | . | |     & ...
| . . . | . . . . | . | | 34. j : Vertices(the_graph)
| . . . | . . . . | . | | 35. (inl(j) ∈ s3)
| . . . | . . . . | . | |     ⇒
| . . . | . . . . | . | |     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | . | | 36. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . . . | . | |     & ¬L1@0-the_graph->*j)
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| . . . | . . . . | . | | 37. i-the_graph->*j
| . . . | . . . . | . | | 38. ¬L1@0-the_graph->*j
| . . . | . . . . | . | | 39. (inr(j) ∈ s2)
| . . . | . . . . | . | | 40. (inl(j) ∈ s2)
| . . . | . . . . | . | | ⊢  (inl(j) ∈ s2) ∨ (inl(j) ∈ s)
| . . . | . . . . | . | |    ∨ inl(j) = inl(i)  by ObviousFrom [-1]
| . . . | . . . . | . | |
| . . . | . . . . | . | .\
| . . . | . . . . | . | . 1. the_graph : Graph
| . . . | . . . . | . | . 2. GraphObject(the_graph)
| . . . | . . . . | . | . 3. Vertices(the_graph) List
| . . . | . . . . | . | . 4. L1 : Vertices(the_graph) List
| . . . | . . . . | . | . 5. s : traversal(the_graph)
| . . . | . . . . | . | . 6. i : Vertices(the_graph)
| . . . | . . . . | . | . 7. paren(Vertices(the_graph);s)
| . . . | . . . . | . | . 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . | .    Vertices(the_graph);s)
| . . . | . . . . | . | . 9. df-traversal(the_graph;s)
| . . . | . . . . | . | . 10. ∀i:Vertices(the_graph).
| . . . | . . . . | . | .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . | . | . 11. ¬(inl(i) ∈ s)
| . . . | . . . . | . | . 12. s2 : traversal(the_graph)
| . . . | . . . . | . | . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . | . | .     Vertices(the_graph);s2;s)
| . . . | . . . . | . | . 14. paren(Vertices(the_graph);s2)
| . . . | . . . . | . | . 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . | .     Vertices(the_graph);s2)
| . . . | . . . . | . | . 16. ¬(inr(i) ∈ s2)
| . . . | . . . . | . | . 17. ∀j:Vertices(the_graph).
| . . . | . . . . | . | .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . | . | . 18. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . | .     , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . | .     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . | .     =
| . . . | . . . . | . | .     (s1 @ [inr(i@0)] @ s2@0)
| . . . | . . . . | . | .     ⇒
| . . . | . . . . | . | .     (∀j:Vertices(the_graph).
| . . . | . . . . | . | .      (inr(j) ∈ s2@0)
| . . . | . . . . | . | .      ⇒
| . . . | . . . . | . | .      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . | . . . . | . | . 19. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . | .     , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . | .     (∀j:Vertices(the_graph).
| . . . | . . . . | . | .      i@0-the_graph->*j
| . . . | . . . . | . | .      ⇒
| . . . | . . . . | . | .      ¬non-trivial-loop(the_graph;j))
| . . . | . . . . | . | .     ⇒
| . . . | . . . . | . | .     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . | .     =
| . . . | . . . . | . | .     (s1 @ [inl(i@0)] @ s2@0)
| . . . | . . . . | . | .     ⇒
| . . . | . . . . | . | .     (∀j:Vertices(the_graph).
| . . . | . . . . | . | .      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . | . . . . | . | . 20. ∀i@0:Vertices(the_graph).
| . . . | . . . . | . | .     L1 @ [i]-the_graph->*i@0
```

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| . . . | . . . . | . | .     ⇒
| . . . | . . . . | . | .     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . | . | . 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . | . | .     L1 = (L1@0 @ L2)
| . . . | . . . . | . | .     ⇒
| . . . | . . . . | . | .     (∃s1,s2:traversal(the_graph).
| . . . | . . . . | . | .      s = (s2 @ s1)
| . . . | . . . . | . | .      & paren(Vertices(the_graph);s1)
| . . . | . . . . | . | .      & paren(Vertices(the_graph);s2)
| . . . | . . . . | . | .      & (∀j:Vertices(the_graph).
| . . . | . . . . | . | .         ...
| . . . | . . . . | . | .         & ((inl(j) ∈ s2)
| . . . | . . . . | . | .            ⟺
| . . . | . . . . | . | .            L2-the_graph->*j
| . . . | . . . . | . | .            & ¬L1@0-the_graph->*j)))
| . . . | . . . . | . | . 22. L1@0 : Vertices(the_graph) List
| . . . | . . . . | . | . 23. L2 : Vertices(the_graph) List
| . . . | . . . . | . | . 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . | . | . 25. e : Vertices(the_graph) List
| . . . | . . . . | . | . 26. L1 = (L1@0 @ e)
| . . . | . . . . | . | . 27. L2 = (e @ [i])
| . . . | . . . . | . | . 28. s1 : traversal(the_graph)
| . . . | . . . . | . | . 29. s3 : traversal(the_graph)
| . . . | . . . . | . | . 30. s = (s3 @ s1)
| . . . | . . . . | . | . 31. paren(Vertices(the_graph);s1)
| . . . | . . . . | . | . 32. paren(Vertices(the_graph);s3)
| . . . | . . . . | . | . 33. ∀j:Vertices(the_graph).
| . . . | . . . . | . | .     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | . | .     & ...
| . . . | . . . . | . | . 34. j : Vertices(the_graph)
| . . . | . . . . | . | . 35. (inl(j) ∈ s3)
| . . . | . . . . | . | .     ⇒
| . . . | . . . . | . | .     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | . | . 36. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . . . | . | .     & ¬L1@0-the_graph->*j)
| . . . | . . . . | . | . 37. i-the_graph->*j
| . . . | . . . . | . | . 38. ¬L1@0-the_graph->*j
| . . . | . . . . | . | . 39. inr(j) = inr(i)
| . . . | . . . . | . | . ⊢  (inl(j) ∈ s2) ∨ (inl(j) ∈ s)
| . . . | . . . . | . | .    ∨ inl(j) = inl(i)  by ObviousFrom [-1]
| . . . | . . . . | . |
| . . . | . . . . | . .\
| . . . | . . . . | . . 1. the_graph : Graph
| . . . | . . . . | . . 2. GraphObject(the_graph)
| . . . | . . . . | . . 3. Vertices(the_graph) List
| . . . | . . . . | . . 4. L1 : Vertices(the_graph) List
| . . . | . . . . | . . 5. s : traversal(the_graph)
| . . . | . . . . | . . 6. i : Vertices(the_graph)
| . . . | . . . . | . . 7. paren(Vertices(the_graph);s)
| . . . | . . . . | . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . . |  Vertices(the_graph);s)
| . . . | . . . . | . . 9. df-traversal(the_graph;s)
| . . . | . . . . | . . 10. ∀i:Vertices(the_graph).
| . . . | . . . . | . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . | . . 11. ¬(inl(i) ∈ s)
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| . . . | . . . . | . . 12. s2 : traversal(the_graph)
| . . . | . . . . | . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . | . . |   Vertices(the_graph);s2;s)
| . . . | . . . . | . . 14. paren(Vertices(the_graph);s2)
| . . . | . . . . | . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . . |   Vertices(the_graph);s2)
| . . . | . . . . | . . 16. ¬(inr(i) ∈ s2)
| . . . | . . . . | . . 17. ∀j:Vertices(the_graph).
| . . . | . . . . | . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . | . . 18. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . . |   , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . . |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . . |   =
| . . . | . . . . | . . |   (s1 @ [inr(i@0)] @ s2@0)
| . . . | . . . . | . . |   ⇒
| . . . | . . . . | . . |   (∀j:Vertices(the_graph).
| . . . | . . . . | . . |    (inr(j) ∈ s2@0)
| . . . | . . . . | . . |    ⇒
| . . . | . . . . | . . |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . | . . . . | . . 19. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . . |   , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . . |   (∀j:Vertices(the_graph).
| . . . | . . . . | . . |    i@0-the_graph->*j
| . . . | . . . . | . . |    ⇒
| . . . | . . . . | . . |    ¬non-trivial-loop(the_graph;j))
| . . . | . . . . | . . |   ⇒
| . . . | . . . . | . . |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . . |   =
| . . . | . . . . | . . |   (s1 @ [inl(i@0)] @ s2@0)
| . . . | . . . . | . . |   ⇒
| . . . | . . . . | . . |   (∀j:Vertices(the_graph).
| . . . | . . . . | . . |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . | . . . . | . . 20. ∀i@0:Vertices(the_graph).
| . . . | . . . . | . . |   L1 @ [i]-the_graph->*i@0
| . . . | . . . . | . . |   ⇒
| . . . | . . . . | . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . | . . 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . | . . |   L1 = (L1@0 @ L2)
| . . . | . . . . | . . |   ⇒
| . . . | . . . . | . . |   (∃s1,s2:traversal(the_graph).
| . . . | . . . . | . . |    s = (s2 @ s1)
| . . . | . . . . | . . |    & paren(Vertices(the_graph);s1)

| . . . | . . . . | . . |    & paren(Vertices(the_graph);s2)
| . . . | . . . . | . . |    & (∀j:Vertices(the_graph).
| . . . | . . . . | . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | . . |       & ((inl(j) ∈ s2)
| . . . | . . . . | . . |          ⟺
| . . . | . . . . | . . |          L2-the_graph->*j
| . . . | . . . . | . . |          & ¬L1@0-the_graph->*j)))
| . . . | . . . . | . . 22. L1@0 : Vertices(the_graph) List
| . . . | . . . . | . . 23. L2 : Vertices(the_graph) List
| . . . | . . . . | . . 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . | . . 25. e : Vertices(the_graph) List
| . . . | . . . . | . . 26. L1 = (L1@0 @ e)
| . . . | . . . . | . . 27. L2 = (e @ [i])
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| . . . | . . . . | . . 28. s1 : traversal(the_graph)
| . . . | . . . . | . . 29. s3 : traversal(the_graph)
| . . . | . . . . | . . 30. s = (s3 @ s1)
| . . . | . . . . | . . 31. paren(Vertices(the_graph);s1)
| . . . | . . . . | . . 32. paren(Vertices(the_graph);s3)
| . . . | . . . . | . . 33. ∀j:Vertices(the_graph).
| . . . | . . . . | . . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | . . |   & ((inl(j) ∈ s3)
| . . . | . . . . | . . |      ⟺
| . . . | . . . . | . . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . | . . 34. j : Vertices(the_graph)
| . . . | . . . . | . . 35. (inl(j) ∈ s3)
| . . . | . . . . | . . |   ⇒
| . . . | . . . . | . . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | . . 36. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . . . | . . |   & ¬L1@0-the_graph->*j)
| . . . | . . . . | . . 37. i-the_graph->*j
| . . . | . . . . | . . 38. ¬L1@0-the_graph->*j
| . . . | . . . . | . . 39. (inr(j) ∈ s)
| . . . | . . . . | . . ⊢  (inl(j) ∈ s2) ∨ (inl(j) ∈ s)
| . . . | . . . . | . . |  ∨ inl(j) = inl(i)
| . . . | . . . . | . . | by FwdThru
| . . . | . . . . | . . |    Thm* ∀s':(T+T) List.
| . . . | . . . . | . . |         paren(T;s')
| . . . | . . . . | . . |         ⇒
| . . . | . . . . | . . |         (∀i:T.
| . . . | . . . . | . . |           (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
| . . . | . . . . | . . |    {paren_balance2}
| . . . | . . . . | . . |    [7;-1]
| . . . | . . . . | . . |
| . . . | . . . . | . . 1. the_graph : Graph
| . . . | . . . . | . . 2. GraphObject(the_graph)
| . . . | . . . . | . . 3. Vertices(the_graph) List
| . . . | . . . . | . . 4. L1 : Vertices(the_graph) List
| . . . | . . . . | . . 5. s : traversal(the_graph)
| . . . | . . . . | . . 6. i : Vertices(the_graph)
| . . . | . . . . | . . 7. paren(Vertices(the_graph);s)
| . . . | . . . . | . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . .    Vertices(the_graph);s)
| . . . | . . . . | . . 9. df-traversal(the_graph;s)
| . . . | . . . . | . . 10. ∀i:Vertices(the_graph).
| . . . | . . . . | . .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . | . . 11. ¬(inl(i) ∈ s)
| . . . | . . . . | . . 12. s2 : traversal(the_graph)
| . . . | . . . . | . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . | . .     Vertices(the_graph);s2;s)
| . . . | . . . . | . . 14. paren(Vertices(the_graph);s2)
| . . . | . . . . | . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . | . .     Vertices(the_graph);s2)
| . . . | . . . . | . . 16. ¬(inr(i) ∈ s2)
| . . . | . . . . | . . 17. ∀j:Vertices(the_graph).
| . . . | . . . . | . .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . | . . 18. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . .     , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . .     (([inl(i)] @ s2 @ [inr(i)]) @ s)
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| . . . | . . . . | . .      =
| . . . | . . . . | . .      (s1 @ [inr(i@0)] @ s2@0)
| . . . | . . . . | . .      ⇒
| . . . | . . . . | . .      (∀j:Vertices(the_graph).
| . . . | . . . . | . .       (inr(j) ∈ s2@0)
| . . . | . . . . | . .       ⇒
| . . . | . . . . | . .       ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . | . . . . | . . 19. ∀i@0:Vertices(the_graph)
| . . . | . . . . | . .      , s1,s2@0:traversal(the_graph).
| . . . | . . . . | . .      (∀j:Vertices(the_graph).
| . . . | . . . . | . .       i@0-the_graph->*j
| . . . | . . . . | . .       ⇒
| . . . | . . . . | . .       ¬non-trivial-loop(the_graph;j))
| . . . | . . . . | . .      ⇒
| . . . | . . . . | . .      (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . | . . . . | . .      =
| . . . | . . . . | . .      (s1 @ [inl(i@0)] @ s2@0)
| . . . | . . . . | . .      ⇒
| . . . | . . . . | . .      (∀j:Vertices(the_graph).
| . . . | . . . . | . .       i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . | . . . . | . . 20. ∀i@0:Vertices(the_graph).
| . . . | . . . . | . .      L1 @ [i]-the_graph->*i@0
| . . . | . . . . | . .      ⇒
| . . . | . . . . | . .      ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . | . . 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . | . .      L1 = (L1@0 @ L2)
| . . . | . . . . | . .      ⇒
| . . . | . . . . | . .      (∃s1,s2:traversal(the_graph).
| . . . | . . . . | . .       s = (s2 @ s1)
| . . . | . . . . | . .       & paren(Vertices(the_graph);s1)
| . . . | . . . . | . .       & paren(Vertices(the_graph);s2)
| . . . | . . . . | . .       & (∀j:Vertices(the_graph).
| . . . | . . . . | . .           ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | . .           & ((inl(j) ∈ s2)
| . . . | . . . . | . .              ⟺
| . . . | . . . . | . .              L2-the_graph->*j
| . . . | . . . . | . .              & ¬L1@0-the_graph->*j)))
| . . . | . . . . | . . 22. L1@0 : Vertices(the_graph) List
| . . . | . . . . | . . 23. L2 : Vertices(the_graph) List
| . . . | . . . . | . . 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . | . . 25. e : Vertices(the_graph) List
| . . . | . . . . | . . 26. L1 = (L1@0 @ e)
| . . . | . . . . | . . 27. L2 = (e @ [i])
| . . . | . . . . | . . 28. s1 : traversal(the_graph)
| . . . | . . . . | . . 29. s3 : traversal(the_graph)
| . . . | . . . . | . . 30. s = (s3 @ s1)
| . . . | . . . . | . . 31. paren(Vertices(the_graph);s1)
| . . . | . . . . | . . 32. paren(Vertices(the_graph);s3)
| . . . | . . . . | . . 33. ∀j:Vertices(the_graph).
| . . . | . . . . | . .      ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . | . .      & ((inl(j) ∈ s3)
| . . . | . . . . | . .         ⟺
| . . . | . . . . | . .         e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . | . . 34. j : Vertices(the_graph)
| . . . | . . . . | . . 35. (inl(j) ∈ s3)
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| . . . | . . . . | . .     ⇒
| . . . | . . . . | . .     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . | . . 36. (inl(j) ∈ s3) ⇐ (e-the_graph->*j
| . . . | . . . . | . .     & ¬L1@0-the_graph->*j)
| . . . | . . . . | . . 37. i-the_graph->*j
| . . . | . . . . | . . 38. ¬L1@0-the_graph->*j
| . . . | . . . . | . . 39. (inr(j) ∈ s)
| . . . | . . . . | . . 40. (inl(j) ∈ s)
| . . . | . . . . | . . ⊢  (inl(j) ∈ s2) ∨ (inl(j) ∈ s)
| . . . | . . . . | . .    ∨ inl(j) = inl(i)  by ObviousFrom [-1]
| . . . | . . . . |
| . . . | . . . . .\
| . . . | . . . . . 1. the_graph : Graph
| . . . | . . . . . 2. GraphObject(the_graph)
| . . . | . . . . . 3. Vertices(the_graph) List
| . . . | . . . . . 4. L1 : Vertices(the_graph) List
| . . . | . . . . . 5. s : traversal(the_graph)
| . . . | . . . . . 6. i : Vertices(the_graph)
| . . . | . . . . . 7. paren(Vertices(the_graph);s)
| . . . | . . . . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . . |  Vertices(the_graph);s)
| . . . | . . . . . 9. df-traversal(the_graph;s)
| . . . | . . . . . 10. ∀i:Vertices(the_graph).
| . . . | . . . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . . 11. ¬(inl(i) ∈ s)
| . . . | . . . . . 12. s2 : traversal(the_graph)
| . . . | . . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . . |   Vertices(the_graph);s2;s)
| . . . | . . . . . 14. paren(Vertices(the_graph);s2)
| . . . | . . . . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . . |   Vertices(the_graph);s2)
| . . . | . . . . . 16. ¬(inr(i) ∈ s2)
| . . . | . . . . . 17. ∀j:Vertices(the_graph).
| . . . | . . . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . . . . |                           @ s2
| . . . | . . . . . |                           @ [inr(i)])
| . . . | . . . . . |   @ s)
| . . . | . . . . . 19. ∀i@0:Vertices(the_graph).
| . . . | . . . . . |   L1 @ [i]-the_graph->*i@0
| . . . | . . . . . |   ⇒
| . . . | . . . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . . |   L1 = (L1@0 @ L2)
| . . . | . . . . . |   ⇒
| . . . | . . . . . |   (∃s1,s2:traversal(the_graph).
| . . . | . . . . . |    s = (s2 @ s1)
| . . . | . . . . . |    & paren(Vertices(the_graph);s1)
| . . . | . . . . . |    & paren(Vertices(the_graph);s2)
| . . . | . . . . . |    & ...)
| . . . | . . . . . 21. L1@0 : Vertices(the_graph) List
| . . . | . . . . . 22. L2 : Vertices(the_graph) List
| . . . | . . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . . 24. e : Vertices(the_graph) List
| . . . | . . . . . 25. L1 = (L1@0 @ e)
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| . . . | . . . . . 26. L2 = (e @ [i])
| . . . | . . . . . 27. s1 : traversal(the_graph)
| . . . | . . . . . 28. s3 : traversal(the_graph)
| . . . | . . . . . 29. s = (s3 @ s1)
| . . . | . . . . . 30. paren(Vertices(the_graph);s1)
| . . . | . . . . . 31. paren(Vertices(the_graph);s3)
| . . . | . . . . . 32. ∀j:Vertices(the_graph).
| . . . | . . . . . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . . |   & ((inl(j) ∈ s3)
| . . . | . . . . . |      ⟺
| . . . | . . . . . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . . 33. j : Vertices(the_graph)
| . . . | . . . . . 34. (inl(j) ∈ s3)
| . . . | . . . . . |   ⟺
| . . . | . . . . . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . . 35. [i]-the_graph->*j
| . . . | . . . . . 36. ¬L1@0-the_graph->*j
| . . . | . . . . . 37. (inl(j) ∈ s2) ∨ (inl(j) ∈ s) ∨ inl(j) = inl(i)
| . . . | . . . . . ⊢  inl(j) = inl(i)
| . . . | . . . . . |  ∨ (inl(j) ∈ s2)
| . . . | . . . . . |  ∨ inl(j) = inr(i)
| . . . | . . . . . |  ∨ e-the_graph->*j & ¬L1@0-the_graph->*j

| . . . | . . . . . | by Analyze -1
| . . . | . . . . . |
| . . . | . . . . . |\
| . . . | . . . . . | 1. the_graph : Graph
| . . . | . . . . . | 2. GraphObject(the_graph)
| . . . | . . . . . | 3. Vertices(the_graph) List
| . . . | . . . . . | 4. L1 : Vertices(the_graph) List
| . . . | . . . . . | 5. s : traversal(the_graph)
| . . . | . . . . . | 6. i : Vertices(the_graph)
| . . . | . . . . . | 7. paren(Vertices(the_graph);s)
| . . . | . . . . . | 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . . | |  Vertices(the_graph);s)
| . . . | . . . . . | 9. df-traversal(the_graph;s)
| . . . | . . . . . | 10. ∀i:Vertices(the_graph).
| . . . | . . . . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . . | 11. ¬(inl(i) ∈ s)
| . . . | . . . . . | 12. s2 : traversal(the_graph)
| . . . | . . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . . | |   Vertices(the_graph);s2;s)
| . . . | . . . . . | 14. paren(Vertices(the_graph);s2)
| . . . | . . . . . | 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . . | |   Vertices(the_graph);s2)
| . . . | . . . . . | 16. ¬(inr(i) ∈ s2)
| . . . | . . . . . | 17. ∀j:Vertices(the_graph).
| . . . | . . . . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . . | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . . . . | |                           @ s2
| . . . | . . . . . | |                           @ [inr(i)])
| . . . | . . . . . | |   @ s)
| . . . | . . . . . | 19. ∀i@0:Vertices(the_graph).
| . . . | . . . . . | |   L1 @ [i]-the_graph->*i@0
| . . . | . . . . . | |   ⇒
| . . . | . . . . . | |   ¬non-trivial-loop(the_graph;i@0)
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| . . . | . . . . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . . . | |   L1 = (L1@0 @ L2)
| . . . | . . . . . . | |   ⇒
| . . . | . . . . . . | |   (∃s1,s2:traversal(the_graph).
| . . . | . . . . . . | |    s = (s2 @ s1)
| . . . | . . . . . . | |    & paren(Vertices(the_graph);s1)
| . . . | . . . . . . | |    & paren(Vertices(the_graph);s2)
| . . . | . . . . . . | |    & (∀j:Vertices(the_graph).
| . . . | . . . . . . | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . . . | |       & ((inl(j) ∈ s2)
| . . . | . . . . . . | |          ⟺
| . . . | . . . . . . | |          L2-the_graph->*j
| . . . | . . . . . . | |          & ¬L1@0-the_graph->*j)))
| . . . | . . . . . . | 21. L1@0 : Vertices(the_graph) List
| . . . | . . . . . . | 22. L2 : Vertices(the_graph) List
| . . . | . . . . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . . . | 24. e : Vertices(the_graph) List
| . . . | . . . . . . | 25. L1 = (L1@0 @ e)
| . . . | . . . . . . | 26. L2 = (e @ [i])
| . . . | . . . . . . | 27. s1 : traversal(the_graph)
| . . . | . . . . . . | 28. s3 : traversal(the_graph)
| . . . | . . . . . . | 29. s = (s3 @ s1)
| . . . | . . . . . . | 30. paren(Vertices(the_graph);s1)
| . . . | . . . . . . | 31. paren(Vertices(the_graph);s3)
| . . . | . . . . . . | 32. ∀j:Vertices(the_graph).
| . . . | . . . . . . | |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . . . | |   & ((inl(j) ∈ s3)
| . . . | . . . . . . | |      ⟺
| . . . | . . . . . . | |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . . . | 33. j : Vertices(the_graph)
| . . . | . . . . . . | 34. (inl(j) ∈ s3)
| . . . | . . . . . . | |   ⟺
| . . . | . . . . . . | |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . . . | 35. [i]-the_graph->*j
| . . . | . . . . . . | 36. ¬L1@0-the_graph->*j
| . . . | . . . . . . | 37. (inl(j) ∈ s2) ∨ (inl(j) ∈ s)
| . . . | . . . . . . | ⊢  inl(j) = inl(i)
| . . . | . . . . . . | |  ∨ (inl(j) ∈ s2)
| . . . | . . . . . . | |  ∨ inl(j) = inr(i)
| . . . | . . . . . . | |  ∨ e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . . . | | by Analyze -1
| . . . | . . . . . . | |
| . . . | . . . . . . | |\
| . . . | . . . . . . | | 1. the_graph : Graph
| . . . | . . . . . . | | 2. GraphObject(the_graph)
| . . . | . . . . . . | | 3. Vertices(the_graph) List
| . . . | . . . . . . | | 4. L1 : Vertices(the_graph) List
| . . . | . . . . . . | | 5. s : traversal(the_graph)
| . . . | . . . . . . | | 6. i : Vertices(the_graph)
| . . . | . . . . . . | | 7. paren(Vertices(the_graph);s)
| . . . | . . . . . . | | 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . . . | |    Vertices(the_graph);s)
| . . . | . . . . . . | | 9. df-traversal(the_graph;s)
| . . . | . . . . . . | | 10. ∀i:Vertices(the_graph).
| . . . | . . . . . . | |      (inl(i) ∈ s) ⇒ L1-the_graph->*i
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| . . . | . . . . . . | | 11. ¬(inl(i) ∈ s)
| . . . | . . . . . . | | 12. s2 : traversal(the_graph)
| . . . | . . . . . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . . . | |     Vertices(the_graph);s2;s)
| . . . | . . . . . . | | 14. paren(Vertices(the_graph);s2)
| . . . | . . . . . . | | 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . . . | |     Vertices(the_graph);s2)
| . . . | . . . . . . | | 16. ¬(inr(i) ∈ s2)
| . . . | . . . . . . | | 17. ∀j:Vertices(the_graph).
| . . . | . . . . . . | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . . . | | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . . . . . | |                             @ s2

| . . . | . . . . . . | |                             @ [inr(i)])
| . . . | . . . . . . | |     @ s)
| . . . | . . . . . . | | 19. ∀i@0:Vertices(the_graph).
| . . . | . . . . . . | |     L1 @ [i]-the_graph->*i@0
| . . . | . . . . . . | |     ⇒
| . . . | . . . . . . | |     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . . . | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . . . | |     L1 = (L1@0 @ L2)
| . . . | . . . . . . | |     ⇒
| . . . | . . . . . . | |     (∃s1,s2:traversal(the_graph).
| . . . | . . . . . . | |      s = (s2 @ s1)
| . . . | . . . . . . | |      & paren(Vertices(the_graph);s1)
| . . . | . . . . . . | |      & paren(Vertices(the_graph);s2)
| . . . | . . . . . . | |      & (∀j:Vertices(the_graph).
| . . . | . . . . . . | |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . . . | |         & ((inl(j) ∈ s2)
| . . . | . . . . . . | |            ⟺
| . . . | . . . . . . | |            L2-the_graph->*j
| . . . | . . . . . . | |            & ¬L1@0-the_graph->*j)))
| . . . | . . . . . . | | 21. L1@0 : Vertices(the_graph) List
| . . . | . . . . . . | | 22. L2 : Vertices(the_graph) List
| . . . | . . . . . . | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . . . | | 24. e : Vertices(the_graph) List
| . . . | . . . . . . | | 25. L1 = (L1@0 @ e)
| . . . | . . . . . . | | 26. L2 = (e @ [i])
| . . . | . . . . . . | | 27. s1 : traversal(the_graph)
| . . . | . . . . . . | | 28. s3 : traversal(the_graph)
| . . . | . . . . . . | | 29. s = (s3 @ s1)
| . . . | . . . . . . | | 30. paren(Vertices(the_graph);s1)
| . . . | . . . . . . | | 31. paren(Vertices(the_graph);s3)
| . . . | . . . . . . | | 32. ∀j:Vertices(the_graph).
| . . . | . . . . . . | |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . . . | |     & ((inl(j) ∈ s3)
| . . . | . . . . . . | |        ⟺
| . . . | . . . . . . | |        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . . . | | 33. j : Vertices(the_graph)
| . . . | . . . . . . | | 34. (inl(j) ∈ s3)
| . . . | . . . . . . | |     ⟺
| . . . | . . . . . . | |     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . . . | | 35. [i]-the_graph->*j
| . . . | . . . . . . | | 36. ¬L1@0-the_graph->*j
| . . . | . . . . . . | | 37. (inl(j) ∈ s2)
| . . . | . . . . . . | | ⊢  inl(j) = inl(i)
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| . . . | . . . . . . | |    ∨ (inl(j) ∈ s2)
| . . . | . . . . . . | |    ∨ inl(j) = inr(i)
| . . . | . . . . . . | |    ∨ e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . . . | |   by ObviousFrom [-1]
| . . . | . . . . . . | |
| . . . | . . . . . . | .\
| . . . | . . . . . . | . 1. the_graph : Graph
| . . . | . . . . . . | . 2. GraphObject(the_graph)
| . . . | . . . . . . | . 3. Vertices(the_graph) List
| . . . | . . . . . . | . 4. L1 : Vertices(the_graph) List
| . . . | . . . . . . | . 5. s : traversal(the_graph)
| . . . | . . . . . . | . 6. i : Vertices(the_graph)
| . . . | . . . . . . | . 7. paren(Vertices(the_graph);s)
| . . . | . . . . . . | . 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . . . | . |  Vertices(the_graph);s)
| . . . | . . . . . . | . 9. df-traversal(the_graph;s)
| . . . | . . . . . . | . 10. ∀i:Vertices(the_graph).
| . . . | . . . . . . | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . . . | . 11. ¬(inl(i) ∈ s)
| . . . | . . . . . . | . 12. s2 : traversal(the_graph)
| . . . | . . . . . . | . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . . . | . |   Vertices(the_graph);s2;s)
| . . . | . . . . . . | . 14. paren(Vertices(the_graph);s2)
| . . . | . . . . . . | . 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . . . | . |   Vertices(the_graph);s2)
| . . . | . . . . . . | . 16. ¬(inr(i) ∈ s2)
| . . . | . . . . . . | . 17. ∀j:Vertices(the_graph).
| . . . | . . . . . . | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . . . | . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . . . . . | . |                           @ s2
| . . . | . . . . . . | . |                           @ [inr(i)])
| . . . | . . . . . . | . |   @ s)
| . . . | . . . . . . | . 19. ∀i@0:Vertices(the_graph).
| . . . | . . . . . . | . |   L1 @ [i]-the_graph->*i@0
| . . . | . . . . . . | . |   ⇒
| . . . | . . . . . . | . |   ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . . . | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . . . | . |   L1 = (L1@0 @ L2)
| . . . | . . . . . . | . |   ⇒
| . . . | . . . . . . | . |   (∃s1,s2:traversal(the_graph).
| . . . | . . . . . . | . |    s = (s2 @ s1)
| . . . | . . . . . . | . |    & paren(Vertices(the_graph);s1)
| . . . | . . . . . . | . |    & paren(Vertices(the_graph);s2)
| . . . | . . . . . . | . |    & (∀j:Vertices(the_graph).
| . . . | . . . . . . | . |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . . . | . |        & ((inl(j) ∈ s2)
| . . . | . . . . . . | . |           ⟺
| . . . | . . . . . . | . |           L2-the_graph->*j
| . . . | . . . . . . | . |           & ¬L1@0-the_graph->*j)))
| . . . | . . . . . . | . 21. L1@0 : Vertices(the_graph) List
| . . . | . . . . . . | . 22. L2 : Vertices(the_graph) List
| . . . | . . . . . . | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . . . | . 24. e : Vertices(the_graph) List
| . . . | . . . . . . | . 25. L1 = (L1@0 @ e)
| . . . | . . . . . . | . 26. L2 = (e @ [i])
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| . . . | . . . . . . | . 27. s1 : traversal(the_graph)
| . . . | . . . . . . | . 28. s3 : traversal(the_graph)
| . . . | . . . . . . | . 29. s = (s3 @ s1)
| . . . | . . . . . . | . 30. paren(Vertices(the_graph);s1)
| . . . | . . . . . . | . 31. paren(Vertices(the_graph);s3)
| . . . | . . . . . . | . 32. ∀j:Vertices(the_graph).
| . . . | . . . . . . | . |   ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . . . | . |   & ((inl(j) ∈ s3)
| . . . | . . . . . . | . |      ⟺
| . . . | . . . . . . | . |      e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . . . | . 33. j : Vertices(the_graph)
| . . . | . . . . . . | . 34. (inl(j) ∈ s3)
| . . . | . . . . . . | . |   ⟺
| . . . | . . . . . . | . |   e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . . . | . 35. [i]-the_graph->*j
| . . . | . . . . . . | . 36. ¬L1@0-the_graph->*j
| . . . | . . . . . . | . 37. (inl(j) ∈ s)
| . . . | . . . . . . | . ⊢  inl(j) = inl(i)
| . . . | . . . . . . | . |  ∨ (inl(j) ∈ s2)
| . . . | . . . . . . | . |  ∨ inl(j) = inr(i)
| . . . | . . . . . . | . |  ∨ e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . . . | . | by (SubstFor s -1)
| . . . | . . . . . . | . |    THEN
| . . . | . . . . . . | . |    (RWO
| . . . | . . . . . . | . |     Thm* ∀x:T, l1,l2:T List.
| . . . | . . . . . . | . |           (x ∈ l1 @ l2)
| . . . | . . . . . . | . |           ⟺
| . . . | . . . . . . | . |           (x ∈ l1) ∨ (x ∈ l2)
| . . . | . . . . . . | . |     {member_append}
| . . . | . . . . . . | . |     -1)
| . . . | . . . . . . | . |    THEN
| . . . | . . . . . . | . |    (Analyze -1)
| . . . | . . . . . . | . |
| . . . | . . . . . . | . |\
| . . . | . . . . . . | . | 1. the_graph : Graph
| . . . | . . . . . . | . | 2. GraphObject(the_graph)
| . . . | . . . . . . | . | 3. Vertices(the_graph) List
| . . . | . . . . . . | . | 4. L1 : Vertices(the_graph) List
| . . . | . . . . . . | . | 5. s : traversal(the_graph)
| . . . | . . . . . . | . | 6. i : Vertices(the_graph)
| . . . | . . . . . . | . | 7. paren(Vertices(the_graph);s)
| . . . | . . . . . . | . | 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . . . | . |    Vertices(the_graph);s)
| . . . | . . . . . . | . | 9. df-traversal(the_graph;s)
| . . . | . . . . . . | . | 10. ∀i:Vertices(the_graph).
| . . . | . . . . . . | . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . . . | . | 11. ¬(inl(i) ∈ s)
| . . . | . . . . . . | . | 12. s2 : traversal(the_graph)
| . . . | . . . . . . | . | 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . . . | . |     Vertices(the_graph);s2;s)
| . . . | . . . . . . | . | 14. paren(Vertices(the_graph);s2)
| . . . | . . . . . . | . | 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . . . | . |     Vertices(the_graph);s2)
| . . . | . . . . . . | . | 16. ¬(inr(i) ∈ s2)
| . . . | . . . . . . | . | 17. ∀j:Vertices(the_graph).
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| . . . | . . . . . . | . . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . . . | . . | 18. df-traversal(the_graph;([inl(i)]
| . . . | . . . . . . | . . |                             @ s2
| . . . | . . . . . . | . . |                             @ [inr(i)])
| . . . | . . . . . . | . . |     @ s)
| . . . | . . . . . . | . . | 19. ∀i@0:Vertices(the_graph).
| . . . | . . . . . . | . . |     L1 @ [i]-the_graph->*i@0
| . . . | . . . . . . | . . |     ⇒
| . . . | . . . . . . | . . |     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . . . | . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . . . | . . |     L1 = (L1@0 @ L2)
| . . . | . . . . . . | . . |     ⇒
| . . . | . . . . . . | . . |     (∃s1,s2:traversal(the_graph).
| . . . | . . . . . . | . . |      s = (s2 @ s1)
| . . . | . . . . . . | . . |      & paren(Vertices(the_graph);s1)
| . . . | . . . . . . | . . |      & paren(Vertices(the_graph);s2)
| . . . | . . . . . . | . . |      & (∀j:Vertices(the_graph).
| . . . | . . . . . . | . . |         ...
| . . . | . . . . . . | . . |         & ((inl(j) ∈ s2)
| . . . | . . . . . . | . . |            ⟺
| . . . | . . . . . . | . . |            L2-the_graph->*j
| . . . | . . . . . . | . . |            & ¬L1@0-the_graph->*j)))
| . . . | . . . . . . | . . | 21. L1@0 : Vertices(the_graph) List
| . . . | . . . . . . | . . | 22. L2 : Vertices(the_graph) List
| . . . | . . . . . . | . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . . . | . . | 24. e : Vertices(the_graph) List
| . . . | . . . . . . | . . | 25. L1 = (L1@0 @ e)
| . . . | . . . . . . | . . | 26. L2 = (e @ [i])
| . . . | . . . . . . | . . | 27. s1 : traversal(the_graph)
| . . . | . . . . . . | . . | 28. s3 : traversal(the_graph)
| . . . | . . . . . . | . . | 29. s = (s3 @ s1)
| . . . | . . . . . . | . . | 30. paren(Vertices(the_graph);s1)
| . . . | . . . . . . | . . | 31. paren(Vertices(the_graph);s3)
| . . . | . . . . . . | . . | 32. ∀j:Vertices(the_graph).
| . . . | . . . . . . | . . |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . . . | . . |     & ...
| . . . | . . . . . . | . . | 33. j : Vertices(the_graph)
| . . . | . . . . . . | . . | 34. (inl(j) ∈ s3)
| . . . | . . . . . . | . . |     ⟺
| . . . | . . . . . . | . . |     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . . . | . . | 35. [i]-the_graph->*j
| . . . | . . . . . . | . . | 36. ¬L1@0-the_graph->*j
| . . . | . . . . . . | . . | 37. (inl(j) ∈ s3)
| . . . | . . . . . . | . . | ⊢  inl(j) = inl(i)
| . . . | . . . . . . | . . |    ∨ (inl(j) ∈ s2)
| . . . | . . . . . . | . . |    ∨ inl(j) = inr(i)
| . . . | . . . . . . | . . |    ∨ e-the_graph->*j & ¬L1@0-the_graph->*j

| . . . | . . . . . . | . . |   by OrRight THEN (Analyze -4)
| . . . | . . . . . . | . . |      THEN
| . . . | . . . . . . | . . |      (Analyze -5)
| . . . | . . . . . . | . . |
| . . . | . . . . . . | . . .\
| . . . | . . . . . . | . . . 1. the_graph : Graph
| . . . | . . . . . . | . . . 2. GraphObject(the_graph)
| . . . | . . . . . . | . . . 3. Vertices(the_graph) List
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| . . . | . . . . . . | . . 4. L1 : Vertices(the_graph) List
| . . . | . . . . . . | . . 5. s : traversal(the_graph)
| . . . | . . . . . . | . . 6. i : Vertices(the_graph)
| . . . | . . . . . . | . . 7. paren(Vertices(the_graph);s)
| . . . | . . . . . . | . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . . . | . .    Vertices(the_graph);s)
| . . . | . . . . . . | . . 9. df-traversal(the_graph;s)
| . . . | . . . . . . | . . 10. ∀i:Vertices(the_graph).
| . . . | . . . . . . | . .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . . . | . . 11. ¬(inl(i) ∈ s)
| . . . | . . . . . . | . . 12. s2 : traversal(the_graph)
| . . . | . . . . . . | . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . . . | . .     Vertices(the_graph);s2;s)
| . . . | . . . . . . | . . 14. paren(Vertices(the_graph);s2)
| . . . | . . . . . . | . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . . . | . .     Vertices(the_graph);s2)
| . . . | . . . . . . | . . 16. ¬(inr(i) ∈ s2)
| . . . | . . . . . . | . . 17. ∀j:Vertices(the_graph).
| . . . | . . . . . . | . .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . . . | . . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . . . . . | . .                             @ s2
| . . . | . . . . . . | . .                             @ [inr(i)])
| . . . | . . . . . . | . .     @ s)
| . . . | . . . . . . | . . 19. ∀i@0:Vertices(the_graph).
| . . . | . . . . . . | . .     L1 @ [i]-the_graph->*i@0
| . . . | . . . . . . | . .     ⇒
| . . . | . . . . . . | . .     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . . . | . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . . . | . .     L1 = (L1@0 @ L2)
| . . . | . . . . . . | . .     ⇒
| . . . | . . . . . . | . .     (∃s1,s2:traversal(the_graph).
| . . . | . . . . . . | . .      s = (s2 @ s1)
| . . . | . . . . . . | . .      & paren(Vertices(the_graph);s1)
| . . . | . . . . . . | . .      & paren(Vertices(the_graph);s2)
| . . . | . . . . . . | . .      & (∀j:Vertices(the_graph).
| . . . | . . . . . . | . .          ...
| . . . | . . . . . . | . .          & ((inl(j) ∈ s2)
| . . . | . . . . . . | . .             ⟺
| . . . | . . . . . . | . .             L2-the_graph->*j
| . . . | . . . . . . | . .             & ¬L1@0-the_graph->*j)))
| . . . | . . . . . . | . . 21. L1@0 : Vertices(the_graph) List
| . . . | . . . . . . | . . 22. L2 : Vertices(the_graph) List
| . . . | . . . . . . | . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . . . | . . 24. e : Vertices(the_graph) List
| . . . | . . . . . . | . . 25. L1 = (L1@0 @ e)
| . . . | . . . . . . | . . 26. L2 = (e @ [i])
| . . . | . . . . . . | . . 27. s1 : traversal(the_graph)
| . . . | . . . . . . | . . 28. s3 : traversal(the_graph)
| . . . | . . . . . . | . . 29. s = (s3 @ s1)
| . . . | . . . . . . | . . 30. paren(Vertices(the_graph);s1)
| . . . | . . . . . . | . . 31. paren(Vertices(the_graph);s3)
| . . . | . . . . . . | . . 32. ∀j:Vertices(the_graph).
| . . . | . . . . . . | . .     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . . . | . .     & ...
| . . . | . . . . . . | . . 33. j : Vertices(the_graph)
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| . . . | . . . . . . | . . 34. (inl(j) ∈ s3)
| . . . | . . . . . . | . .     ⟺
| . . . | . . . . . . | . .     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . . . | . . 35. [i]-the_graph->*j
| . . . | . . . . . . | . . 36. ¬L1@0-the_graph->*j
| . . . | . . . . . . | . . 37. (inl(j) ∈ s1)
| . . . | . . . . . . | . . ⊢  inl(j) = inl(i)
| . . . | . . . . . . | . .    ∨ (inl(j) ∈ s2)
| . . . | . . . . . . | . .    ∨ inl(j) = inr(i)
| . . . | . . . . . . | . .    ∨ e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . . . | . .   by (InstHyp [j] -6) THEN (Analyze -1)
| . . . | . . . . . . | . .      THEN
| . . . | . . . . . . | . .      (Analyze -2)
| . . . | . . . . . . | . .      THEN
| . . . | . . . . . . | . .      (Analyze -3)
| . . . | . . . . . . |
| . . . | . . . . . . .\
| . . . | . . . . . . . 1. the_graph : Graph
| . . . | . . . . . . . 2. GraphObject(the_graph)
| . . . | . . . . . . . 3. Vertices(the_graph) List
| . . . | . . . . . . . 4. L1 : Vertices(the_graph) List
| . . . | . . . . . . . 5. s : traversal(the_graph)
| . . . | . . . . . . . 6. i : Vertices(the_graph)
| . . . | . . . . . . . 7. paren(Vertices(the_graph);s)
| . . . | . . . . . . . 8. no_repeats(Vertices(the_graph)+
| . . . | . . . . . . .    Vertices(the_graph);s)
| . . . | . . . . . . . 9. df-traversal(the_graph;s)
| . . . | . . . . . . . 10. ∀i:Vertices(the_graph).
| . . . | . . . . . . .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . | . . . . . . . 11. ¬(inl(i) ∈ s)
| . . . | . . . . . . . 12. s2 : traversal(the_graph)
| . . . | . . . . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . | . . . . . . .     Vertices(the_graph);s2;s)
| . . . | . . . . . . . 14. paren(Vertices(the_graph);s2)
| . . . | . . . . . . . 15. no_repeats(Vertices(the_graph)+
| . . . | . . . . . . .     Vertices(the_graph);s2)
| . . . | . . . . . . . 16. ¬(inr(i) ∈ s2)
| . . . | . . . . . . . 17. ∀j:Vertices(the_graph).
| . . . | . . . . . . .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . | . . . . . . . 18. df-traversal(the_graph;([inl(i)]
| . . . | . . . . . . .                             @ s2
| . . . | . . . . . . .                             @ [inr(i)])
| . . . | . . . . . . .     @ s)
| . . . | . . . . . . . 19. ∀i@0:Vertices(the_graph).
| . . . | . . . . . . .     L1 @ [i]-the_graph->*i@0
| . . . | . . . . . . .     ⇒
| . . . | . . . . . . .     ¬non-trivial-loop(the_graph;i@0)
| . . . | . . . . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . | . . . . . . .     L1 = (L1@0 @ L2)
| . . . | . . . . . . .     ⇒
| . . . | . . . . . . .     (∃s1,s2:traversal(the_graph).
| . . . | . . . . . . .      s = (s2 @ s1)
| . . . | . . . . . . .      & paren(Vertices(the_graph);s1)
| . . . | . . . . . . .      & paren(Vertices(the_graph);s2)
| . . . | . . . . . . .      & (∀j:Vertices(the_graph).
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| . . . | . . . . . .         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . . .         & ((inl(j) ∈ s2)
| . . . | . . . . . .            ⟺
| . . . | . . . . . .            L2-the_graph->*j
| . . . | . . . . . .            & ¬L1@0-the_graph->*j)))
| . . . | . . . . . . 21. L1@0 : Vertices(the_graph) List
| . . . | . . . . . . 22. L2 : Vertices(the_graph) List
| . . . | . . . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . | . . . . . . 24. e : Vertices(the_graph) List
| . . . | . . . . . . 25. L1 = (L1@0 @ e)
| . . . | . . . . . . 26. L2 = (e @ [i])
| . . . | . . . . . . 27. s1 : traversal(the_graph)
| . . . | . . . . . . 28. s3 : traversal(the_graph)
| . . . | . . . . . . 29. s = (s3 @ s1)
| . . . | . . . . . . 30. paren(Vertices(the_graph);s1)
| . . . | . . . . . . 31. paren(Vertices(the_graph);s3)
| . . . | . . . . . . 32. ∀j:Vertices(the_graph).
| . . . | . . . . . .     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . | . . . . . .     & ((inl(j) ∈ s3)
| . . . | . . . . . .        ⟺
| . . . | . . . . . .        e-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . | . . . . . . 33. j : Vertices(the_graph)
| . . . | . . . . . . 34. (inl(j) ∈ s3)
| . . . | . . . . . .     ⟺
| . . . | . . . . . .     e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . . . 35. [i]-the_graph->*j
| . . . | . . . . . . 36. ¬L1@0-the_graph->*j
| . . . | . . . . . . 37. inl(j) = inl(i)
| . . . | . . . . . . ⊢  inl(j) = inl(i)
| . . . | . . . . . .    ∨ (inl(j) ∈ s2)
| . . . | . . . . . .    ∨ inl(j) = inr(i)
| . . . | . . . . . .    ∨ e-the_graph->*j & ¬L1@0-the_graph->*j
| . . . | . . . . . .   by ObviousFrom [-1]
| . . . |
| . . . .\
| . . . . 1. the_graph : Graph
| . . . . 2. GraphObject(the_graph)
| . . . . 3. Vertices(the_graph) List
| . . . . 4. L1 : Vertices(the_graph) List
| . . . . 5. s : traversal(the_graph)
| . . . . 6. i : Vertices(the_graph)
| . . . . 7. paren(Vertices(the_graph);s)
| . . . . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . . 9. df-traversal(the_graph;s)
| . . . . 10. ∀i:Vertices(the_graph). (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . 11. ¬(inl(i) ∈ s)
| . . . . 12. s2 : traversal(the_graph)
| . . . . 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s2;s)
| . . . . 14. paren(Vertices(the_graph);s2)
| . . . . 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . . . 16. ¬(inr(i) ∈ s2)
| . . . . 17. ∀j:Vertices(the_graph). (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . 19. ∀i@0:Vertices(the_graph).
| . . . . |   L1 @ [i]-the_graph->*i@0
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| . . . . |   ⇒
| . . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . |   L1 = (L1@0 @ L2)
| . . . . |   ⇒
| . . . . |   (∃s1,s2:traversal(the_graph).
| . . . . |    s = (s2 @ s1)
| . . . . |    & paren(Vertices(the_graph);s1)
| . . . . |    & paren(Vertices(the_graph);s2)
| . . . . |    & (∀j:Vertices(the_graph).
| . . . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . |       & ((inl(j) ∈ s2)
| . . . . |          ⟺
| . . . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . 21. L1@0 : Vertices(the_graph) List
| . . . . 22. L2 : Vertices(the_graph) List
| . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . 24. e : Vertices(the_graph) List
| . . . . 25. L1@0 = (L1 @ e)
| . . . . 26. [i] = (e @ L2)
| . . . . ⊢  ∃s1,s2@0:traversal(the_graph).
| . . . . |  (([inl(i)] @ s2 @ [inr(i)]) @ s) = (s2@0 @ s1)
| . . . . |  & paren(Vertices(the_graph);s1)
| . . . . |  & paren(Vertices(the_graph);s2@0)
| . . . . |  & (∀j:Vertices(the_graph).
| . . . . |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . |     & ((inl(j) ∈ s2@0)
| . . . . |        ⟺
| . . . . |        L2-the_graph->*j & ¬L1@0-the_graph->*j))
| . . . . | by Assert
| . . . . |    (∀i:Vertices(the_graph).
| . . . . |     (inl(i) ∈ s) ⟺ L1-the_graph->*i)
| . . . . |
| . . . . |\
| . . . . | .....assertion .....
| . . . . | 1. the_graph : Graph
| . . . . | 2. GraphObject(the_graph)
| . . . . | 3. Vertices(the_graph) List
| . . . . | 4. L1 : Vertices(the_graph) List
| . . . . | 5. s : traversal(the_graph)
| . . . . | 6. i : Vertices(the_graph)
| . . . . | 7. paren(Vertices(the_graph);s)
| . . . . | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . . | 9. df-traversal(the_graph;s)
| . . . . | 10. ∀i:Vertices(the_graph).
| . . . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . | 11. ¬(inl(i) ∈ s)
| . . . . | 12. s2 : traversal(the_graph)
| . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . | |   Vertices(the_graph);s2;s)
| . . . . | 14. paren(Vertices(the_graph);s2)
| . . . . | 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . . . | 16. ¬(inr(i) ∈ s2)
| . . . . | 17. ∀j:Vertices(the_graph).
| . . . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
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| . . . . | 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . | 19. ∀i@0:Vertices(the_graph).
| . . . . | |   L1 @ [i]-the_graph->*i@0
| . . . . | |   ⇒
| . . . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . | |   L1 = (L1@0 @ L2)
| . . . . | |   ⇒
| . . . . | |   (∃s1,s2:traversal(the_graph).
| . . . . | |    s = (s2 @ s1)
| . . . . | |    & paren(Vertices(the_graph);s1)
| . . . . | |    & paren(Vertices(the_graph);s2)
| . . . . | |    & (∀j:Vertices(the_graph).
| . . . . | |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . | |        & ((inl(j) ∈ s2)
| . . . . | |           ⟺
| . . . . | |           L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . | 21. L1@0 : Vertices(the_graph) List
| . . . . | 22. L2 : Vertices(the_graph) List
| . . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . | 24. e : Vertices(the_graph) List
| . . . . | 25. L1@0 = (L1 @ e)
| . . . . | 26. [i] = (e @ L2)
| . . . . | ⊢  ∀i:Vertices(the_graph).
| . . . . | |  (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . | | by ((InstHyp [L1;nil] -7)
| . . . . | |     THENA
| . . . . | |     (Auto
| . . . . | |      THEN
| . . . . | |      (RWO
| . . . . | |       Thm* ∀l:T List. (l @ nil) ~ l    {append_nil_sq}
| . . . . | |       0)))
| . . . . | |    THEN
| . . . . | |    ExRepDHyps
| . . . . | |
| . . . . | 1. the_graph : Graph
| . . . . | 2. GraphObject(the_graph)
| . . . . | 3. Vertices(the_graph) List
| . . . . | 4. L1 : Vertices(the_graph) List
| . . . . | 5. s : traversal(the_graph)
| . . . . | 6. i : Vertices(the_graph)
| . . . . | 7. paren(Vertices(the_graph);s)
| . . . . | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . . | 9. df-traversal(the_graph;s)
| . . . . | 10. ∀i:Vertices(the_graph).
| . . . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . | 11. ¬(inl(i) ∈ s)
| . . . . | 12. s2 : traversal(the_graph)
| . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . | |   Vertices(the_graph);s2;s)
| . . . . | 14. paren(Vertices(the_graph);s2)
| . . . . | 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . . . | 16. ¬(inr(i) ∈ s2)
| . . . . | 17. ∀j:Vertices(the_graph).
| . . . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
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| . . . . | 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . | 19. ∀i@0:Vertices(the_graph).
| . . . . | |   L1 @ [i]-the_graph->*i@0
| . . . . | |   ⇒
| . . . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . | |   L1 = (L1@0 @ L2)
| . . . . | |   ⇒
| . . . . | |   (∃s1,s2:traversal(the_graph).
| . . . . | |    s = (s2 @ s1)
| . . . . | |    & paren(Vertices(the_graph);s1)
| . . . . | |    & paren(Vertices(the_graph);s2)
| . . . . | |    & (∀j:Vertices(the_graph).
| . . . . | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . | |       & ((inl(j) ∈ s2)
| . . . . | |          ⟺
| . . . . | |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . | 21. L1@0 : Vertices(the_graph) List
| . . . . | 22. L2 : Vertices(the_graph) List
| . . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . | 24. e : Vertices(the_graph) List
| . . . . | 25. L1@0 = (L1 @ e)
| . . . . | 26. [i] = (e @ L2)
| . . . . | 27. s1 : traversal(the_graph)
| . . . . | 28. s3 : traversal(the_graph)
| . . . . | 29. s = (s3 @ s1)
| . . . . | 30. paren(Vertices(the_graph);s1)
| . . . . | 31. paren(Vertices(the_graph);s3)
| . . . . | 32. ∀j:Vertices(the_graph).
| . . . . | |   (inl(j) ∈ s1) ⟺ L1-the_graph->*j
| . . . . | 33. ∀j:Vertices(the_graph).
| . . . . | |   (inl(j) ∈ s3) ⟺ nil-the_graph->*j & ¬L1-the_graph->*j
| . . . . | ⊢  ∀i:Vertices(the_graph).
| . . . . | |  (inl(i) ∈ s) ⟺ L1-the_graph->*i  by Assert (s3 = nil)
| . . . . | |
| . . . . | |\
| . . . . | | .....assertion .....
| . . . . | | 1. the_graph : Graph
| . . . . | | 2. GraphObject(the_graph)
| . . . . | | 3. Vertices(the_graph) List
| . . . . | | 4. L1 : Vertices(the_graph) List
| . . . . | | 5. s : traversal(the_graph)
| . . . . | | 6. i : Vertices(the_graph)
| . . . . | | 7. paren(Vertices(the_graph);s)
| . . . . | | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . . | | 9. df-traversal(the_graph;s)
| . . . . | | 10. ∀i:Vertices(the_graph).
| . . . . | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . | | 11. ¬(inl(i) ∈ s)
| . . . . | | 12. s2 : traversal(the_graph)
| . . . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . . | | |   Vertices(the_graph);s2;s)
| . . . . | | 14. paren(Vertices(the_graph);s2)
| . . . . | | 15. no_repeats(Vertices(the_graph)+
| . . . . | | |   Vertices(the_graph);s2)
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| . . . . | | 16. ¬(inr(i) ∈ s2)
| . . . . | | 17. ∀j:Vertices(the_graph).
| . . . . | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . | | 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . . | | |   @ s)
| . . . . | | 19. ∀i@0:Vertices(the_graph).
| . . . . | | |   L1 @ [i]-the_graph->*i@0
| . . . . | | |   ⇒
| . . . . | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . | | |   L1 = (L1@0 @ L2)
| . . . . | | |   ⇒
| . . . . | | |   (∃s1,s2:traversal(the_graph).
| . . . . | | |    s = (s2 @ s1)
| . . . . | | |    & paren(Vertices(the_graph);s1)
| . . . . | | |    & paren(Vertices(the_graph);s2)
| . . . . | | |    & (∀j:Vertices(the_graph).
| . . . . | | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . | | |       & ((inl(j) ∈ s2)
| . . . . | | |          ⟺
| . . . . | | |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . | | 21. L1@0 : Vertices(the_graph) List
| . . . . | | 22. L2 : Vertices(the_graph) List
| . . . . | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . | | 24. e : Vertices(the_graph) List
| . . . . | | 25. L1@0 = (L1 @ e)
| . . . . | | 26. [i] = (e @ L2)
| . . . . | | 27. s1 : traversal(the_graph)
| . . . . | | 28. s3 : traversal(the_graph)
| . . . . | | 29. s = (s3 @ s1)
| . . . . | | 30. paren(Vertices(the_graph);s1)
| . . . . | | 31. paren(Vertices(the_graph);s3)
| . . . . | | 32. ∀j:Vertices(the_graph).
| . . . . | | |   (inl(j) ∈ s1) ⟺ L1-the_graph->*j
| . . . . | | 33. ∀j:Vertices(the_graph).
| . . . . | | |   (inl(j) ∈ s3)
| . . . . | | |   ⟺
| . . . . | | |   nil-the_graph->*j & ¬L1-the_graph->*j
| . . . . | | ⊢  s3 = nil
| . . . . | | | by Assert
| . . . . | | |    (∀j:Vertices(the_graph). (inl(j) ∈ s3) ⇒ False)
| . . . . | | |
| . . . . | | |\
| . . . . | | | .....assertion .....
| . . . . | | | 1. the_graph : Graph
| . . . . | | | 2. GraphObject(the_graph)
| . . . . | | | 3. Vertices(the_graph) List
| . . . . | | | 4. L1 : Vertices(the_graph) List
| . . . . | | | 5. s : traversal(the_graph)
| . . . . | | | 6. i : Vertices(the_graph)
| . . . . | | | 7. paren(Vertices(the_graph);s)
| . . . . | | | 8. no_repeats(Vertices(the_graph)+
| . . . . | | | |  Vertices(the_graph);s)
| . . . . | | | 9. df-traversal(the_graph;s)
| . . . . | | | 10. ∀i:Vertices(the_graph).
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| . . . . | | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . | | | 11. ¬(inl(i) ∈ s)
| . . . . | | | 12. s2 : traversal(the_graph)
| . . . . | | | 13. l_disjoint(Vertices(the_graph)+
| . . . . | | | |   Vertices(the_graph);s2;s)
| . . . . | | | 14. paren(Vertices(the_graph);s2)
| . . . . | | | 15. no_repeats(Vertices(the_graph)+
| . . . . | | | |   Vertices(the_graph);s2)
| . . . . | | | 16. ¬(inr(i) ∈ s2)
| . . . . | | | 17. ∀j:Vertices(the_graph).
| . . . . | | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . | | | 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . . | | | |   @ s)
| . . . . | | | 19. ∀i@0:Vertices(the_graph).
| . . . . | | | |   L1 @ [i]-the_graph->*i@0
| . . . . | | | |   ⇒
| . . . . | | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . | | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . | | | |   L1 = (L1@0 @ L2)
| . . . . | | | |   ⇒
| . . . . | | | |   (∃s1,s2:traversal(the_graph).
| . . . . | | | |    s = (s2 @ s1)
| . . . . | | | |    & paren(Vertices(the_graph);s1)
| . . . . | | | |    & paren(Vertices(the_graph);s2)
| . . . . | | | |    & (∀j:Vertices(the_graph).
| . . . . | | | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . | | | |       & ((inl(j) ∈ s2)
| . . . . | | | |          ⟺
| . . . . | | | |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . | | | 21. L1@0 : Vertices(the_graph) List
| . . . . | | | 22. L2 : Vertices(the_graph) List
| . . . . | | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . | | | 24. e : Vertices(the_graph) List
| . . . . | | | 25. L1@0 = (L1 @ e)
| . . . . | | | 26. [i] = (e @ L2)
| . . . . | | | 27. s1 : traversal(the_graph)
| . . . . | | | 28. s3 : traversal(the_graph)
| . . . . | | | 29. s = (s3 @ s1)
| . . . . | | | 30. paren(Vertices(the_graph);s1)
| . . . . | | | 31. paren(Vertices(the_graph);s3)
| . . . . | | | 32. ∀j:Vertices(the_graph).
| . . . . | | | |   (inl(j) ∈ s1) ⟺ L1-the_graph->*j
| . . . . | | | 33. ∀j:Vertices(the_graph).
| . . . . | | | |   (inl(j) ∈ s3)
| . . . . | | | |   ⟺
| . . . . | | | |   nil-the_graph->*j & ¬L1-the_graph->*j
| . . . . | | | ⊢  ∀j:Vertices(the_graph). (inl(j) ∈ s3) ⇒ False
| . . . . | | | | by (ParallelOp -1) THEN ThinTrivial
| . . . . | | | |    THEN
| . . . . | | | |    (Analyze -2)
| . . . . | | | |    THEN
| . . . . | | | |    (Analyze -2)
| . . . . | | | |
| . . . . | | | 1. the_graph : Graph
| . . . . | | | 2. GraphObject(the_graph)
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| . . . . | | | 3. Vertices(the_graph) List
| . . . . | | | 4. L1 : Vertices(the_graph) List
| . . . . | | | 5. s : traversal(the_graph)
| . . . . | | | 6. i : Vertices(the_graph)
| . . . . | | | 7. paren(Vertices(the_graph);s)
| . . . . | | | 8. no_repeats(Vertices(the_graph)+
| . . . . | | |    Vertices(the_graph);s)
| . . . . | | | 9. df-traversal(the_graph;s)
| . . . . | | | 10. ∀i:Vertices(the_graph).
| . . . . | | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . | | | 11. ¬(inl(i) ∈ s)
| . . . . | | | 12. s2 : traversal(the_graph)
| . . . . | | | 13. l_disjoint(Vertices(the_graph)+
| . . . . | | |     Vertices(the_graph);s2;s)
| . . . . | | | 14. paren(Vertices(the_graph);s2)
| . . . . | | | 15. no_repeats(Vertices(the_graph)+
| . . . . | | |     Vertices(the_graph);s2)
| . . . . | | | 16. ¬(inr(i) ∈ s2)
| . . . . | | | 17. ∀j:Vertices(the_graph).
| . . . . | | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . | | | 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . . | | |     @ s)
| . . . . | | | 19. ∀i@0:Vertices(the_graph).
| . . . . | | |     L1 @ [i]-the_graph->*i@0
| . . . . | | |     ⇒
| . . . . | | |     ¬non-trivial-loop(the_graph;i@0)
| . . . . | | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . | | |     L1 = (L1@0 @ L2)
| . . . . | | |     ⇒
| . . . . | | |     (∃s1,s2:traversal(the_graph).
| . . . . | | |      s = (s2 @ s1)
| . . . . | | |      & paren(Vertices(the_graph);s1)
| . . . . | | |      & paren(Vertices(the_graph);s2)
| . . . . | | |      & (∀j:Vertices(the_graph).
| . . . . | | |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . | | |         & ((inl(j) ∈ s2)
| . . . . | | |            ⟺
| . . . . | | |            L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . | | | 21. L1@0 : Vertices(the_graph) List
| . . . . | | | 22. L2 : Vertices(the_graph) List
| . . . . | | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . | | | 24. e : Vertices(the_graph) List
| . . . . | | | 25. L1@0 = (L1 @ e)
| . . . . | | | 26. [i] = (e @ L2)
| . . . . | | | 27. s1 : traversal(the_graph)
| . . . . | | | 28. s3 : traversal(the_graph)
| . . . . | | | 29. s = (s3 @ s1)
| . . . . | | | 30. paren(Vertices(the_graph);s1)
| . . . . | | | 31. paren(Vertices(the_graph);s3)
| . . . . | | | 32. ∀j:Vertices(the_graph).
| . . . . | | |     (inl(j) ∈ s1) ⟺ L1-the_graph->*j
| . . . . | | | 33. ∀j:Vertices(the_graph).
| . . . . | | |     (inl(j) ∈ s3)
| . . . . | | |     ⟺
| . . . . | | |     nil-the_graph->*j & ¬L1-the_graph->*j
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| . . . . | | | 34. j : Vertices(the_graph)
| . . . . | | | 35. (inl(j) ∈ s3) ⇐ (nil-the_graph->*j
| . . . . | | |     & ¬L1-the_graph->*j)
| . . . . | | | 36. (inl(j) ∈ s3)
| . . . . | | | 37. y : Vertices(the_graph)
| . . . . | | | 38. (y ∈ nil)
| . . . . | | | 39. y-the_graph->*j
| . . . . | | | 40. ¬L1-the_graph->*j
| . . . . | | | ⊢  False  by ObviousFrom [-3]
| . . . . | | |
| . . . . | | .\
| . . . . | | . 1. the_graph : Graph
| . . . . | | . 2. GraphObject(the_graph)
| . . . . | | . 3. Vertices(the_graph) List
| . . . . | | . 4. L1 : Vertices(the_graph) List
| . . . . | | . 5. s : traversal(the_graph)
| . . . . | | . 6. i : Vertices(the_graph)
| . . . . | | . 7. paren(Vertices(the_graph);s)
| . . . . | | . 8. no_repeats(Vertices(the_graph)+
| . . . . | | . |  Vertices(the_graph);s)
| . . . . | | . 9. df-traversal(the_graph;s)
| . . . . | | . 10. ∀i:Vertices(the_graph).
| . . . . | | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . | | . 11. ¬(inl(i) ∈ s)
| . . . . | | . 12. s2 : traversal(the_graph)
| . . . . | | . 13. l_disjoint(Vertices(the_graph)+
| . . . . | | . |   Vertices(the_graph);s2;s)
| . . . . | | . 14. paren(Vertices(the_graph);s2)
| . . . . | | . 15. no_repeats(Vertices(the_graph)+
| . . . . | | . |   Vertices(the_graph);s2)
| . . . . | | . 16. ¬(inr(i) ∈ s2)
| . . . . | | . 17. ∀j:Vertices(the_graph).
| . . . . | | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . | | . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . . | | . |   @ s)
| . . . . | | . 19. ∀i@0:Vertices(the_graph).
| . . . . | | . |   L1 @ [i]-the_graph->*i@0
| . . . . | | . |   ⇒
| . . . . | | . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . | | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . | | . |   L1 = (L1@0 @ L2)
| . . . . | | . |   ⇒
| . . . . | | . |   (∃s1,s2:traversal(the_graph).
| . . . . | | . |    s = (s2 @ s1)
| . . . . | | . |    & paren(Vertices(the_graph);s1)
| . . . . | | . |    & paren(Vertices(the_graph);s2)
| . . . . | | . |    & (∀j:Vertices(the_graph).
| . . . . | | . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . | | . |       & ((inl(j) ∈ s2)
| . . . . | | . |          ⟺
| . . . . | | . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . | | . 21. L1@0 : Vertices(the_graph) List
| . . . . | | . 22. L2 : Vertices(the_graph) List
| . . . . | | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . | | . 24. e : Vertices(the_graph) List
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| . . . . . | | . 25. L1@0 = (L1 @ e)
| . . . . . | | . 26. [i] = (e @ L2)
| . . . . . | | . 27. s1 : traversal(the_graph)
| . . . . . | | . 28. s3 : traversal(the_graph)
| . . . . . | | . 29. s = (s3 @ s1)
| . . . . . | | . 30. paren(Vertices(the_graph);s1)
| . . . . . | | . 31. paren(Vertices(the_graph);s3)
| . . . . . | | . 32. ∀j:Vertices(the_graph).
| . . . . . | | . |   (inl(j) ∈ s1) ⟺ L1-the_graph->*j
| . . . . . | | . 33. ∀j:Vertices(the_graph).
| . . . . . | | . |   (inl(j) ∈ s3)
| . . . . . | | . |   ⟺
| . . . . . | | . |   nil-the_graph->*j & ¬L1-the_graph->*j
| . . . . . | | . 34. ∀j:Vertices(the_graph). (inl(j) ∈ s3) ⇒ False
| . . . . . | | . ⊢  s3 = nil
| . . . . . | | . | by Assert
| . . . . . | | . |    (∀j:Vertices(the_graph). (inr(j) ∈ s3) ⇒ False)
| . . . . . | | . |
| . . . . . | | . |\
| . . . . . | | . | .....assertion .....
| . . . . . | | . | 1. the_graph : Graph
| . . . . . | | . | 2. GraphObject(the_graph)
| . . . . . | | . | 3. Vertices(the_graph) List
| . . . . . | | . | 4. L1 : Vertices(the_graph) List
| . . . . . | | . | 5. s : traversal(the_graph)
| . . . . . | | . | 6. i : Vertices(the_graph)
| . . . . . | | . | 7. paren(Vertices(the_graph);s)
| . . . . . | | . | 8. no_repeats(Vertices(the_graph)+
| . . . . . | | . |    Vertices(the_graph);s)
| . . . . . | | . | 9. df-traversal(the_graph;s)
| . . . . . | | . | 10. ∀i:Vertices(the_graph).
| . . . . . | | . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . | | . | 11. ¬(inl(i) ∈ s)
| . . . . . | | . | 12. s2 : traversal(the_graph)
| . . . . . | | . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . | | . |     Vertices(the_graph);s2;s)
| . . . . . | | . | 14. paren(Vertices(the_graph);s2)
| . . . . . | | . | 15. no_repeats(Vertices(the_graph)+
| . . . . . | | . |     Vertices(the_graph);s2)
| . . . . . | | . | 16. ¬(inr(i) ∈ s2)
| . . . . . | | . | 17. ∀j:Vertices(the_graph).
| . . . . . | | . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . | | . | 18. df-traversal(the_graph;([inl(i)]
| . . . . . | | . |                             @ s2
| . . . . . | | . |                             @ [inr(i)])
| . . . . . | | . |     @ s)
| . . . . . | | . | 19. ∀i@0:Vertices(the_graph).
| . . . . . | | . |     L1 @ [i]-the_graph->*i@0
| . . . . . | | . |     ⇒
| . . . . . | | . |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . | | . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . | | . |     L1 = (L1@0 @ L2)
| . . . . . | | . |     ⇒
| . . . . . | | . |     (∃s1,s2:traversal(the_graph).
| . . . . . | | . |      s = (s2 @ s1)
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| . . . . | | . |      & paren(Vertices(the_graph);s1)
| . . . . | | . |      & paren(Vertices(the_graph);s2)
| . . . . | | . |      & (∀j:Vertices(the_graph).
| . . . . | | . |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . | | . |         & ((inl(j) ∈ s2)
| . . . . | | . |            ⟺
| . . . . | | . |            L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . | | . | 21. L1@0 : Vertices(the_graph) List
| . . . . | | . | 22. L2 : Vertices(the_graph) List
| . . . . | | . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . | | . | 24. e : Vertices(the_graph) List
| . . . . | | . | 25. L1@0 = (L1 @ e)
| . . . . | | . | 26. [i] = (e @ L2)
| . . . . | | . | 27. s1 : traversal(the_graph)
| . . . . | | . | 28. s3 : traversal(the_graph)
| . . . . | | . | 29. s = (s3 @ s1)
| . . . . | | . | 30. paren(Vertices(the_graph);s1)
| . . . . | | . | 31. paren(Vertices(the_graph);s3)
| . . . . | | . | 32. ∀j:Vertices(the_graph).
| . . . . | | . |     (inl(j) ∈ s1) ⟺ L1-the_graph->*j
| . . . . | | . | 33. ∀j:Vertices(the_graph).
| . . . . | | . |     (inl(j) ∈ s3)
| . . . . | | . |     ⟺
| . . . . | | . |     nil-the_graph->*j & ¬L1-the_graph->*j
| . . . . | | . | 34. ∀j:Vertices(the_graph). (inl(j) ∈ s3) ⇒ False
| . . . . | | . | ⊢  ∀j:Vertices(the_graph). (inr(j) ∈ s3) ⇒ False
| . . . . | | . |   by (RepeatFor 2 (ParallelOp -1))
| . . . . | | . |      THEN
| . . . . | | . |      (BackThru
| . . . . | | . |       Thm* ∀s':(T+T) List.
| . . . . | | . |            paren(T;s')
| . . . . | | . |            ⇒
| . . . . | | . |            (∀i:T. (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
| . . . . | | . |       {paren_balance2})
| . . . . | | . |
| . . . . | | . .\
| . . . . | | . . 1. the_graph : Graph
| . . . . | | . . 2. GraphObject(the_graph)
| . . . . | | . . 3. Vertices(the_graph) List
| . . . . | | . . 4. L1 : Vertices(the_graph) List
| . . . . | | . . 5. s : traversal(the_graph)
| . . . . | | . . 6. i : Vertices(the_graph)
| . . . . | | . . 7. paren(Vertices(the_graph);s)
| . . . . | | . . 8. no_repeats(Vertices(the_graph)+
| . . . . | | . . |  Vertices(the_graph);s)
| . . . . | | . . 9. df-traversal(the_graph;s)
| . . . . | | . . 10. ∀i:Vertices(the_graph).
| . . . . | | . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . | | . . 11. ¬(inl(i) ∈ s)
| . . . . | | . . 12. s2 : traversal(the_graph)
| . . . . | | . . 13. l_disjoint(Vertices(the_graph)+
| . . . . | | . . |   Vertices(the_graph);s2;s)
| . . . . | | . . 14. paren(Vertices(the_graph);s2)
| . . . . | | . . 15. no_repeats(Vertices(the_graph)+
| . . . . | | . . |   Vertices(the_graph);s2)
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| . . . . . | | . . 16. ¬(inr(i) ∈ s2)
| . . . . . | | . . 17. ∀j:Vertices(the_graph).
| . . . . . | | . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . | | . . 18. df-traversal(the_graph;([inl(i)]
| . . . . . | | . . |                         @ s2
| . . . . . | | . . |                         @ [inr(i)])
| . . . . . | | . . |   @ s)
| . . . . . | | . . 19. ∀i@0:Vertices(the_graph).
| . . . . . | | . . |   L1 @ [i]-the_graph->*i@0
| . . . . . | | . . |   ⇒
| . . . . . | | . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . | | . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . | | . . |   L1 = (L1@0 @ L2)
| . . . . . | | . . |   ⇒
| . . . . . | | . . |   (∃s1,s2:traversal(the_graph).
| . . . . . | | . . |    s = (s2 @ s1)
| . . . . . | | . . |    & paren(Vertices(the_graph);s1)
| . . . . . | | . . |    & paren(Vertices(the_graph);s2)
| . . . . . | | . . |    & (∀j:Vertices(the_graph).
| . . . . . | | . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . | | . . |       & ((inl(j) ∈ s2)
| . . . . . | | . . |          ⟺
| . . . . . | | . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . . | | . . 21. L1@0 : Vertices(the_graph) List
| . . . . . | | . . 22. L2 : Vertices(the_graph) List
| . . . . . | | . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . | | . . 24. e : Vertices(the_graph) List
| . . . . . | | . . 25. L1@0 = (L1 @ e)
| . . . . . | | . . 26. [i] = (e @ L2)
| . . . . . | | . . 27. s1 : traversal(the_graph)
| . . . . . | | . . 28. s3 : traversal(the_graph)
| . . . . . | | . . 29. s = (s3 @ s1)
| . . . . . | | . . 30. paren(Vertices(the_graph);s1)
| . . . . . | | . . 31. paren(Vertices(the_graph);s3)
| . . . . . | | . . 32. ∀j:Vertices(the_graph).
| . . . . . | | . . |   (inl(j) ∈ s1) ⟺ L1-the_graph->*j
| . . . . . | | . . 33. ∀j:Vertices(the_graph).
| . . . . . | | . . |   (inl(j) ∈ s3)
| . . . . . | | . . |   ⟺
| . . . . . | | . . |   nil-the_graph->*j & ¬L1-the_graph->*j
| . . . . . | | . . 34. ∀j:Vertices(the_graph). (inl(j) ∈ s3) ⇒ False
| . . . . . | | . . 35. ∀j:Vertices(the_graph). (inr(j) ∈ s3) ⇒ False
| . . . . . | | . . ⊢  s3 = nil  by (Analyze -8) THEN (Analyze -9)
| . . . . . | | . . |
| . . . . . | | . . |\
| . . . . . | | . . | 1. the_graph : Graph
| . . . . . | | . . | 2. GraphObject(the_graph)
| . . . . . | | . . | 3. Vertices(the_graph) List
| . . . . . | | . . | 4. L1 : Vertices(the_graph) List
| . . . . . | | . . | 5. s : traversal(the_graph)
| . . . . . | | . . | 6. i : Vertices(the_graph)
| . . . . . | | . . | 7. paren(Vertices(the_graph);s)
| . . . . . | | . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . | | . . | |  Vertices(the_graph);s)
| . . . . . | | . . | 9. df-traversal(the_graph;s)
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| . . . . | | . . | 10. ∀i:Vertices(the_graph).
| . . . . | | . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . | | . . | 11. ¬(inl(i) ∈ s)
| . . . . | | . . | 12. s2 : traversal(the_graph)
| . . . . | | . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . | | . . | |   Vertices(the_graph);s2;s)
| . . . . | | . . | 14. paren(Vertices(the_graph);s2)
| . . . . | | . . | 15. no_repeats(Vertices(the_graph)+
| . . . . | | . . | |   Vertices(the_graph);s2)
| . . . . | | . . | 16. ¬(inr(i) ∈ s2)
| . . . . | | . . | 17. ∀j:Vertices(the_graph).
| . . . . | | . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . | | . . | 18. df-traversal(the_graph;([inl(i)]
| . . . . | | . . | |                         @ s2
| . . . . | | . . | |                         @ [inr(i)])
| . . . . | | . . | |   @ s)
| . . . . | | . . | 19. ∀i@0:Vertices(the_graph).
| . . . . | | . . | |   L1 @ [i]-the_graph->*i@0
| . . . . | | . . | |   ⇒
| . . . . | | . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . | | . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . | | . . | |   L1 = (L1@0 @ L2)
| . . . . | | . . | |   ⇒
| . . . . | | . . | |   (∃s1,s2:traversal(the_graph).
| . . . . | | . . | |    s = (s2 @ s1)
| . . . . | | . . | |    & paren(Vertices(the_graph);s1)
| . . . . | | . . | |    & paren(Vertices(the_graph);s2)
| . . . . | | . . | |    & ...)
| . . . . | | . . | 21. L1@0 : Vertices(the_graph) List
| . . . . | | . . | 22. L2 : Vertices(the_graph) List
| . . . . | | . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . | | . . | 24. e : Vertices(the_graph) List
| . . . . | | . . | 25. L1@0 = (L1 @ e)
| . . . . | | . . | 26. [i] = (e @ L2)
| . . . . | | . . | 27. s1 : traversal(the_graph)
| . . . . | | . . | 28. x : Vertices(the_graph)
| . . . . | | . . | 29. v :
| . . . . | | . . | |   (Vertices(the_graph)+Vertices(the_graph)) List
| . . . . | | . . | 30. s = ([inl(x) / v] @ s1)
| . . . . | | . . | 31. paren(Vertices(the_graph);s1)
| . . . . | | . . | 32. paren(Vertices(the_graph);[inl(x) / v])
| . . . . | | . . | 33. ∀j:Vertices(the_graph).
| . . . . | | . . | |   (inl(j) ∈ s1) ⟺ L1-the_graph->*j
| . . . . | | . . | 34. ∀j:Vertices(the_graph).
| . . . . | | . . | |   (inl(j) ∈ [inl(x) / v])
| . . . . | | . . | |   ⟺
| . . . . | | . . | |   nil-the_graph->*j & ¬L1-the_graph->*j
| . . . . | | . . | 35. ∀j:Vertices(the_graph).
| . . . . | | . . | |   (inl(j) ∈ [inl(x) / v]) ⇒ False
| . . . . | | . . | 36. ∀j:Vertices(the_graph).
| . . . . | | . . | |   (inr(j) ∈ [inl(x) / v]) ⇒ False
| . . . . | | . . | ⊢  [inl(x) / v] = nil  by InstHyp [x] -2
| . . . . | | . . | |
| . . . . | | . . | .....antecedent .....
| . . . . | | . . | 1. the_graph : Graph
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| . . . . | | . . | 2. GraphObject(the_graph)
| . . . . | | . . | 3. Vertices(the_graph) List
| . . . . | | . . | 4. L1 : Vertices(the_graph) List
| . . . . | | . . | 5. s : traversal(the_graph)
| . . . . | | . . | 6. i : Vertices(the_graph)
| . . . . | | . . | 7. paren(Vertices(the_graph);s)
| . . . . | | . . | 8. no_repeats(Vertices(the_graph)+
| . . . . | | . . |    Vertices(the_graph);s)
| . . . . | | . . | 9. df-traversal(the_graph;s)
| . . . . | | . . | 10. ∀i:Vertices(the_graph).
| . . . . | | . . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . | | . . | 11. ¬(inl(i) ∈ s)
| . . . . | | . . | 12. s2 : traversal(the_graph)
| . . . . | | . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . | | . . |     Vertices(the_graph);s2;s)
| . . . . | | . . | 14. paren(Vertices(the_graph);s2)
| . . . . | | . . | 15. no_repeats(Vertices(the_graph)+
| . . . . | | . . |     Vertices(the_graph);s2)
| . . . . | | . . | 16. ¬(inr(i) ∈ s2)
| . . . . | | . . | 17. ∀j:Vertices(the_graph).
| . . . . | | . . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . | | . . | 18. df-traversal(the_graph;([inl(i)]
| . . . . | | . . |                                 @ s2
| . . . . | | . . |                                 @ [inr(i)])
| . . . . | | . . |     @ s)
| . . . . | | . . | 19. ∀i@0:Vertices(the_graph).
| . . . . | | . . |     L1 @ [i]-the_graph->*i@0
| . . . . | | . . |     ⇒
| . . . . | | . . |     ¬non-trivial-loop(the_graph;i@0)
| . . . . | | . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . | | . . |     L1 = (L1@0 @ L2)
| . . . . | | . . |     ⇒
| . . . . | | . . |     (∃s1,s2:traversal(the_graph).
| . . . . | | . . |      s = (s2 @ s1)
| . . . . | | . . |      & paren(Vertices(the_graph);s1)
| . . . . | | . . |      & paren(Vertices(the_graph);s2)
| . . . . | | . . |      & ...)
| . . . . | | . . | 21. L1@0 : Vertices(the_graph) List
| . . . . | | . . | 22. L2 : Vertices(the_graph) List
| . . . . | | . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . | | . . | 24. e : Vertices(the_graph) List
| . . . . | | . . | 25. L1@0 = (L1 @ e)
| . . . . | | . . | 26. [i] = (e @ L2)
| . . . . | | . . | 27. s1 : traversal(the_graph)
| . . . . | | . . | 28. x : Vertices(the_graph)
| . . . . | | . . | 29. v :
| . . . . | | . . |     (Vertices(the_graph)+Vertices(the_graph)) List
| . . . . | | . . | 30. s = ([inl(x) / v] @ s1)
| . . . . | | . . | 31. paren(Vertices(the_graph);s1)
| . . . . | | . . | 32. paren(Vertices(the_graph);[inl(x) / v])
| . . . . | | . . | 33. ∀j:Vertices(the_graph).
| . . . . | | . . |     (inl(j) ∈ s1) ⟺ L1-the_graph->*j
| . . . . | | . . | 34. ∀j:Vertices(the_graph).
| . . . . | | . . |     (inl(j) ∈ [inl(x) / v])
| . . . . | | . . |     ⟺
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| . . . . | | . . |     nil-the_graph->*j & ¬L1-the_graph->*j
| . . . . | | . . | 35. ∀j:Vertices(the_graph).
| . . . . | | . . |     (inl(j) ∈ [inl(x) / v]) ⇒ False
| . . . . | | . . | 36. ∀j:Vertices(the_graph).
| . . . . | | . . |     (inr(j) ∈ [inl(x) / v]) ⇒ False
| . . . . | | . . | ⊢  (inl(x) ∈ [inl(x) / v])  by ObviousConcl
| . . . . | | . . |
| . . . . | | . . .\
| . . . . | | . . . 1. the_graph : Graph
| . . . . | | . . . 2. GraphObject(the_graph)
| . . . . | | . . . 3. Vertices(the_graph) List
| . . . . | | . . . 4. L1 : Vertices(the_graph) List
| . . . . | | . . . 5. s : traversal(the_graph)
| . . . . | | . . . 6. i : Vertices(the_graph)
| . . . . | | . . . 7. paren(Vertices(the_graph);s)
| . . . . | | . . . 8. no_repeats(Vertices(the_graph)+
| . . . . | | . . . |  Vertices(the_graph);s)
| . . . . | | . . . 9. df-traversal(the_graph;s)
| . . . . | | . . . 10. ∀i:Vertices(the_graph).
| . . . . | | . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . | | . . . 11. ¬(inl(i) ∈ s)
| . . . . | | . . . 12. s2 : traversal(the_graph)
| . . . . | | . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . | | . . . |   Vertices(the_graph);s2;s)
| . . . . | | . . . 14. paren(Vertices(the_graph);s2)
| . . . . | | . . . 15. no_repeats(Vertices(the_graph)+
| . . . . | | . . . |   Vertices(the_graph);s2)
| . . . . | | . . . 16. ¬(inr(i) ∈ s2)
| . . . . | | . . . 17. ∀j:Vertices(the_graph).
| . . . . | | . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . | | . . . 18. df-traversal(the_graph;([inl(i)]
| . . . . | | . . . |                            @ s2
| . . . . | | . . . |                            @ [inr(i)])
| . . . . | | . . . |   @ s)
| . . . . | | . . . 19. ∀i@0:Vertices(the_graph).
| . . . . | | . . . |   L1 @ [i]-the_graph->*i@0
| . . . . | | . . . |   ⇒
| . . . . | | . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . | | . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . | | . . . |   L1 = (L1@0 @ L2)
| . . . . | | . . . |   ⇒
| . . . . | | . . . |   (∃s1,s2:traversal(the_graph).
| . . . . | | . . . |    s = (s2 @ s1)
| . . . . | | . . . |    & paren(Vertices(the_graph);s1)
| . . . . | | . . . |    & paren(Vertices(the_graph);s2)
| . . . . | | . . . |    & ...)
| . . . . | | . . . 21. L1@0 : Vertices(the_graph) List
| . . . . | | . . . 22. L2 : Vertices(the_graph) List
| . . . . | | . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . | | . . . 24. e : Vertices(the_graph) List
| . . . . | | . . . 25. L1@0 = (L1 @ e)
| . . . . | | . . . 26. [i] = (e @ L2)
| . . . . | | . . . 27. s1 : traversal(the_graph)
| . . . . | | . . . 28. y : Vertices(the_graph)
| . . . . | | . . . 29. v :
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| . . . . | | . . . |   (Vertices(the_graph)+Vertices(the_graph)) List
| . . . . | | . . . 30. s = ([inr(y) / v] @ s1)
| . . . . | | . . . 31. paren(Vertices(the_graph);s1)
| . . . . | | . . . 32. paren(Vertices(the_graph);[inr(y) / v])
| . . . . | | . . . 33. ∀j:Vertices(the_graph).
| . . . . | | . . . |   (inl(j) ∈ s1) ⟺ L1-the_graph->*j
| . . . . | | . . . 34. ∀j:Vertices(the_graph).
| . . . . | | . . . |   (inl(j) ∈ [inr(y) / v])
| . . . . | | . . . |   ⟺
| . . . . | | . . . |   nil-the_graph->*j & ¬L1-the_graph->*j
| . . . . | | . . . 35. ∀j:Vertices(the_graph).
| . . . . | | . . . |   (inl(j) ∈ [inr(y) / v]) ⇒ False
| . . . . | | . . . 36. ∀j:Vertices(the_graph).
| . . . . | | . . . |   (inr(j) ∈ [inr(y) / v]) ⇒ False
| . . . . | | . . . ⊢  [inr(y) / v] = nil  by InstHyp [y] -1
| . . . . | | . . . |
| . . . . | | . . . .....antecedent .....
| . . . . | | . . . 1. the_graph : Graph
| . . . . | | . . . 2. GraphObject(the_graph)
| . . . . | | . . . 3. Vertices(the_graph) List
| . . . . | | . . . 4. L1 : Vertices(the_graph) List
| . . . . | | . . . 5. s : traversal(the_graph)
| . . . . | | . . . 6. i : Vertices(the_graph)
| . . . . | | . . . 7. paren(Vertices(the_graph);s)
| . . . . | | . . . 8. no_repeats(Vertices(the_graph)+
| . . . . | | . . .    Vertices(the_graph);s)
| . . . . | | . . . 9. df-traversal(the_graph;s)
| . . . . | | . . . 10. ∀i:Vertices(the_graph).
| . . . . | | . . .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . | | . . . 11. ¬(inl(i) ∈ s)
| . . . . | | . . . 12. s2 : traversal(the_graph)
| . . . . | | . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . | | . . .     Vertices(the_graph);s2;s)
| . . . . | | . . . 14. paren(Vertices(the_graph);s2)
| . . . . | | . . . 15. no_repeats(Vertices(the_graph)+
| . . . . | | . . .     Vertices(the_graph);s2)
| . . . . | | . . . 16. ¬(inr(i) ∈ s2)
| . . . . | | . . . 17. ∀j:Vertices(the_graph).
| . . . . | | . . .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . | | . . . 18. df-traversal(the_graph;([inl(i)]
| . . . . | | . . .                              @ s2
| . . . . | | . . .                              @ [inr(i)])
| . . . . | | . . .     @ s)
| . . . . | | . . . 19. ∀i@0:Vertices(the_graph).
| . . . . | | . . .     L1 @ [i]-the_graph->*i@0
| . . . . | | . . .     ⇒
| . . . . | | . . .     ¬non-trivial-loop(the_graph;i@0)
| . . . . | | . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . | | . . .     L1 = (L1@0 @ L2)
| . . . . | | . . .     ⇒
| . . . . | | . . .     (∃s1,s2:traversal(the_graph).
| . . . . | | . . .      s = (s2 @ s1)
| . . . . | | . . .      & paren(Vertices(the_graph);s1)
| . . . . | | . . .      & paren(Vertices(the_graph);s2)
| . . . . | | . . .      & ...)
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| . . . . | | . . . 21. L1@0 : Vertices(the_graph) List
| . . . . | | . . . 22. L2 : Vertices(the_graph) List
| . . . . | | . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . | | . . . 24. e : Vertices(the_graph) List
| . . . . | | . . . 25. L1@0 = (L1 @ e)
| . . . . | | . . . 26. [i] = (e @ L2)
| . . . . | | . . . 27. s1 : traversal(the_graph)
| . . . . | | . . . 28. y : Vertices(the_graph)
| . . . . | | . . . 29. v :
| . . . . | | . . .     (Vertices(the_graph)+Vertices(the_graph)) List
| . . . . | | . . . 30. s = ([inr(y) / v] @ s1)
| . . . . | | . . . 31. paren(Vertices(the_graph);s1)
| . . . . | | . . . 32. paren(Vertices(the_graph);[inr(y) / v])
| . . . . | | . . . 33. ∀j:Vertices(the_graph).
| . . . . | | . . .     (inl(j) ∈ s1) ⟺ L1-the_graph->*j
| . . . . | | . . . 34. ∀j:Vertices(the_graph).
| . . . . | | . . .     (inl(j) ∈ [inr(y) / v])
| . . . . | | . . .     ⟺
| . . . . | | . . .     nil-the_graph->*j & ¬L1-the_graph->*j
| . . . . | | . . . 35. ∀j:Vertices(the_graph).
| . . . . | | . . .     (inl(j) ∈ [inr(y) / v]) ⇒ False
| . . . . | | . . . 36. ∀j:Vertices(the_graph).
| . . . . | | . . .     (inr(j) ∈ [inr(y) / v]) ⇒ False
| . . . . | | . . . ⊢  (inr(y) ∈ [inr(y) / v])  by ObviousConcl
| . . . . | |
| . . . . | .\
| . . . . | . 1. the_graph : Graph
| . . . . | . 2. GraphObject(the_graph)
| . . . . | . 3. Vertices(the_graph) List
| . . . . | . 4. L1 : Vertices(the_graph) List
| . . . . | . 5. s : traversal(the_graph)
| . . . . | . 6. i : Vertices(the_graph)
| . . . . | . 7. paren(Vertices(the_graph);s)
| . . . . | . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . . | . 9. df-traversal(the_graph;s)
| . . . . | . 10. ∀i:Vertices(the_graph).
| . . . . | .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . | . 11. ¬(inl(i) ∈ s)
| . . . . | . 12. s2 : traversal(the_graph)
| . . . . | . 13. l_disjoint(Vertices(the_graph)+
| . . . . | .     Vertices(the_graph);s2;s)
| . . . . | . 14. paren(Vertices(the_graph);s2)
| . . . . | . 15. no_repeats(Vertices(the_graph)+
| . . . . | .     Vertices(the_graph);s2)
| . . . . | . 16. ¬(inr(i) ∈ s2)
| . . . . | . 17. ∀j:Vertices(the_graph).
| . . . . | .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . | . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . . | .     @ s)
| . . . . | . 19. ∀i@0:Vertices(the_graph).
| . . . . | .     L1 @ [i]-the_graph->*i@0
| . . . . | .     ⇒
| . . . . | .     ¬non-trivial-loop(the_graph;i@0)
| . . . . | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . | .     L1 = (L1@0 @ L2)
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| . . . . | .     ⇒
| . . . . | .     (∃s1,s2:traversal(the_graph).
| . . . . | .      s = (s2 @ s1)
| . . . . | .      & paren(Vertices(the_graph);s1)
| . . . . | .      & paren(Vertices(the_graph);s2)
| . . . . | .      & (∀j:Vertices(the_graph).
| . . . . | .         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . | .         & ((inl(j) ∈ s2)
| . . . . | .            ⟺
| . . . . | .            L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . | . 21. L1@0 : Vertices(the_graph) List
| . . . . | . 22. L2 : Vertices(the_graph) List
| . . . . | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . | . 24. e : Vertices(the_graph) List
| . . . . | . 25. L1@0 = (L1 @ e)
| . . . . | . 26. [i] = (e @ L2)
| . . . . | . 27. s1 : traversal(the_graph)
| . . . . | . 28. s3 : traversal(the_graph)
| . . . . | . 29. s = (s3 @ s1)
| . . . . | . 30. paren(Vertices(the_graph);s1)
| . . . . | . 31. paren(Vertices(the_graph);s3)
| . . . . | . 32. ∀j:Vertices(the_graph).
| . . . . | .     (inl(j) ∈ s1) ⟺ L1-the_graph->*j
| . . . . | . 33. ∀j:Vertices(the_graph).
| . . . . | .     (inl(j) ∈ s3)
| . . . . | .     ⟺
| . . . . | .     nil-the_graph->*j & ¬L1-the_graph->*j
| . . . . | . 34. s3 = nil
| . . . . | . ⊢  ∀i:Vertices(the_graph).
| . . . . | .    (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . | .   by (SubstFor s 0) THEN (SubstFor s3 0) THEN (Reduce 0)
| . . . . | .      THEN
| . . . . | .      Trivial
| . . . . |
| . . . . .\
| . . . . . 1. the_graph : Graph
| . . . . . 2. GraphObject(the_graph)
| . . . . . 3. Vertices(the_graph) List
| . . . . . 4. L1 : Vertices(the_graph) List
| . . . . . 5. s : traversal(the_graph)
| . . . . . 6. i : Vertices(the_graph)
| . . . . . 7. paren(Vertices(the_graph);s)
| . . . . . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . . . 9. df-traversal(the_graph;s)
| . . . . . 10. ∀i:Vertices(the_graph).
| . . . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . 11. ¬(inl(i) ∈ s)
| . . . . . 12. s2 : traversal(the_graph)
| . . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . |   Vertices(the_graph);s2;s)
| . . . . . 14. paren(Vertices(the_graph);s2)
| . . . . . 15. no_repeats(Vertices(the_graph)+Vertices(the_graph);s2)
| . . . . . 16. ¬(inr(i) ∈ s2)
| . . . . . 17. ∀j:Vertices(the_graph).
| . . . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
```

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| . . . . . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . 19. ∀i@0:Vertices(the_graph).
| . . . . . |   L1 @ [i]-the_graph->*i@0
| . . . . . |   ⇒
| . . . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . |   L1 = (L1@0 @ L2)
| . . . . . |   ⇒
| . . . . . |   (∃s1,s2:traversal(the_graph).
| . . . . . |    s = (s2 @ s1)
| . . . . . |    & paren(Vertices(the_graph);s1)
| . . . . . |    & paren(Vertices(the_graph);s2)
| . . . . . |    & (∀j:Vertices(the_graph).
| . . . . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . |       & ((inl(j) ∈ s2)
| . . . . . |          ⟺
| . . . . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . . 21. L1@0 : Vertices(the_graph) List
| . . . . . 22. L2 : Vertices(the_graph) List
| . . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . 24. e : Vertices(the_graph) List
| . . . . . 25. L1@0 = (L1 @ e)
| . . . . . 26. [i] = (e @ L2)
| . . . . . 27. ∀i:Vertices(the_graph).
| . . . . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . ⊢  ∃s1,s2@0:traversal(the_graph).
| . . . . . |  (([inl(i)] @ s2 @ [inr(i)]) @ s) = (s2@0 @ s1)
| . . . . . |  & paren(Vertices(the_graph);s1)
| . . . . . |  & paren(Vertices(the_graph);s2@0)
| . . . . . |  & (∀j:Vertices(the_graph).
| . . . . . |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . |     & ((inl(j) ∈ s2@0)
| . . . . . |        ⟺
| . . . . . |        L2-the_graph->*j & ¬L1@0-the_graph->*j))
| . . . . . | by Assert (L2 = nil ∨ e = nil)
| . . . . . |
| . . . . . |\
| . . . . . | .....assertion .....
| . . . . . | 1. the_graph : Graph
| . . . . . | 2. GraphObject(the_graph)
| . . . . . | 3. Vertices(the_graph) List
| . . . . . | 4. L1 : Vertices(the_graph) List
| . . . . . | 5. s : traversal(the_graph)
| . . . . . | 6. i : Vertices(the_graph)
| . . . . . | 7. paren(Vertices(the_graph);s)
| . . . . . | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . . . | 9. df-traversal(the_graph;s)
| . . . . . | 10. ∀i:Vertices(the_graph).
| . . . . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . | 11. ¬(inl(i) ∈ s)
| . . . . . | 12. s2 : traversal(the_graph)
| . . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . | |   Vertices(the_graph);s2;s)
| . . . . . | 14. paren(Vertices(the_graph);s2)
| . . . . . | 15. no_repeats(Vertices(the_graph)+
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| . . . . . . | |   Vertices(the_graph);s2)
| . . . . . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . | 17. ∀j:Vertices(the_graph).
| . . . . . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . | 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . . . . | |   @ s)
| . . . . . . | 19. ∀i@0:Vertices(the_graph).
| . . . . . . | |   L1 @ [i]-the_graph->*i@0
| . . . . . . | |   ⇒
| . . . . . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . | |   L1 = (L1@0 @ L2)
| . . . . . . | |   ⇒
| . . . . . . | |   (∃s1,s2:traversal(the_graph).
| . . . . . . | |    s = (s2 @ s1)
| . . . . . . | |    & paren(Vertices(the_graph);s1)
| . . . . . . | |    & paren(Vertices(the_graph);s2)
| . . . . . . | |    & (∀j:Vertices(the_graph).
| . . . . . . | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . | |       & ((inl(j) ∈ s2)
| . . . . . . | |          ⟺
| . . . . . . | |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . . . | 21. L1@0 : Vertices(the_graph) List
| . . . . . . | 22. L2 : Vertices(the_graph) List
| . . . . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . | 24. e : Vertices(the_graph) List
| . . . . . . | 25. L1@0 = (L1 @ e)
| . . . . . . | 26. [i] = (e @ L2)
| . . . . . . | 27. ∀i:Vertices(the_graph).
| . . . . . . | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . | ⊢  L2 = nil ∨ e = nil
| . . . . . . | | by (((Lemmaize [-2]) THEN (Analyze -2))
| . . . . . . | |     THEN
| . . . . . . | |     (Analyze 3))
| . . . . . . | |    THEN
| . . . . . . | |    (All Reduce)
| . . . . . . | |
| . . . . . . | |\
| . . . . . . | | 1. the_graph : Graph
| . . . . . . | | 2. i : Vertices(the_graph)
| . . . . . . | | 3. [i] = nil
| . . . . . . | | ⊢  nil = nil ∨ nil = nil  by Obvious
| . . . . . . | |
| . . . . . . | |\
| . . . . . . | | 1. the_graph : Graph
| . . . . . . | | 2. i : Vertices(the_graph)
| . . . . . . | | 3. u : Vertices(the_graph)
| . . . . . . | | 4. v : Vertices(the_graph) List
| . . . . . . | | 5. [i] = [u / v]
| . . . . . . | | ⊢  [u / v] = nil ∨ nil = nil  by Obvious
| . . . . . . | |
| . . . . . . | |\
| . . . . . . | | 1. the_graph : Graph
| . . . . . . | | 2. i : Vertices(the_graph)
| . . . . . . | | 3. u : Vertices(the_graph)
```

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| . . . . . . | | 4. v : Vertices(the_graph) List
| . . . . . . | | 5. [i] = [u / (v @ nil)]
| . . . . . . | | ⊢  nil = nil ∨ [u / v] = nil  by Obvious
| . . . . . . | |
| . . . . . . | .\
| . . . . . . | . 1. the_graph : Graph
| . . . . . . | . 2. i : Vertices(the_graph)
| . . . . . . | . 3. u1 : Vertices(the_graph)
| . . . . . . | . 4. v1 : Vertices(the_graph) List
| . . . . . . | . 5. u : Vertices(the_graph)
| . . . . . . | . 6. v : Vertices(the_graph) List
| . . . . . . | . 7. [i] = [u / (v @ [u1 / v1])]
| . . . . . . | . ⊢  [u1 / v1] = nil ∨ [u / v] = nil  by Obvious
| . . . . . . |
| . . . . . . .\
| . . . . . . . 1. the_graph : Graph
| . . . . . . . 2. GraphObject(the_graph)
| . . . . . . . 3. Vertices(the_graph) List
| . . . . . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . 5. s : traversal(the_graph)
| . . . . . . . 6. i : Vertices(the_graph)
| . . . . . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
| . . . . . . . 9. df-traversal(the_graph;s)
| . . . . . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . 12. s2 : traversal(the_graph)
| . . . . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . |   Vertices(the_graph);s2;s)
| . . . . . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . |   Vertices(the_graph);s2)
| . . . . . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . . . . . |   @ s)
| . . . . . . . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . |   ⇒
| . . . . . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . |   L1 = (L1@0 @ L2)
| . . . . . . . |   ⇒
| . . . . . . . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . |    s = (s2 @ s1)
| . . . . . . . |    & paren(Vertices(the_graph);s1)
| . . . . . . . |    & paren(Vertices(the_graph);s2)
| . . . . . . . |    & (∀j:Vertices(the_graph).
| . . . . . . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . |       & ((inl(j) ∈ s2)
| . . . . . . . |          ⟺
| . . . . . . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . . . . 21. L1@0 : Vertices(the_graph) List
```

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| . . . . . . . 22. L2 : Vertices(the_graph) List
| . . . . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . 24. e : Vertices(the_graph) List
| . . . . . . . 25. L1@0 = (L1 @ e)
| . . . . . . . 26. [i] = (e @ L2)
| . . . . . . . 27. ∀i:Vertices(the_graph).
| . . . . . . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . 28. L2 = nil ∨ e = nil
| . . . . . . . ⊢  ∃s1,s2@0:traversal(the_graph).
| . . . . . . . |  (([inl(i)] @ s2 @ [inr(i)]) @ s) = (s2@0 @ s1)
| . . . . . . . |  & paren(Vertices(the_graph);s1)
| . . . . . . . |  & paren(Vertices(the_graph);s2@0)
| . . . . . . . |  & (∀j:Vertices(the_graph).
| . . . . . . . |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . |     & ((inl(j) ∈ s2@0)
| . . . . . . . |        ⟺
| . . . . . . . |        L2-the_graph->*j & ¬L1@0-the_graph->*j))
| . . . . . . . | by Analyze -1
| . . . . . . . |
| . . . . . . . |\
| . . . . . . . | 1. the_graph : Graph
| . . . . . . . | 2. GraphObject(the_graph)
| . . . . . . . | 3. Vertices(the_graph) List
| . . . . . . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . | 5. s : traversal(the_graph)
| . . . . . . . | 6. i : Vertices(the_graph)
| . . . . . . . | 7. paren(Vertices(the_graph);s)
| . . . . . . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | |  Vertices(the_graph);s)
| . . . . . . . | 9. df-traversal(the_graph;s)
| . . . . . . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . | 12. s2 : traversal(the_graph)
| . . . . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | |   Vertices(the_graph);s2;s)
| . . . . . . . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | |   Vertices(the_graph);s2)
| . . . . . . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . . . . . | |   @ s)
| . . . . . . . | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | |   ⇒
| . . . . . . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | |   L1 = (L1@0 @ L2)
| . . . . . . . | |   ⇒
| . . . . . . . | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | |    s = (s2 @ s1)
| . . . . . . . | |    & paren(Vertices(the_graph);s1)
| . . . . . . . | |    & paren(Vertices(the_graph);s2)
```

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| . . . . . . . | |    & (∀j:Vertices(the_graph).
| . . . . . . . | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | |       & ((inl(j) ∈ s2)
| . . . . . . . | |          ⟺
| . . . . . . . | |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . . . . | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | 22. L2 : Vertices(the_graph) List
| . . . . . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | 24. e : Vertices(the_graph) List
| . . . . . . . | 25. L1@0 = (L1 @ e)
| . . . . . . . | 26. [i] = (e @ L2)
| . . . . . . . | 27. ∀i:Vertices(the_graph).
| . . . . . . . | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | 28. L2 = nil
| . . . . . . . | ⊢  ∃s1,s2@0:traversal(the_graph).
| . . . . . . . | |  (([inl(i)] @ s2 @ [inr(i)]) @ s) = (s2@0 @ s1)
| . . . . . . . | |  & paren(Vertices(the_graph);s1)
| . . . . . . . | |  & paren(Vertices(the_graph);s2@0)
| . . . . . . . | |  & (∀j:Vertices(the_graph).
| . . . . . . . | |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | |     & ((inl(j) ∈ s2@0)
| . . . . . . . | |        ⟺
| . . . . . . . | |        L2-the_graph->*j & ¬L1@0-the_graph->*j))
| . . . . . . . | | by ((HypSubst -1 0)
| . . . . . . . | |     THEN
| . . . . . . . | |     (InstConcl [(([inl(i)] @ s2 @ [inr(i)]) @ s;nil])
| . . . . . . . | |     THEN
| . . . . . . . | |     (Analyze 0))
| . . . . . . . | |    THENL
| . . . . . . . | |    [Reduce 0;Analyze 0]
| . . . . . . . | |
| . . . . . . . | |\
| . . . . . . . | | 1. the_graph : Graph
| . . . . . . . | | 2. GraphObject(the_graph)
| . . . . . . . | | 3. Vertices(the_graph) List
| . . . . . . . | | 4. L1 : Vertices(the_graph) List
| . . . . . . . | | 5. s : traversal(the_graph)
| . . . . . . . | | 6. i : Vertices(the_graph)
| . . . . . . . | | 7. paren(Vertices(the_graph);s)
| . . . . . . . | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | | |  Vertices(the_graph);s)
| . . . . . . . | | 9. df-traversal(the_graph;s)
| . . . . . . . | | 10. ∀i:Vertices(the_graph).
| . . . . . . . | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | | 11. ¬(inl(i) ∈ s)
| . . . . . . . | | 12. s2 : traversal(the_graph)
| . . . . . . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | | |   Vertices(the_graph);s2;s)
| . . . . . . . | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | | |   Vertices(the_graph);s2)
| . . . . . . . | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | | 17. ∀j:Vertices(the_graph).
| . . . . . . . | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | | 18. df-traversal(the_graph;([inl(i)]
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| . . . . . . . | | |                            @ s2
| . . . . . . . | | |                            @ [inr(i)])
| . . . . . . . | | |   @ s)
| . . . . . . . | | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | | |   ⇒
| . . . . . . . | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | | |   L1 = (L1@0 @ L2)
| . . . . . . . | | |   ⇒
| . . . . . . . | | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | | |    s = (s2 @ s1)
| . . . . . . . | | |    & paren(Vertices(the_graph);s1)
| . . . . . . . | | |    & paren(Vertices(the_graph);s2)
| . . . . . . . | | |    & (∀j:Vertices(the_graph).
| . . . . . . . | | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | | |       & ((inl(j) ∈ s2)
| . . . . . . . | | |          ⟺
| . . . . . . . | | |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . . . . | | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | | 22. L2 : Vertices(the_graph) List
| . . . . . . . | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | | 24. e : Vertices(the_graph) List
| . . . . . . . | | 25. L1@0 = (L1 @ e)
| . . . . . . . | | 26. [i] = (e @ L2)
| . . . . . . . | | 27. ∀i:Vertices(the_graph).
| . . . . . . . | | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | | 28. L2 = nil
| . . . . . . . | | ⊢  paren(Vertices(the_graph);([inl(i)]
| . . . . . . . | | |                              @ s2
| . . . . . . . | | |                              @ [inr(i)])
| . . . . . . . | | |  @ s)
| . . . . . . . | | |  & paren(Vertices(the_graph);nil)  by Auto
| . . . . . . . | | |
| . . . . . . . | | |\
| . . . . . . . | | | 1. the_graph : Graph
| . . . . . . . | | | 2. GraphObject(the_graph)
| . . . . . . . | | | 3. Vertices(the_graph) List
| . . . . . . . | | | 4. L1 : Vertices(the_graph) List
| . . . . . . . | | | 5. s : traversal(the_graph)
| . . . . . . . | | | 6. i : Vertices(the_graph)
| . . . . . . . | | | 7. paren(Vertices(the_graph);s)
| . . . . . . . | | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | | | |  Vertices(the_graph);s)
| . . . . . . . | | | 9. df-traversal(the_graph;s)
| . . . . . . . | | | 10. ∀i:Vertices(the_graph).
| . . . . . . . | | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | | | 11. ¬(inl(i) ∈ s)
| . . . . . . . | | | 12. s2 : traversal(the_graph)
| . . . . . . . | | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | | | |   Vertices(the_graph);s2;s)
| . . . . . . . | | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | | | |   Vertices(the_graph);s2)
| . . . . . . . | | | 16. ¬(inr(i) ∈ s2)
```

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| . . . . . . . | | | 17. ∀j:Vertices(the_graph).
| . . . . . . . | | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | | | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | | | |                         @ s2
| . . . . . . . | | | |                         @ [inr(i)])
| . . . . . . . | | | |   @ s)
| . . . . . . . | | | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | | | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | | | |   ⇒
| . . . . . . . | | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | | | |   L1 = (L1@0 @ L2)
| . . . . . . . | | | |   ⇒
| . . . . . . . | | | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | | | |    s = (s2 @ s1)
| . . . . . . . | | | |    & paren(Vertices(the_graph);s1)
| . . . . . . . | | | |    & paren(Vertices(the_graph);s2)
| . . . . . . . | | | |    & ...)
| . . . . . . . | | | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | | | 22. L2 : Vertices(the_graph) List
| . . . . . . . | | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | | | 24. e : Vertices(the_graph) List
| . . . . . . . | | | 25. L1@0 = (L1 @ e)
| . . . . . . . | | | 26. [i] = (e @ L2)
| . . . . . . . | | | 27. ∀i:Vertices(the_graph).
| . . . . . . . | | | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | | | 28. L2 = nil
| . . . . . . . | | | ⊢  paren(Vertices(the_graph);([inl(i)]
| . . . . . . . | | | |                           @ s2
| . . . . . . . | | | |                           @ [inr(i)])
| . . . . . . . | | | |  @ s)
| . . . . . . . | | | | by BackThru
| . . . . . . . | | | |    Thm* ∀s',s3:(T+T) List.
| . . . . . . . | | | |         paren(T;s')
| . . . . . . . | | | |         ⇒
| . . . . . . . | | | |         paren(T;s3) ⇒ paren(T;s3 @ s')
| . . . . . . . | | | |    {append_paren}
| . . . . . . . | | | |
| . . . . . . . | | | 1. the_graph : Graph
| . . . . . . . | | | 2. GraphObject(the_graph)
| . . . . . . . | | | 3. Vertices(the_graph) List
| . . . . . . . | | | 4. L1 : Vertices(the_graph) List
| . . . . . . . | | | 5. s : traversal(the_graph)
| . . . . . . . | | | 6. i : Vertices(the_graph)
| . . . . . . . | | | 7. paren(Vertices(the_graph);s)
| . . . . . . . | | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | | |    Vertices(the_graph);s)
| . . . . . . . | | | 9. df-traversal(the_graph;s)
| . . . . . . . | | | 10. ∀i:Vertices(the_graph).
| . . . . . . . | | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | | | 11. ¬(inl(i) ∈ s)
| . . . . . . . | | | 12. s2 : traversal(the_graph)
| . . . . . . . | | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | | |     Vertices(the_graph);s2;s)
| . . . . . . . | | | 14. paren(Vertices(the_graph);s2)
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| . . . . . . . | | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | | |     Vertices(the_graph);s2)
| . . . . . . . | | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | | | 17. ∀j:Vertices(the_graph).
| . . . . . . . | | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | | | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | | |                              @ s2
| . . . . . . . | | |                              @ [inr(i)])
| . . . . . . . | | |     @ s)
| . . . . . . . | | | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | | |     L1 @ [i]-the_graph->*i@0
| . . . . . . . | | |     ⇒
| . . . . . . . | | |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | | |     L1 = (L1@0 @ L2)
| . . . . . . . | | |     ⇒
| . . . . . . . | | |     (∃s1,s2:traversal(the_graph).
| . . . . . . . | | |      s = (s2 @ s1)
| . . . . . . . | | |      & paren(Vertices(the_graph);s1)
| . . . . . . . | | |      & paren(Vertices(the_graph);s2)
| . . . . . . . | | |      & ...)
| . . . . . . . | | | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | | | 22. L2 : Vertices(the_graph) List
| . . . . . . . | | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | | | 24. e : Vertices(the_graph) List
| . . . . . . . | | | 25. L1@0 = (L1 @ e)
| . . . . . . . | | | 26. [i] = (e @ L2)
| . . . . . . . | | | 27. ∀i:Vertices(the_graph).
| . . . . . . . | | |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | | | 28. L2 = nil
| . . . . . . . | | | ⊢  paren(Vertices(the_graph);[inl(i)]
| . . . . . . . | | |    @ s2
| . . . . . . . | | |    @ [inr(i)])
| . . . . . . . | | |   by (((RecUnfold ‘paren‘ 0) THEN OrRight)
| . . . . . . . | | |       THEN
| . . . . . . . | | |       OrLeft)
| . . . . . . . | | |      THEN
| . . . . . . . | | |      (InstConcl [i;s2])
| . . . . . . . | | |
| . . . . . . . | | .\
| . . . . . . . | | . 1. the_graph : Graph
| . . . . . . . | | . 2. GraphObject(the_graph)
| . . . . . . . | | . 3. Vertices(the_graph) List
| . . . . . . . | | . 4. L1 : Vertices(the_graph) List
| . . . . . . . | | . 5. s : traversal(the_graph)
| . . . . . . . | | . 6. i : Vertices(the_graph)
| . . . . . . . | | . 7. paren(Vertices(the_graph);s)
| . . . . . . . | | . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | | .    Vertices(the_graph);s)
| . . . . . . . | | . 9. df-traversal(the_graph;s)
| . . . . . . . | | . 10. ∀i:Vertices(the_graph).
| . . . . . . . | | .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | | . 11. ¬(inl(i) ∈ s)
| . . . . . . . | | . 12. s2 : traversal(the_graph)
| . . . . . . . | | . 13. l_disjoint(Vertices(the_graph)+
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| . . . . . . . | | .     Vertices(the_graph);s2;s)
| . . . . . . . | | . 14. paren(Vertices(the_graph);s2)
| . . . . . . . | | . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | | .     Vertices(the_graph);s2)
| . . . . . . . | | . 16. ¬(inr(i) ∈ s2)
| . . . . . . . | | . 17. ∀j:Vertices(the_graph).
| . . . . . . . | | .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | | . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | | .                             @ s2
| . . . . . . . | | .                             @ [inr(i)])
| . . . . . . . | | .     @ s)
| . . . . . . . | | . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | | .     L1 @ [i]-the_graph->*i@0
| . . . . . . . | | .     ⇒
| . . . . . . . | | .     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | | .     L1 = (L1@0 @ L2)
| . . . . . . . | | .     ⇒
| . . . . . . . | | .     (∃s1,s2:traversal(the_graph).
| . . . . . . . | | .      s = (s2 @ s1)
| . . . . . . . | | .      & paren(Vertices(the_graph);s1)
| . . . . . . . | | .      & paren(Vertices(the_graph);s2)
| . . . . . . . | | .      & ...)
| . . . . . . . | | . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | | . 22. L2 : Vertices(the_graph) List
| . . . . . . . | | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | | . 24. e : Vertices(the_graph) List
| . . . . . . . | | . 25. L1@0 = (L1 @ e)
| . . . . . . . | | . 26. [i] = (e @ L2)
| . . . . . . . | | . 27. ∀i:Vertices(the_graph).
| . . . . . . . | | .     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | | . 28. L2 = nil
| . . . . . . . | | . ⊢  paren(Vertices(the_graph);nil)
| . . . . . . . | | .   by ObviousConcl
| . . . . . . . | |
| . . . . . . . | .\
| . . . . . . . | . 1. the_graph : Graph
| . . . . . . . | . 2. GraphObject(the_graph)
| . . . . . . . | . 3. Vertices(the_graph) List
| . . . . . . . | . 4. L1 : Vertices(the_graph) List
| . . . . . . . | . 5. s : traversal(the_graph)
| . . . . . . . | . 6. i : Vertices(the_graph)
| . . . . . . . | . 7. paren(Vertices(the_graph);s)
| . . . . . . . | . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . |  Vertices(the_graph);s)
| . . . . . . . | . 9. df-traversal(the_graph;s)
| . . . . . . . | . 10. ∀i:Vertices(the_graph).
| . . . . . . . | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . 11. ¬(inl(i) ∈ s)
| . . . . . . . | . 12. s2 : traversal(the_graph)
| . . . . . . . | . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . |   Vertices(the_graph);s2;s)
| . . . . . . . | . 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . |   Vertices(the_graph);s2)
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| . . . . . . . | . 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . 17. ∀j:Vertices(the_graph).
| . . . . . . . | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . |                           @ s2
| . . . . . . . | . |                           @ [inr(i)])
| . . . . . . . | . |   @ s)
| . . . . . . . | . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . |   ⇒
| . . . . . . . | . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . |   L1 = (L1@0 @ L2)
| . . . . . . . | . |   ⇒
| . . . . . . . | . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . |    s = (s2 @ s1)
| . . . . . . . | . |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . |    & (∀j:Vertices(the_graph).
| . . . . . . . | . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | . |       & ((inl(j) ∈ s2)
| . . . . . . . | . |          ⟺
| . . . . . . . | . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . . . . | . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . 22. L2 : Vertices(the_graph) List
| . . . . . . . | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . 24. e : Vertices(the_graph) List
| . . . . . . . | . 25. L1@0 = (L1 @ e)
| . . . . . . . | . 26. [i] = (e @ L2)
| . . . . . . . | . 27. ∀i:Vertices(the_graph).
| . . . . . . . | . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . 28. L2 = nil
| . . . . . . . | . ⊢  ∀j:Vertices(the_graph).
| . . . . . . . | . |  ((inl(j) ∈ ([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . |   ⟺
| . . . . . . . | . |   L1@0-the_graph->*j)
| . . . . . . . | . |  & ((inl(j) ∈ nil)
| . . . . . . . | . |     ⟺
| . . . . . . . | . |     nil-the_graph->*j & ¬L1@0-the_graph->*j)
| . . . . . . . | . | by Subst (L1@0 = (L1 @ [i])) 0
| . . . . . . . | . |
| . . . . . . . | . |\
| . . . . . . . | . | .....equality .....
| . . . . . . . | . | 1. the_graph : Graph
| . . . . . . . | . | 2. GraphObject(the_graph)
| . . . . . . . | . | 3. Vertices(the_graph) List
| . . . . . . . | . | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . | 5. s : traversal(the_graph)
| . . . . . . . | . | 6. i : Vertices(the_graph)
| . . . . . . . | . | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . |    Vertices(the_graph);s)

| . . . . . . . | . | 9. df-traversal(the_graph;s)
| . . . . . . . | . | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
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| . . . . . . . | . | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . | 12. s2 : traversal(the_graph)
| . . . . . . . | . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . |     Vertices(the_graph);s2;s)
| . . . . . . . | . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . |     Vertices(the_graph);s2)
| . . . . . . . | . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . |                             @ s2
| . . . . . . . | . |                             @ [inr(i)])
| . . . . . . . | . |     @ s)
| . . . . . . . | . | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . |     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . |     ⇒
| . . . . . . . | . |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . |     L1 = (L1@0 @ L2)
| . . . . . . . | . |     ⇒
| . . . . . . . | . |     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . |      s = (s2 @ s1)
| . . . . . . . | . |      & paren(Vertices(the_graph);s1)
| . . . . . . . | . |      & paren(Vertices(the_graph);s2)
| . . . . . . . | . |      & ...)
| . . . . . . . | . | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . | 22. L2 : Vertices(the_graph) List
| . . . . . . . | . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . | 24. e : Vertices(the_graph) List
| . . . . . . . | . | 25. L1@0 = (L1 @ e)
| . . . . . . . | . | 26. [i] = (e @ L2)
| . . . . . . . | . | 27. ∀i:Vertices(the_graph).
| . . . . . . . | . |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . | 28. L2 = nil
| . . . . . . . | . | ⊢  L1@0 = (L1 @ [i])
| . . . . . . . | . |   by (((HypSubst -4 0) THEN Analyze)
| . . . . . . . | . |       THEN
| . . . . . . . | . |       (HypSubst -1 -3))
| . . . . . . . | . |      THEN
| . . . . . . . | . |      (RWO
| . . . . . . . | . |       Thm* ∀l:T List. (l @ nil) ~ l
| . . . . . . . | . |       {append_nil_sq}
| . . . . . . . | . |       -3)
| . . . . . . . | . |
| . . . . . . . | . .\
| . . . . . . . | . . 1. the_graph : Graph
| . . . . . . . | . . 2. GraphObject(the_graph)
| . . . . . . . | . . 3. Vertices(the_graph) List
| . . . . . . . | . . 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . 5. s : traversal(the_graph)
| . . . . . . . | . . 6. i : Vertices(the_graph)
| . . . . . . . | . . 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . |  Vertices(the_graph);s)
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| . . . . . . . | . . 9. df-traversal(the_graph;s)
| . . . . . . . | . . 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . 12. s2 : traversal(the_graph)
| . . . . . . . | . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . |   Vertices(the_graph);s2;s)
| . . . . . . . | . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . |   Vertices(the_graph);s2)
| . . . . . . . | . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . |                            @ s2
| . . . . . . . | . . |                            @ [inr(i)])
| . . . . . . . | . . |   @ s)
| . . . . . . . | . . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . |   ⇒
| . . . . . . . | . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . |   L1 = (L1@0 @ L2)
| . . . . . . . | . . |   ⇒
| . . . . . . . | . . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . |    s = (s2 @ s1)
| . . . . . . . | . . |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . |    & ...)
| . . . . . . . | . . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . 24. e : Vertices(the_graph) List
| . . . . . . . | . . 25. L1@0 = (L1 @ e)
| . . . . . . . | . . 26. [i] = (e @ L2)
| . . . . . . . | . . 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . 28. L2 = nil
| . . . . . . . | . . ⊢  ∀j:Vertices(the_graph).
| . . . . . . . | . . |  ((inl(j) ∈ ([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . |   ⟺
| . . . . . . . | . . |   L1 @ [i]-the_graph->*j)

| . . . . . . . | . . |  & ((inl(j) ∈ nil)
| . . . . . . . | . . |     ⟺
| . . . . . . . | . . |     nil-the_graph->*j & ¬L1 @ [i]-the_graph->*j)
| . . . . . . . | . . | by (Analyze 0) THEN (Analyze 0)
| . . . . . . . | . . |
| . . . . . . . | . . |\
| . . . . . . . | . . | 1. the_graph : Graph
| . . . . . . . | . . | 2. GraphObject(the_graph)
| . . . . . . . | . . | 3. Vertices(the_graph) List
| . . . . . . . | . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | 5. s : traversal(the_graph)
| . . . . . . . | . . | 6. i : Vertices(the_graph)
| . . . . . . . | . . | 7. paren(Vertices(the_graph);s)
```

```
| . . . . . . . | . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | |  Vertices(the_graph);s)
| . . . . . . . | . . | 9. df-traversal(the_graph;s)
| . . . . . . . | . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | 12. s2 : traversal(the_graph)
| . . . . . . . | . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | |   Vertices(the_graph);s2;s)
| . . . . . . . | . . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | |   Vertices(the_graph);s2)
| . . . . . . . | . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . | |                             @ s2
| . . . . . . . | . . | |                             @ [inr(i)])
| . . . . . . . | . . | |   @ s)
| . . . . . . . | . . | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | |   ⇒
| . . . . . . . | . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | |   L1 = (L1@0 @ L2)
| . . . . . . . | . . | |   ⇒
| . . . . . . . | . . | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | |    s = (s2 @ s1)
| . . . . . . . | . . | |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | |    & (∀j:Vertices(the_graph).
| . . . . . . . | . . | |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | . . | |        & ((inl(j) ∈ s2)
| . . . . . . . | . . | |           ⟺
| . . . . . . . | . . | |           L2-the_graph->*j
| . . . . . . . | . . | |           & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | 24. e : Vertices(the_graph) List
| . . . . . . . | . . | 25. L1@0 = (L1 @ e)
| . . . . . . . | . . | 26. [i] = (e @ L2)
| . . . . . . . | . . | 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | 28. L2 = nil
| . . . . . . . | . . | 29. j : Vertices(the_graph)
| . . . . . . . | . . | ⊢  (inl(j) ∈ ([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . | |  ⟺
| . . . . . . . | . . | |  L1 @ [i]-the_graph->*j
| . . . . . . . | . . | | by ((RWO
| . . . . . . . | . . | |      Thm* For any graph
| . . . . . . . | . . | |           ∀A,B:V List, i:V.
| . . . . . . . | . . | |           A @ B-->*i ⟺ A-->*i ∨ B-->*i
| . . . . . . . | . . | |      {list_DASH_connect_DASH_append}
| . . . . . . . | . . | |      0)
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```
| . . . . . . . | . . | |     THEN
| . . . . . . . | . . | |     (RW (SweepDnC (RevHypC -3)) 0)
| . . . . . . . | . . | |     THEN
| . . . . . . . | . . | |     (RWW
| . . . . . . . | . . | |      Thm* ∀x:T, l1,l2:T List.
| . . . . . . . | . . | |           (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| . . . . . . . | . . | |      {member_append}
| . . . . . . . | . . | |      0))
| . . . . . . . | . . | |    THEN
| . . . . . . . | . . | |    (RWO
| . . . . . . . | . . | |     Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a
| . . . . . . . | . . | |     {member_singleton}
| . . . . . . . | . . | |     0)
| . . . . . . . | . . | |    THEN
| . . . . . . . | . . | |    (RWO
| . . . . . . . | . . | |     Thm* For any graph
| . . . . . . . | . . | |          ∀i,j:V. [i]-->*j ⟺ i-the_graph->*j
| . . . . . . . | . . | |     {list_DASH_connect_DASH_singleton}
| . . . . . . . | . . | |     0)
| . . . . . . . | . . | |
| . . . . . . . | . . | 1. the_graph : Graph
| . . . . . . . | . . | 2. GraphObject(the_graph)
| . . . . . . . | . . | 3. Vertices(the_graph) List
| . . . . . . . | . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | 5. s : traversal(the_graph)
| . . . . . . . | . . | 6. i : Vertices(the_graph)
| . . . . . . . | . . | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | |  Vertices(the_graph);s)
| . . . . . . . | . . | 9. df-traversal(the_graph;s)
| . . . . . . . | . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | 12. s2 : traversal(the_graph)
| . . . . . . . | . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | |   Vertices(the_graph);s2;s)
| . . . . . . . | . . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | |   Vertices(the_graph);s2)
| . . . . . . . | . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . | |                           @ s2
| . . . . . . . | . . | |                           @ [inr(i)])
| . . . . . . . | . . | |   @ s)
| . . . . . . . | . . | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | |   ⇒
| . . . . . . . | . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | |   L1 = (L1@0 @ L2)
| . . . . . . . | . . | |   ⇒
| . . . . . . . | . . | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | |    s = (s2 @ s1)
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| . . . . . . . | . . | |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | |    & (∀j:Vertices(the_graph).
| . . . . . . . | . . | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | . . | |       & ((inl(j) ∈ s2)
| . . . . . . . | . . | |          ⟺
| . . . . . . . | . . | |          L2-the_graph->*j
| . . . . . . . | . . | |          & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | 24. e : Vertices(the_graph) List
| . . . . . . . | . . | 25. L1@0 = (L1 @ e)
| . . . . . . . | . . | 26. [i] = (e @ L2)
| . . . . . . . | . . | 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | 28. L2 = nil
| . . . . . . . | . . | 29. j : Vertices(the_graph)
| . . . . . . . | . . | ⊢  inl(j) = inl(i)
| . . . . . . . | . . | |  ∨ (inl(j) ∈ s2)
| . . . . . . . | . . | |  ∨ inl(j) = inr(i)
| . . . . . . . | . . | |  ∨ (inl(j) ∈ s)
| . . . . . . . | . . | |  ⟺
| . . . . . . . | . . | |  (inl(j) ∈ s) ∨ i-the_graph->*j
| . . . . . . . | . . | | by (Analyze 0) THEN (Analyze 0)
| . . . . . . . | . . | |    THEN
| . . . . . . . | . . | |    (Analyze -1)
| . . . . . . . | . . | |
| . . . . . . . | . . | |\
| . . . . . . . | . . | | 1. the_graph : Graph
| . . . . . . . | . . | | 2. GraphObject(the_graph)
| . . . . . . . | . . | | 3. Vertices(the_graph) List
| . . . . . . . | . . | | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | | 5. s : traversal(the_graph)
| . . . . . . . | . . | | 6. i : Vertices(the_graph)
| . . . . . . . | . . | | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | | |  Vertices(the_graph);s)
| . . . . . . . | . . | | 9. df-traversal(the_graph;s)
| . . . . . . . | . . | | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | | 12. s2 : traversal(the_graph)
| . . . . . . . | . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | | |   Vertices(the_graph);s2;s)
| . . . . . . . | . . | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | | |   Vertices(the_graph);s2)
| . . . . . . . | . . | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . | | |                            @ s2
| . . . . . . . | . . | | |                            @ [inr(i)])
| . . . . . . . | . . | | |   @ s)
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| . . . . . . . | . . | | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | | |   ⇒
| . . . . . . . | . . | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | | |   L1 = (L1@0 @ L2)
| . . . . . . . | . . | | |   ⇒
| . . . . . . . | . . | | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | | |    s = (s2 @ s1)
| . . . . . . . | . . | | |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | | |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | | |    & (∀j:Vertices(the_graph).
| . . . . . . . | . . | | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | . . | | |       & ((inl(j) ∈ s2)
| . . . . . . . | . . | | |          ⟺
| . . . . . . . | . . | | |          L2-the_graph->*j
| . . . . . . . | . . | | |          & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | | 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | | 24. e : Vertices(the_graph) List
| . . . . . . . | . . | | 25. L1@0 = (L1 @ e)
| . . . . . . . | . . | | 26. [i] = (e @ L2)
| . . . . . . . | . . | | 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . | | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | | 28. L2 = nil
| . . . . . . . | . . | | 29. j : Vertices(the_graph)
| . . . . . . . | . . | | 30. inl(j) = inl(i)
| . . . . . . . | . . | | |   ∨ (inl(j) ∈ s2)
| . . . . . . . | . . | | |   ∨ inl(j) = inr(i)
| . . . . . . . | . . | | ⊢  (inl(j) ∈ s) ∨ i-the_graph->*j
| . . . . . . . | . . | | | by Analyze -1
| . . . . . . . | . . | | |
| . . . . . . . | . . | | |\
| . . . . . . . | . . | | | 1. the_graph : Graph
| . . . . . . . | . . | | | 2. GraphObject(the_graph)
| . . . . . . . | . . | | | 3. Vertices(the_graph) List
| . . . . . . . | . . | | | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | | | 5. s : traversal(the_graph)
| . . . . . . . | . . | | | 6. i : Vertices(the_graph)
| . . . . . . . | . . | | | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | | | |  Vertices(the_graph);s)
| . . . . . . . | . . | | | 9. df-traversal(the_graph;s)
| . . . . . . . | . . | | | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | | | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | | | 12. s2 : traversal(the_graph)
| . . . . . . . | . . | | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | | | |   Vertices(the_graph);s2;s)
| . . . . . . . | . . | | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | | | |   Vertices(the_graph);s2)
| . . . . . . . | . . | | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | | | 17. ∀j:Vertices(the_graph).
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| . . . . . . . | . . | | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | | | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . | | | |                         @ s2
| . . . . . . . | . . | | | |                         @ [inr(i)])
| . . . . . . . | . . | | | |   @ s)
| . . . . . . . | . . | | | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | | | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | | | |   ⇒
| . . . . . . . | . . | | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | | | |   L1 = (L1@0 @ L2)
| . . . . . . . | . . | | | |   ⇒
| . . . . . . . | . . | | | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | | | |    s = (s2 @ s1)
| . . . . . . . | . . | | | |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | | | |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | | | |    & (∀j:Vertices(the_graph).
| . . . . . . . | . . | | | |       ...
| . . . . . . . | . . | | | |       & ((inl(j) ∈ s2)
| . . . . . . . | . . | | | |          ⟺
| . . . . . . . | . . | | | |          L2-the_graph->*j
| . . . . . . . | . . | | | |          & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | | | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | | | 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . | | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | | | 24. e : Vertices(the_graph) List
| . . . . . . . | . . | | | 25. L1@0 = (L1 @ e)
| . . . . . . . | . . | | | 26. [i] = (e @ L2)
| . . . . . . . | . . | | | 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . | | | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | | | 28. L2 = nil
| . . . . . . . | . . | | | 29. j : Vertices(the_graph)
| . . . . . . . | . . | | | 30. inl(j) = inl(i)
| . . . . . . . | . . | | | ⊢  (inl(j) ∈ s) ∨ i-the_graph->*j
| . . . . . . . | . . | | | | by Subst (j = i) 0
| . . . . . . . | . . | | | |
| . . . . . . . | . . | | | |\
| . . . . . . . | . . | | | | .....equality .....
| . . . . . . . | . . | | | | 1. the_graph : Graph
| . . . . . . . | . . | | | | 2. GraphObject(the_graph)
| . . . . . . . | . . | | | | 3. Vertices(the_graph) List
| . . . . . . . | . . | | | | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | | | | 5. s : traversal(the_graph)
| . . . . . . . | . . | | | | 6. i : Vertices(the_graph)
| . . . . . . . | . . | | | | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | | | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | | | |    Vertices(the_graph);s)
| . . . . . . . | . . | | | | 9. df-traversal(the_graph;s)
| . . . . . . . | . . | | | | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | | | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | | | | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | | | | 12. s2 : traversal(the_graph)
| . . . . . . . | . . | | | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | | | |     Vertices(the_graph);s2;s)
| . . . . . . . | . . | | | | 14. paren(Vertices(the_graph);s2)
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| . . . . . . . | . . | | | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | | | |     Vertices(the_graph);s2)
| . . . . . . . | . . | | | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | | | | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | | | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | | | | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . | | | |                             @ s2
| . . . . . . . | . . | | | |                             @ [inr(i)])
| . . . . . . . | . . | | | |     @ s)
| . . . . . . . | . . | | | | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | | | |     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | | | |     ⇒
| . . . . . . . | . . | | | |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | | | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | | | |     L1 = (L1@0 @ L2)
| . . . . . . . | . . | | | |     ⇒
| . . . . . . . | . . | | | |     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | | | |      s = (s2 @ s1)
| . . . . . . . | . . | | | |      & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | | | |      & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | | | |      & (∀j:Vertices(the_graph).
| . . . . . . . | . . | | | |          ((inl(j) ∈ s1)
| . . . . . . . | . . | | | |           ⟺
| . . . . . . . | . . | | | |           L1@0-the_graph->*j)
| . . . . . . . | . . | | | |          & ((inl(j) ∈ s2)
| . . . . . . . | . . | | | |             ⟺
| . . . . . . . | . . | | | |             L2-the_graph->*j
| . . . . . . . | . . | | | |             & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | | | | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | | | | 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . | | | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | | | | 24. e : Vertices(the_graph) List
| . . . . . . . | . . | | | | 25. L1@0 = (L1 @ e)
| . . . . . . . | . . | | | | 26. [i] = (e @ L2)
| . . . . . . . | . . | | | | 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . | | | |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | | | | 28. L2 = nil
| . . . . . . . | . . | | | | 29. j : Vertices(the_graph)
| . . . . . . . | . . | | | | 30. inl(j) = inl(i)
| . . . . . . . | . . | | | | ⊢  j = i  by ObviousFrom [-1]
| . . . . . . . | . . | | | |
| . . . . . . . | . . | | | .\
| . . . . . . . | . . | | | . 1. the_graph : Graph
| . . . . . . . | . . | | | . 2. GraphObject(the_graph)
| . . . . . . . | . . | | | . 3. Vertices(the_graph) List
| . . . . . . . | . . | | | . 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | | | . 5. s : traversal(the_graph)
| . . . . . . . | . . | | | . 6. i : Vertices(the_graph)
| . . . . . . . | . . | | | . 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | | | . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | | | .    Vertices(the_graph);s)
| . . . . . . . | . . | | | . 9. df-traversal(the_graph;s)
| . . . . . . . | . . | | | . 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | | | .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | | | . 11. ¬(inl(i) ∈ s)
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| . . . . . . | . . | | | . 12. s2 : traversal(the_graph)
| . . . . . . | . . | | | . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . | . . | | | .     Vertices(the_graph);s2;s)
| . . . . . . | . . | | | . 14. paren(Vertices(the_graph);s2)
| . . . . . . | . . | | | . 15. no_repeats(Vertices(the_graph)+
| . . . . . . | . . | | | .     Vertices(the_graph);s2)
| . . . . . . | . . | | | . 16. ¬(inr(i) ∈ s2)
| . . . . . . | . . | | | . 17. ∀j:Vertices(the_graph).
| . . . . . . | . . | | | .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . | . . | | | . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . | . . | | | .                             @ s2
| . . . . . . | . . | | | .                             @ [inr(i)])
| . . . . . . | . . | | | .     @ s)
| . . . . . . | . . | | | . 19. ∀i@0:Vertices(the_graph).
| . . . . . . | . . | | | .     L1 @ [i]-the_graph->*i@0
| . . . . . . | . . | | | .     ⇒
| . . . . . . | . . | | | .     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . | . . | | | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . | . . | | | .     L1 = (L1@0 @ L2)
| . . . . . . | . . | | | .     ⇒
| . . . . . . | . . | | | .     (∃s1,s2:traversal(the_graph).
| . . . . . . | . . | | | .      s = (s2 @ s1)
| . . . . . . | . . | | | .      & paren(Vertices(the_graph);s1)
| . . . . . . | . . | | | .      & paren(Vertices(the_graph);s2)
| . . . . . . | . . | | | .      & (∀j:Vertices(the_graph).
| . . . . . . | . . | | | .         ((inl(j) ∈ s1)
| . . . . . . | . . | | | .          ⟺
| . . . . . . | . . | | | .          L1@0-the_graph->*j)
| . . . . . . | . . | | | .         & ((inl(j) ∈ s2)
| . . . . . . | . . | | | .            ⟺
| . . . . . . | . . | | | .            L2-the_graph->*j
| . . . . . . | . . | | | .            & ¬L1@0-the_graph->*j)))
| . . . . . . | . . | | | . 21. L1@0 : Vertices(the_graph) List
| . . . . . . | . . | | | . 22. L2 : Vertices(the_graph) List
| . . . . . . | . . | | | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . | . . | | | . 24. e : Vertices(the_graph) List
| . . . . . . | . . | | | . 25. L1@0 = (L1 @ e)
| . . . . . . | . . | | | . 26. [i] = (e @ L2)
| . . . . . . | . . | | | . 27. ∀i:Vertices(the_graph).
| . . . . . . | . . | | | .     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . | . . | | | . 28. L2 = nil
| . . . . . . | . . | | | . 29. j : Vertices(the_graph)
| . . . . . . | . . | | | . 30. inl(j) = inl(i)
| . . . . . . | . . | | | . ⊢  (inl(i) ∈ s) ∨ i-the_graph->*i
| . . . . . . | . . | | | .   by OrRight THEN Connect
| . . . . . . | . . | | |
| . . . . . . | . . | | .\
| . . . . . . | . . | | . 1. the_graph : Graph
| . . . . . . | . . | | . 2. GraphObject(the_graph)
| . . . . . . | . . | | . 3. Vertices(the_graph) List
| . . . . . . | . . | | . 4. L1 : Vertices(the_graph) List
| . . . . . . | . . | | . 5. s : traversal(the_graph)
| . . . . . . | . . | | . 6. i : Vertices(the_graph)
| . . . . . . | . . | | . 7. paren(Vertices(the_graph);s)
| . . . . . . | . . | | . 8. no_repeats(Vertices(the_graph)+
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| . . . . . . . | . . . | | . |  Vertices(the_graph);s)
| . . . . . . . | . . . | | . 9. df-traversal(the_graph;s)

| . . . . . . . | . . . | | . 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . . | | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . . | | . 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . . | | . 12. s2 : traversal(the_graph)
| . . . . . . . | . . . | | . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . . | | . |   Vertices(the_graph);s2;s)
| . . . . . . . | . . . | | . 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | | . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . | | . |   Vertices(the_graph);s2)
| . . . . . . . | . . . | | . 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . . | | . 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . . | | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . . | | . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . . | | . |                             @ s2
| . . . . . . . | . . . | | . |                             @ [inr(i)])
| . . . . . . . | . . . | | . |   @ s)
| . . . . . . . | . . . | | . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . . | | . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . . | | . |   ⇒
| . . . . . . . | . . . | | . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . . | | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . . | | . |   L1 = (L1@0 @ L2)
| . . . . . . . | . . . | | . |   ⇒
| . . . . . . . | . . . | | . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . . | | . |    s = (s2 @ s1)
| . . . . . . . | . . . | | . |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . . | | . |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | | . |    & (∀j:Vertices(the_graph).
| . . . . . . . | . . . | | . |       ...
| . . . . . . . | . . . | | . |       & ((inl(j) ∈ s2)
| . . . . . . . | . . . | | . |          ⟺
| . . . . . . . | . . . | | . |          L2-the_graph->*j
| . . . . . . . | . . . | | . |          & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . . | | . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . . | | . 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . . | | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . . | | . 24. e : Vertices(the_graph) List
| . . . . . . . | . . . | | . 25. L1@0 = (L1 @ e)
| . . . . . . . | . . . | | . 26. [i] = (e @ L2)
| . . . . . . . | . . . | | . 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . . | | . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . . | | . 28. L2 = nil
| . . . . . . . | . . . | | . 29. j : Vertices(the_graph)
| . . . . . . . | . . . | | . 30. (inl(j) ∈ s2) ∨ inl(j) = inr(i)
| . . . . . . . | . . . | | . ⊢  (inl(j) ∈ s) ∨ i-the_graph->*j
| . . . . . . . | . . . | | . | by Analyze -1
| . . . . . . . | . . . | | . |
| . . . . . . . | . . . | | . |\
| . . . . . . . | . . . | | . | 1. the_graph : Graph
| . . . . . . . | . . . | | . | 2. GraphObject(the_graph)
| . . . . . . . | . . . | | . | 3. Vertices(the_graph) List
| . . . . . . . | . . . | | . | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . . | | . | 5. s : traversal(the_graph)
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| . . . . . . . | . . | | . | 6. i : Vertices(the_graph)
| . . . . . . . | . . | | . | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | | . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | | . |    Vertices(the_graph);s)
| . . . . . . . | . . | | . | 9. df-traversal(the_graph;s)
| . . . . . . . | . . | | . | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | | . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | | . | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | | . | 12. s2 : traversal(the_graph)
| . . . . . . . | . . | | . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | | . |     Vertices(the_graph);s2;s)
| . . . . . . . | . . | | . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | | . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | | . |     Vertices(the_graph);s2)
| . . . . . . . | . . | | . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | | . | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | | . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | | . | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . | | . |                             @ s2
| . . . . . . . | . . | | . |                             @ [inr(i)])
| . . . . . . . | . . | | . |     @ s)
| . . . . . . . | . . | | . | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | | . |     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | | . |     ⇒
| . . . . . . . | . . | | . |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | | . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | | . |     L1 = (L1@0 @ L2)
| . . . . . . . | . . | | . |     ⇒
| . . . . . . . | . . | | . |     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | | . |      s = (s2 @ s1)
| . . . . . . . | . . | | . |      & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | | . |      & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | | . |      & (∀j:Vertices(the_graph).
| . . . . . . . | . . | | . |         ((inl(j) ∈ s1)
| . . . . . . . | . . | | . |          ⟺
| . . . . . . . | . . | | . |          L1@0-the_graph->*j)
| . . . . . . . | . . | | . |         & ((inl(j) ∈ s2)
| . . . . . . . | . . | | . |            ⟺
| . . . . . . . | . . | | . |            L2-the_graph->*j
| . . . . . . . | . . | | . |            & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | | . | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | | . | 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . | | . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | | . | 24. e : Vertices(the_graph) List
| . . . . . . . | . . | | . | 25. L1@0 = (L1 @ e)
| . . . . . . . | . . | | . | 26. [i] = (e @ L2)
| . . . . . . . | . . | | . | 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . | | . |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | | . | 28. L2 = nil
| . . . . . . . | . . | | . | 29. j : Vertices(the_graph)
| . . . . . . . | . . | | . | 30. (inl(j) ∈ s2)
| . . . . . . . | . . | | . | ⊢  (inl(j) ∈ s) ∨ i-the_graph->*j
| . . . . . . . | . . | | . |   by OrRight THEN BackThruSomeHyp
| . . . . . . . | . . | | . |      THEN
| . . . . . . . | . . | | . |      (BackThru
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| . . . . . . . | . . | | . |       Thm* ∀s':(T+T) List.
| . . . . . . . | . . | | . |            paren(T;s')
| . . . . . . . | . . | | . |            ⇒
| . . . . . . . | . . | | . |            ...
| . . . . . . . | . . | | . |       {paren_balance1})
| . . . . . . . | . . | | . |
| . . . . . . . | . . | | . .\
| . . . . . . . | . . | | . . 1. the_graph : Graph
| . . . . . . . | . . | | . . 2. GraphObject(the_graph)
| . . . . . . . | . . | | . . 3. Vertices(the_graph) List
| . . . . . . . | . . | | . . 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | | . . 5. s : traversal(the_graph)
| . . . . . . . | . . | | . . 6. i : Vertices(the_graph)
| . . . . . . . | . . | | . . 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | | . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | | . .    Vertices(the_graph);s)
| . . . . . . . | . . | | . . 9. df-traversal(the_graph;s)
| . . . . . . . | . . | | . . 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | | . .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | | . . 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | | . . 12. s2 : traversal(the_graph)
| . . . . . . . | . . | | . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | | . .     Vertices(the_graph);s2;s)
| . . . . . . . | . . | | . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | | . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | | . .     Vertices(the_graph);s2)
| . . . . . . . | . . | | . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | | . . 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | | . .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | | . . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . | | . .                             @ s2
| . . . . . . . | . . | | . .                             @ [inr(i)])
| . . . . . . . | . . | | . .     @ s)
| . . . . . . . | . . | | . . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | | . .     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | | . .     ⇒
| . . . . . . . | . . | | . .     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | | . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | | . .     L1 = (L1@0 @ L2)
| . . . . . . . | . . | | . .     ⇒
| . . . . . . . | . . | | . .     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | | . .      s = (s2 @ s1)
| . . . . . . . | . . | | . .      & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | | . .      & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | | . .      & (∀j:Vertices(the_graph).
| . . . . . . . | . . | | . .         ((inl(j) ∈ s1)
| . . . . . . . | . . | | . .          ⟺
| . . . . . . . | . . | | . .          L1@0-the_graph->*j)
| . . . . . . . | . . | | . .         & ((inl(j) ∈ s2)
| . . . . . . . | . . | | . .            ⟺
| . . . . . . . | . . | | . .            L2-the_graph->*j
| . . . . . . . | . . | | . .            & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | | . . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | | . . 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . | | . . 23. (L1 @ [i]) = (L1@0 @ L2)
```

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| . . . . . . . | . . | | . . 24. e : Vertices(the_graph) List
| . . . . . . . | . . | | . . 25. L1@0 = (L1 @ e)
| . . . . . . . | . . | | . . 26. [i] = (e @ L2)
| . . . . . . . | . . | | . . 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . | | . .     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | | . . 28. L2 = nil
| . . . . . . . | . . | | . . 29. j : Vertices(the_graph)
| . . . . . . . | . . | | . . 30. inl(j) = inr(i)
| . . . . . . . | . . | | . . ⊢  (inl(j) ∈ s) ∨ i-the_graph->*j
| . . . . . . . | . . | | . .   by ObviousContradiction [-1]
| . . . . . . . | . . | |
| . . . . . . . | . . | |\
| . . . . . . . | . . | | 1. the_graph : Graph
| . . . . . . . | . . | | 2. GraphObject(the_graph)
| . . . . . . . | . . | | 3. Vertices(the_graph) List
| . . . . . . . | . . | | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | | 5. s : traversal(the_graph)
| . . . . . . . | . . | | 6. i : Vertices(the_graph)
| . . . . . . . | . . | | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | |    Vertices(the_graph);s)
| . . . . . . . | . . | | 9. df-traversal(the_graph;s)
| . . . . . . . | . . | | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | | 12. s2 : traversal(the_graph)
| . . . . . . . | . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | |     Vertices(the_graph);s2;s)
| . . . . . . . | . . | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | |     Vertices(the_graph);s2)
| . . . . . . . | . . | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . | |                             @ s2
| . . . . . . . | . . | |                             @ [inr(i)])
| . . . . . . . | . . | |     @ s)
| . . . . . . . | . . | | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | |     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | |     ⇒
| . . . . . . . | . . | |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | |     L1 = (L1@0 @ L2)
| . . . . . . . | . . | |     ⇒
| . . . . . . . | . . | |     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | |      s = (s2 @ s1)
| . . . . . . . | . . | |      & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | |      & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | |      & (∀j:Vertices(the_graph).
| . . . . . . . | . . | |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | . . | |         & ((inl(j) ∈ s2)
| . . . . . . . | . . | |            ⟺
| . . . . . . . | . . | |            L2-the_graph->*j
| . . . . . . . | . . | |            & ¬L1@0-the_graph->*j)))
```

```
| . . . . . . . | . . | | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | | 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | | 24. e : Vertices(the_graph) List
| . . . . . . . | . . | | 25. L1@0 = (L1 @ e)
| . . . . . . . | . . | | 26. [i] = (e @ L2)
| . . . . . . . | . . | | 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . | |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | | 28. L2 = nil
| . . . . . . . | . . | | 29. j : Vertices(the_graph)
| . . . . . . . | . . | | 30. (inl(j) ∈ s)
| . . . . . . . | . . | | ⊢  (inl(j) ∈ s) ∨ i-the_graph->*j
| . . . . . . . | . . | |   by ObviousFrom [-1]
| . . . . . . . | . . | |
| . . . . . . . | . . | |\
| . . . . . . . | . . | | 1. the_graph : Graph
| . . . . . . . | . . | | 2. GraphObject(the_graph)
| . . . . . . . | . . | | 3. Vertices(the_graph) List
| . . . . . . . | . . | | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | | 5. s : traversal(the_graph)
| . . . . . . . | . . | | 6. i : Vertices(the_graph)
| . . . . . . . | . . | | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | |    Vertices(the_graph);s)
| . . . . . . . | . . | | 9. df-traversal(the_graph;s)
| . . . . . . . | . . | | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | | 12. s2 : traversal(the_graph)
| . . . . . . . | . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | |     Vertices(the_graph);s2;s)
| . . . . . . . | . . | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | |     Vertices(the_graph);s2)
| . . . . . . . | . . | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . | |                             @ s2
| . . . . . . . | . . | |                             @ [inr(i)])
| . . . . . . . | . . | |     @ s)
| . . . . . . . | . . | | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | |     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | |     ⇒
| . . . . . . . | . . | |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | |     L1 = (L1@0 @ L2)
| . . . . . . . | . . | |     ⇒
| . . . . . . . | . . | |     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | |      s = (s2 @ s1)
| . . . . . . . | . . | |      & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | |      & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | |      & (∀j:Vertices(the_graph).
| . . . . . . . | . . | |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | . . | |         & ((inl(j) ∈ s2)
```

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| . . . . . . . | . . | |           ⟺
| . . . . . . . | . . | |           L2-the_graph->*j
| . . . . . . . | . . | |           & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | | 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | | 24. e : Vertices(the_graph) List
| . . . . . . . | . . | | 25. L1@0 = (L1 @ e)
| . . . . . . . | . . | | 26. [i] = (e @ L2)
| . . . . . . . | . . | | 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . | |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | | 28. L2 = nil
| . . . . . . . | . . | | 29. j : Vertices(the_graph)
| . . . . . . . | . . | | 30. (inl(j) ∈ s)
| . . . . . . . | . . | | ⊢  inl(j) = inl(i)
| . . . . . . . | . . | |    ∨ (inl(j) ∈ s2)
| . . . . . . . | . . | |    ∨ inl(j) = inr(i)
| . . . . . . . | . . | |    ∨ (inl(j) ∈ s)  by ObviousFrom [-1]
| . . . . . . . | . . | |
| . . . . . . . | . . | .\
| . . . . . . . | . . | . 1. the_graph : Graph
| . . . . . . . | . . | . 2. GraphObject(the_graph)
| . . . . . . . | . . | . 3. Vertices(the_graph) List
| . . . . . . . | . . | . 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | . 5. s : traversal(the_graph)
| . . . . . . . | . . | . 6. i : Vertices(the_graph)
| . . . . . . . | . . | . 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | . |  Vertices(the_graph);s)
| . . . . . . . | . . | . 9. df-traversal(the_graph;s)
| . . . . . . . | . . | . 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | . 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | . 12. s2 : traversal(the_graph)
| . . . . . . . | . . | . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | . |   Vertices(the_graph);s2;s)
| . . . . . . . | . . | . 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | . |   Vertices(the_graph);s2)
| . . . . . . . | . . | . 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | . 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . | . |                         @ s2
| . . . . . . . | . . | . |                         @ [inr(i)])
| . . . . . . . | . . | . |   @ s)
| . . . . . . . | . . | . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | . |   ⇒
| . . . . . . . | . . | . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | . |   L1 = (L1@0 @ L2)
| . . . . . . . | . . | . |   ⇒
| . . . . . . . | . . | . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | . |    s = (s2 @ s1)
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| . . . . . . . | . . . | . |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . . | . |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | . |    & (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | . . . | . |       & ((inl(j) ∈ s2)
| . . . . . . . | . . . | . |          ⟺
| . . . . . . . | . . . | . |          L2-the_graph->*j
| . . . . . . . | . . . | . |          & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . . | . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . . | . 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . . | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . . | . 24. e : Vertices(the_graph) List
| . . . . . . . | . . . | . 25. L1@0 = (L1 @ e)
| . . . . . . . | . . . | . 26. [i] = (e @ L2)
| . . . . . . . | . . . | . 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . . | . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . . | . 28. L2 = nil
| . . . . . . . | . . . | . 29. j : Vertices(the_graph)
| . . . . . . . | . . . | . 30. i-the_graph->*j
| . . . . . . . | . . . | . ⊢  inl(j) = inl(i)
| . . . . . . . | . . . | . |  ∨ (inl(j) ∈ s2)
| . . . . . . . | . . . | . |  ∨ inl(j) = inr(i)
| . . . . . . . | . . . | . |  ∨ (inl(j) ∈ s)
| . . . . . . . | . . . | . | by (Unfold `df-traversal` 18)
| . . . . . . . | . . . | . |    THEN
| . . . . . . . | . . . | . |    (Analyze 18)
| . . . . . . . | . . . | . |    THEN
| . . . . . . . | . . . | . |    (InstHyp [i;nil;(s2 @ [inr(i)]) @ s;j]
| . . . . . . . | . . . | . |     19)
| . . . . . . . | . . . | . |
| . . . . . . . | . . . | . |\
| . . . . . . . | . . . | . | 1. the_graph : Graph
| . . . . . . . | . . . | . | 2. GraphObject(the_graph)
| . . . . . . . | . . . | . | 3. Vertices(the_graph) List
| . . . . . . . | . . . | . | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . . | . | 5. s : traversal(the_graph)
| . . . . . . . | . . . | . | 6. i : Vertices(the_graph)
| . . . . . . . | . . . | . | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . . | . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . | . | |  Vertices(the_graph);s)
| . . . . . . . | . . . | . | 9. df-traversal(the_graph;s)
| . . . . . . . | . . . | . | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . . | . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . . | . | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . . | . | 12. s2 : traversal(the_graph)
| . . . . . . . | . . . | . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . . | . | |   Vertices(the_graph);s2;s)
| . . . . . . . | . . . | . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . | . | |   Vertices(the_graph);s2)
| . . . . . . . | . . . | . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . . | . | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . . | . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . . | . | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . . | . | |   , s1,s2@0:traversal(the_graph).
```

```
| . . . . . . . | . . . | . | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . . | . | |   =
| . . . . . . . | . . . | . | |   (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . | . . . | . | |   ⇒
| . . . . . . . | . . . | . | |   (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . | |    (inr(j) ∈ s2@0)
| . . . . . . . | . . . | . | |    ⇒
| . . . . . . . | . . . | . | |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . | . . . | . | 19. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . . | . | |   , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . . | . | |   (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . | |    i@0-the_graph->*j
| . . . . . . . | . . . | . | |    ⇒
| . . . . . . . | . . . | . | |    ¬non-trivial-loop(the_graph;j))
| . . . . . . . | . . . | . | |   ⇒
| . . . . . . . | . . . | . | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . . | . | |   =
| . . . . . . . | . . . | . | |   (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . | . . . | . | |   ⇒
| . . . . . . . | . . . | . | |   (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . | |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . | . . . | . | 20. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . . | . | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . . | . | |   ⇒
| . . . . . . . | . . . | . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . . | . | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . . | . | |   L1 = (L1@0 @ L2)
| . . . . . . . | . . . | . | |   ⇒
| . . . . . . . | . . . | . | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . . | . | |    s = (s2 @ s1)
| . . . . . . . | . . . | . | |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . . | . | |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | . | |    & (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . | |        ...
| . . . . . . . | . . . | . | |        & ((inl(j) ∈ s2)
| . . . . . . . | . . . | . | |           ⟺
| . . . . . . . | . . . | . | |           L2-the_graph->*j
| . . . . . . . | . . . | . | |           & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . . | . | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . . | . | 23. L2 : Vertices(the_graph) List
| . . . . . . . | . . . | . | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . . | . | 25. e : Vertices(the_graph) List
| . . . . . . . | . . . | . | 26. L1@0 = (L1 @ e)
| . . . . . . . | . . . | . | 27. [i] = (e @ L2)
| . . . . . . . | . . . | . | 28. ∀i:Vertices(the_graph).
| . . . . . . . | . . . | . | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . . | . | 29. L2 = nil
| . . . . . . . | . . . | . | 30. j : Vertices(the_graph)
| . . . . . . . | . . . | . | 31. i-the_graph->*j
| . . . . . . . | . . . | . | 32. j1 : Vertices(the_graph)
| . . . . . . . | . . . | . | 33. i-the_graph->*j1
| . . . . . . . | . . . | . | ⊢  ¬non-trivial-loop(the_graph;j1)
| . . . . . . . | . . . | . | | by (BackThruSomeHyp
| . . . . . . . | . . . | . | |     THEN
| . . . . . . . | . . . | . | |     (RWO
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| . . . . . . . | . . . | . | |      Thm* For any graph
| . . . . . . . | . . . | . | |           ∀A,B:V List, i:V.
| . . . . . . . | . . . | . | |           A @ B-->*i ⟺ A-->*i ∨ B-->*i
| . . . . . . . | . . . | . | |      {list_DASH_connect_DASH_append}
| . . . . . . . | . . . | . | |      0))
| . . . . . . . | . . . | . | |    THEN
| . . . . . . . | . . . | . | |    (RWO
| . . . . . . . | . . . | . | |     Thm* For any graph
| . . . . . . . | . . . | . | |          ∀i,j:V.
| . . . . . . . | . . . | . | |          [i]-->*j ⟺ i-the_graph->*j
| . . . . . . . | . . . | . | |     {list_DASH_connect_DASH_singleton}
| . . . . . . . | . . . | . | |     0)
| . . . . . . . | . . . | . | |
| . . . . . . . | . . . | . | 1. the_graph : Graph
| . . . . . . . | . . . | . | 2. GraphObject(the_graph)
| . . . . . . . | . . . | . | 3. Vertices(the_graph) List
| . . . . . . . | . . . | . | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . . | . | 5. s : traversal(the_graph)
| . . . . . . . | . . . | . | 6. i : Vertices(the_graph)
| . . . . . . . | . . . | . | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . . | . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . | . |    Vertices(the_graph);s)
| . . . . . . . | . . . | . | 9. df-traversal(the_graph;s)
| . . . . . . . | . . . | . | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . . | . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . . | . | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . . | . | 12. s2 : traversal(the_graph)
| . . . . . . . | . . . | . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . . | . |     Vertices(the_graph);s2;s)
| . . . . . . . | . . . | . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . | . |     Vertices(the_graph);s2)
| . . . . . . . | . . . | . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . . | . | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . . | . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . . | . | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . . | . |     , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . . | . |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . . | . |     =
| . . . . . . . | . . . | . |     (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . | . . . | . |     ⇒
| . . . . . . . | . . . | . |     (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . |      (inr(j) ∈ s2@0)
| . . . . . . . | . . . | . |      ⇒
| . . . . . . . | . . . | . |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . | . . . | . | 19. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . . | . |     , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . . | . |     (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . |      i@0-the_graph->*j
| . . . . . . . | . . . | . |      ⇒
| . . . . . . . | . . . | . |      ¬non-trivial-loop(the_graph;j))
| . . . . . . . | . . . | . |     ⇒
| . . . . . . . | . . . | . |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . . | . |     =
| . . . . . . . | . . . | . |     (s1 @ [inl(i@0)] @ s2@0)
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| . . . . . . . | . . . | . |     ⇒
| . . . . . . . | . . . | . |     (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . |      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . | . . . | . | 20. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . . | . |     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . . | . |     ⇒
| . . . . . . . | . . . | . |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . . | . | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . . | . |     L1 = (L1@0 @ L2)
| . . . . . . . | . . . | . |     ⇒
| . . . . . . . | . . . | . |     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . . | . |      s = (s2 @ s1)
| . . . . . . . | . . . | . |      & paren(Vertices(the_graph);s1)
| . . . . . . . | . . . | . |      & paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | . |      & (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . |         ...
| . . . . . . . | . . . | . |         & ((inl(j) ∈ s2)
| . . . . . . . | . . . | . |            ⟺
| . . . . . . . | . . . | . |            L2-the_graph->*j
| . . . . . . . | . . . | . |            & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . . | . | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . . | . | 23. L2 : Vertices(the_graph) List
| . . . . . . . | . . . | . | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . . | . | 25. e : Vertices(the_graph) List
| . . . . . . . | . . . | . | 26. L1@0 = (L1 @ e)
| . . . . . . . | . . . | . | 27. [i] = (e @ L2)
| . . . . . . . | . . . | . | 28. ∀i:Vertices(the_graph).
| . . . . . . . | . . . | . |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . . | . | 29. L2 = nil
| . . . . . . . | . . . | . | 30. j : Vertices(the_graph)
| . . . . . . . | . . . | . | 31. i-the_graph->*j
| . . . . . . . | . . . | . | 32. j1 : Vertices(the_graph)
| . . . . . . . | . . . | . | 33. i-the_graph->*j1
| . . . . . . . | . . . | . | ⊢  L1-the_graph->*j1 ∨ i-the_graph->*j1
| . . . . . . . | . . . | . |   by ObviousFrom [-1]
| . . . . . . . | . . . | . |
| . . . . . . . | . . . | . |\
| . . . . . . . | . . . | . | .....antecedent .....
| . . . . . . . | . . . | . | 1. the_graph : Graph
| . . . . . . . | . . . | . | 2. GraphObject(the_graph)
| . . . . . . . | . . . | . | 3. Vertices(the_graph) List
| . . . . . . . | . . . | . | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . . | . | 5. s : traversal(the_graph)
| . . . . . . . | . . . | . | 6. i : Vertices(the_graph)
| . . . . . . . | . . . | . | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . . | . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . | . |    Vertices(the_graph);s)
| . . . . . . . | . . . | . | 9. df-traversal(the_graph;s)
| . . . . . . . | . . . | . | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . . | . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . . | . | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . . | . | 12. s2 : traversal(the_graph)
| . . . . . . . | . . . | . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . . | . |     Vertices(the_graph);s2;s)
| . . . . . . . | . . . | . | 14. paren(Vertices(the_graph);s2)
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| . . . . . . . | . . . | . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . | . |     Vertices(the_graph);s2)
| . . . . . . . | . . . | . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . . | . | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . . | . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . . | . | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . . | . |     , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . . | . |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . . | . |     =
| . . . . . . . | . . . | . |     (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . | . . . | . |     ⇒
| . . . . . . . | . . . | . |     (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . |      (inr(j) ∈ s2@0)
| . . . . . . . | . . . | . |      ⇒
| . . . . . . . | . . . | . |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . | . . . | . | 19. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . . | . |     , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . . | . |     (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . |      i@0-the_graph->*j
| . . . . . . . | . . . | . |      ⇒
| . . . . . . . | . . . | . |      ¬non-trivial-loop(the_graph;j))
| . . . . . . . | . . . | . |     ⇒
| . . . . . . . | . . . | . |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . . | . |     =
| . . . . . . . | . . . | . |     (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . | . . . | . |     ⇒
| . . . . . . . | . . . | . |     (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . |      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . | . . . | . | 20. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . . | . |     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . . | . |     ⇒
| . . . . . . . | . . . | . |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . . | . | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . . | . |     L1 = (L1@0 @ L2)
| . . . . . . . | . . . | . |     ⇒
| . . . . . . . | . . . | . |     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . . | . |      s = (s2 @ s1)
| . . . . . . . | . . . | . |      & paren(Vertices(the_graph);s1)
| . . . . . . . | . . . | . |      & paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | . |      & (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . |         ...
| . . . . . . . | . . . | . |         & ((inl(j) ∈ s2)
| . . . . . . . | . . . | . |            ⟺
| . . . . . . . | . . . | . |            L2-the_graph->*j
| . . . . . . . | . . . | . |            & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . . | . | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . . | . | 23. L2 : Vertices(the_graph) List
| . . . . . . . | . . . | . | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . . | . | 25. e : Vertices(the_graph) List
| . . . . . . . | . . . | . | 26. L1@0 = (L1 @ e)
| . . . . . . . | . . . | . | 27. [i] = (e @ L2)
| . . . . . . . | . . . | . | 28. ∀i:Vertices(the_graph).

| . . . . . . . | . . . | . |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . . | . | 29. L2 = nil
| . . . . . . . | . . . | . | 30. j : Vertices(the_graph)
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| . . . . . . . | . . . | . . | 31. i-the_graph->*j
| . . . . . . . | . . . | . . | ⊢  (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . . | . . |    =
| . . . . . . . | . . . | . . |    (nil @ [inl(i)] @ (s2 @ [inr(i)]) @ s)
| . . . . . . . | . . . | . . |   by Reduce 0
| . . . . . . . | . . . | . . |
| . . . . . . . | . . . | . .\
| . . . . . . . | . . . | . . . 1. the_graph : Graph
| . . . . . . . | . . . | . . . 2. GraphObject(the_graph)
| . . . . . . . | . . . | . . . 3. Vertices(the_graph) List
| . . . . . . . | . . . | . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . . | . . . 5. s : traversal(the_graph)
| . . . . . . . | . . . | . . . 6. i : Vertices(the_graph)
| . . . . . . . | . . . | . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . . | . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . | . . . |  Vertices(the_graph);s)
| . . . . . . . | . . . | . . . 9. df-traversal(the_graph;s)
| . . . . . . . | . . . | . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . . | . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . . | . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . . | . . . 12. s2 : traversal(the_graph)
| . . . . . . . | . . . | . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . . | . . . |   Vertices(the_graph);s2;s)
| . . . . . . . | . . . | . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . | . . . |   Vertices(the_graph);s2)
| . . . . . . . | . . . | . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . . | . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . . | . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . . | . . . 18. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . . | . . . |   , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . . | . . . |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . . | . . . |   =
| . . . . . . . | . . . | . . . |   (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . | . . . | . . . |   ⇒
| . . . . . . . | . . . | . . . |   (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . . . |    (inr(j) ∈ s2@0)
| . . . . . . . | . . . | . . . |    ⇒
| . . . . . . . | . . . | . . . |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . | . . . | . . . 19. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . . | . . . |   , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . . | . . . |   (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . . . |    i@0-the_graph->*j
| . . . . . . . | . . . | . . . |    ⇒
| . . . . . . . | . . . | . . . |    ¬non-trivial-loop(the_graph;j))
| . . . . . . . | . . . | . . . |   ⇒
| . . . . . . . | . . . | . . . |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . . | . . . |   =
| . . . . . . . | . . . | . . . |   (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . | . . . | . . . |   ⇒
| . . . . . . . | . . . | . . . |   (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . . . |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . | . . . | . . . 20. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . . | . . . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . . | . . . |   ⇒
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| . . . . . . . | . . | . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | . . 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | . . |   L1 = (L1@0 @ L2)
| . . . . . . . | . . | . . |   ⇒
| . . . . . . . | . . | . . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | . . |    s = (s2 @ s1)
| . . . . . . . | . . | . . |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | . . |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | . . |    & (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . |       ...
| . . . . . . . | . . | . . |       & ((inl(j) ∈ s2)
| . . . . . . . | . . | . . |          ⟺
| . . . . . . . | . . | . . |          L2-the_graph->*j
| . . . . . . . | . . | . . |          & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | . . 22. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | . . 23. L2 : Vertices(the_graph) List
| . . . . . . . | . . | . . 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | . . 25. e : Vertices(the_graph) List
| . . . . . . . | . . | . . 26. L1@0 = (L1 @ e)
| . . . . . . . | . . | . . 27. [i] = (e @ L2)
| . . . . . . . | . . | . . 28. ∀i:Vertices(the_graph).
| . . . . . . . | . . | . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | . . 29. L2 = nil
| . . . . . . . | . . | . . 30. j : Vertices(the_graph)
| . . . . . . . | . . | . . 31. i-the_graph->*j
| . . . . . . . | . . | . . 32. (inr(j) ∈ (s2 @ [inr(i)]) @ s)
| . . . . . . . | . . | . . ⊢  inl(j) = inl(i)
| . . . . . . . | . . | . . |  ∨ (inl(j) ∈ s2)
| . . . . . . . | . . | . . |  ∨ inl(j) = inr(i)
| . . . . . . . | . . | . . |  ∨ (inl(j) ∈ s)
| . . . . . . . | . . | . . | by ((RWW
| . . . . . . . | . . | . . |      Thm* ∀x:T, l1,l2:T List.
| . . . . . . . | . . | . . |           (x ∈ l1 @ l2)
| . . . . . . . | . . | . . |           ⟺
| . . . . . . . | . . | . . |           (x ∈ l1) ∨ (x ∈ l2)
| . . . . . . . | . . | . . |      {member_append}
| . . . . . . . | . . | . . |      -1)
| . . . . . . . | . . | . . |     THEN
| . . . . . . . | . . | . . |     (RWO
| . . . . . . . | . . | . . |      Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a
| . . . . . . . | . . | . . |      {member_singleton}
| . . . . . . . | . . | . . |      -1))
| . . . . . . . | . . | . . |    THEN
| . . . . . . . | . . | . . |    (Analyze -1)
| . . . . . . . | . . | . . |
| . . . . . . . | . . | . . |\
| . . . . . . . | . . | . . | 1. the_graph : Graph
| . . . . . . . | . . | . . | 2. GraphObject(the_graph)
| . . . . . . . | . . | . . | 3. Vertices(the_graph) List
| . . . . . . . | . . | . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | . . | 5. s : traversal(the_graph)
| . . . . . . . | . . | . . | 6. i : Vertices(the_graph)
| . . . . . . . | . . | . . | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | . . | |  Vertices(the_graph);s)
```

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| . . . . . . . | . . . | . . | 9. df-traversal(the_graph;s)
| . . . . . . . | . . . | . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . . | . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . . | . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . . | . . | 12. s2 : traversal(the_graph)
| . . . . . . . | . . . | . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . . | . . | |   Vertices(the_graph);s2;s)
| . . . . . . . | . . . | . . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | . . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . | . . | |   Vertices(the_graph);s2)
| . . . . . . . | . . . | . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . . | . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . . | . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . . | . . | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . . | . . | |   , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . . | . . | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . . | . . | |   =
| . . . . . . . | . . . | . . | |   (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . | . . . | . . | |   ⇒
| . . . . . . . | . . . | . . | |   ...
| . . . . . . . | . . . | . . | 19. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . . | . . | |   , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . . | . . | |   (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . . | |    i@0-the_graph->*j
| . . . . . . . | . . . | . . | |    ⇒
| . . . . . . . | . . . | . . | |    ¬non-trivial-loop(the_graph;j))
| . . . . . . . | . . . | . . | |   ⇒
| . . . . . . . | . . . | . . | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . . | . . | |   =
| . . . . . . . | . . . | . . | |   (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . | . . . | . . | |   ⇒
| . . . . . . . | . . . | . . | |   (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . . | |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . | . . . | . . | 20. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . . | . . | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . . | . . | |   ⇒
| . . . . . . . | . . . | . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . . | . . | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . . | . . | |   L1 = (L1@0 @ L2)
| . . . . . . . | . . . | . . | |   ⇒
| . . . . . . . | . . . | . . | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . . | . . | |    s = (s2 @ s1)
| . . . . . . . | . . . | . . | |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . . | . . | |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | . . | |    & (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . . | |        ((inl(j) ∈ s1)
| . . . . . . . | . . . | . . | |         ⟺
| . . . . . . . | . . . | . . | |         L1@0-the_graph->*j)
| . . . . . . . | . . . | . . | |        & ((inl(j) ∈ s2)
| . . . . . . . | . . . | . . | |           ⟺
| . . . . . . . | . . . | . . | |           L2-the_graph->*j
| . . . . . . . | . . . | . . | |           & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . . | . . | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . . | . . | 23. L2 : Vertices(the_graph) List
| . . . . . . . | . . . | . . | 24. (L1 @ [i]) = (L1@0 @ L2)
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| . . . . . . . | . . | . . | 25. e : Vertices(the_graph) List
| . . . . . . . | . . | . . | 26. L1@0 = (L1 @ e)
| . . . . . . . | . . | . . | 27. [i] = (e @ L2)
| . . . . . . . | . . | . . | 28. ∀i:Vertices(the_graph).
| . . . . . . . | . . | . . | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | . . | 29. L2 = nil
| . . . . . . . | . . | . . | 30. j : Vertices(the_graph)
| . . . . . . . | . . | . . | 31. i-the_graph->*j
| . . . . . . . | . . | . . | 32. (inr(j) ∈ s2) ∨ inr(j) = inr(i)
| . . . . . . . | . . | . . | ⊢  inl(j) = inl(i)
| . . . . . . . | . . | . . | |  ∨ (inl(j) ∈ s2)
| . . . . . . . | . . | . . | |  ∨ inl(j) = inr(i)
| . . . . . . . | . . | . . | |  ∨ (inl(j) ∈ s)  by Analyze -1
| . . . . . . . | . . | . . | |
| . . . . . . . | . . | . . | |\
| . . . . . . . | . . | . . | | 1. the_graph : Graph
| . . . . . . . | . . | . . | | 2. GraphObject(the_graph)
| . . . . . . . | . . | . . | | 3. Vertices(the_graph) List
| . . . . . . . | . . | . . | | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | . . | | 5. s : traversal(the_graph)
| . . . . . . . | . . | . . | | 6. i : Vertices(the_graph)
| . . . . . . . | . . | . . | | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | . . | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | . . | | |  Vertices(the_graph);s)
| . . . . . . . | . . | . . | | 9. df-traversal(the_graph;s)
| . . . . . . . | . . | . . | | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | . . | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | . . | | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | . . | | 12. s2 : traversal(the_graph)
| . . . . . . . | . . | . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | . . | | |   Vertices(the_graph);s2;s)
| . . . . . . . | . . | . . | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | . . | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | . . | | |   Vertices(the_graph);s2)
| . . . . . . . | . . | . . | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | . . | | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | . . | | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . | . . | | |   , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . | . . | | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . | . . | | |   =
| . . . . . . . | . . | . . | | |   (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . | . . | . . | | |   ⇒
| . . . . . . . | . . | . . | | |   (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | | |    (inr(j) ∈ s2@0)
| . . . . . . . | . . | . . | | |    ⇒
| . . . . . . . | . . | . . | | |    ¬(inl(j) ∈ s2@0)
| . . . . . . . | . . | . . | | |    ⇒
| . . . . . . . | . . | . . | | |    j-the_graph->*i@0)
| . . . . . . . | . . | . . | | 19. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . | . . | | |   , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . | . . | | |   (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | | |    i@0-the_graph->*j
| . . . . . . . | . . | . . | | |    ⇒
| . . . . . . . | . . | . . | | |    ¬non-trivial-loop(the_graph;j))
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| . . . . . . . | . . | . . | | |   ⇒
| . . . . . . . | . . | . . | | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . | . . | | |   =
| . . . . . . . | . . | . . | | |   (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . | . . | . . | | |   ⇒
| . . . . . . . | . . | . . | | |   (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | | |    i@0-the_graph->*j
| . . . . . . . | . . | . . | | |    ⇒
| . . . . . . . | . . | . . | | |    (inr(j) ∈ s2@0))
| . . . . . . . | . . | . . | | 20. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | . . | | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | . . | | |   ⇒
| . . . . . . . | . . | . . | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | . . | | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | . . | | |   L1 = (L1@0 @ L2)
| . . . . . . . | . . | . . | | |   ⇒
| . . . . . . . | . . | . . | | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | . . | | |    s = (s2 @ s1)
| . . . . . . . | . . | . . | | |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | . . | | |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | . . | | |    & (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | | |       ((inl(j) ∈ s1)
| . . . . . . . | . . | . . | | |        ⟺
| . . . . . . . | . . | . . | | |        L1@0-the_graph->*j)
| . . . . . . . | . . | . . | | |       & ((inl(j) ∈ s2)
| . . . . . . . | . . | . . | | |          ⟺
| . . . . . . . | . . | . . | | |          L2-the_graph->*j
| . . . . . . . | . . | . . | | |          & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | . . | | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | . . | | 23. L2 : Vertices(the_graph) List
| . . . . . . . | . . | . . | | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | . . | | 25. e : Vertices(the_graph) List
| . . . . . . . | . . | . . | | 26. L1@0 = (L1 @ e)
| . . . . . . . | . . | . . | | 27. [i] = (e @ L2)
| . . . . . . . | . . | . . | | 28. ∀i:Vertices(the_graph).
| . . . . . . . | . . | . . | | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | . . | | 29. L2 = nil
| . . . . . . . | . . | . . | | 30. j : Vertices(the_graph)
| . . . . . . . | . . | . . | | 31. i-the_graph->*j
| . . . . . . . | . . | . . | | 32. (inr(j) ∈ s2)
| . . . . . . . | . . | . . | | ⊢  inl(j) = inl(i)
| . . . . . . . | . . | . . | | |  ∨ (inl(j) ∈ s2)
| . . . . . . . | . . | . . | | |  ∨ inl(j) = inr(i)
| . . . . . . . | . . | . . | | |  ∨ (inl(j) ∈ s)
| . . . . . . . | . . | . . | | | by FwdThru
| . . . . . . . | . . | . . | | |    Thm* ∀s':(T+T) List.
| . . . . . . . | . . | . . | | |          paren(T;s')
| . . . . . . . | . . | . . | | |          ⇒
| . . . . . . . | . . | . . | | |          (∀i:T.
| . . . . . . . | . . | . . | | |           (inr(i) ∈ s')
| . . . . . . . | . . | . . | | |           ⇒
| . . . . . . . | . . | . . | | |           (inl(i) ∈ s'))
| . . . . . . . | . . | . . | | |    {paren_balance2}
| . . . . . . . | . . | . . | | |    [14;-1]
| . . . . . . . | . . | . . | | |
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| . . . . . . . | . . | . . | | 1. the_graph : Graph
| . . . . . . . | . . | . . | | 2. GraphObject(the_graph)
| . . . . . . . | . . | . . | | 3. Vertices(the_graph) List
| . . . . . . . | . . | . . | | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | . . | | 5. s : traversal(the_graph)
| . . . . . . . | . . | . . | | 6. i : Vertices(the_graph)
| . . . . . . . | . . | . . | | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | . . | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | . . | |    Vertices(the_graph);s)
| . . . . . . . | . . | . . | | 9. df-traversal(the_graph;s)
| . . . . . . . | . . | . . | | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | . . | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | . . | | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | . . | | 12. s2 : traversal(the_graph)
| . . . . . . . | . . | . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | . . | |     Vertices(the_graph);s2;s)
| . . . . . . . | . . | . . | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | . . | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | . . | |     Vertices(the_graph);s2)
| . . . . . . . | . . | . . | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | . . | | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | . . | | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . | . . | |     , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . | . . | |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . | . . | |     =
| . . . . . . . | . . | . . | |     (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . | . . | . . | |     ⇒
| . . . . . . . | . . | . . | |     (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | |      (inr(j) ∈ s2@0)
| . . . . . . . | . . | . . | |      ⇒
| . . . . . . . | . . | . . | |      ¬(inl(j) ∈ s2@0)
| . . . . . . . | . . | . . | |      ⇒
| . . . . . . . | . . | . . | |      j-the_graph->*i@0)
| . . . . . . . | . . | . . | | 19. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . | . . | |     , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . | . . | |     (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | |      i@0-the_graph->*j
| . . . . . . . | . . | . . | |      ⇒
| . . . . . . . | . . | . . | |      ¬non-trivial-loop(the_graph;j))
| . . . . . . . | . . | . . | |     ⇒
| . . . . . . . | . . | . . | |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . | . . | |     =
| . . . . . . . | . . | . . | |     (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . | . . | . . | |     ⇒
| . . . . . . . | . . | . . | |     (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | |      i@0-the_graph->*j
| . . . . . . . | . . | . . | |      ⇒
| . . . . . . . | . . | . . | |      (inr(j) ∈ s2@0))
| . . . . . . . | . . | . . | | 20. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | . . | |     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | . . | |     ⇒
| . . . . . . . | . . | . . | |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | . . | | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | . . | |     L1 = (L1@0 @ L2)
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| . . . . . . . | . . . | . . . | |      ⇒
| . . . . . . . | . . . | . . . | |     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . . | . . . | |      s = (s2 @ s1)
| . . . . . . . | . . . | . . . | |      & paren(Vertices(the_graph);s1)
| . . . . . . . | . . . | . . . | |      & paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | . . . | |      & (∀j:Vertices(the_graph).
| . . . . . . . | . . . | . . . | |         ((inl(j) ∈ s1)
| . . . . . . . | . . . | . . . | |          ⟺
| . . . . . . . | . . . | . . . | |          L1@0-the_graph->*j)
| . . . . . . . | . . . | . . . | |         & ((inl(j) ∈ s2)
| . . . . . . . | . . . | . . . | |            ⟺
| . . . . . . . | . . . | . . . | |            L2-the_graph->*j
| . . . . . . . | . . . | . . . | |            & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . . | . . . | | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . . | . . . | | 23. L2 : Vertices(the_graph) List
| . . . . . . . | . . . | . . . | | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . . | . . . | | 25. e : Vertices(the_graph) List
| . . . . . . . | . . . | . . . | | 26. L1@0 = (L1 @ e)
| . . . . . . . | . . . | . . . | | 27. [i] = (e @ L2)
| . . . . . . . | . . . | . . . | | 28. ∀i:Vertices(the_graph).
| . . . . . . . | . . . | . . . | |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . . | . . . | | 29. L2 = nil
| . . . . . . . | . . . | . . . | | 30. j : Vertices(the_graph)
| . . . . . . . | . . . | . . . | | 31. i-the_graph->*j
| . . . . . . . | . . . | . . . | | 32. (inr(j) ∈ s2)
| . . . . . . . | . . . | . . . | | 33. (inl(j) ∈ s2)
| . . . . . . . | . . . | . . . | | ⊢  inl(j) = inl(i)
| . . . . . . . | . . . | . . . | |    ∨ (inl(j) ∈ s2)
| . . . . . . . | . . . | . . . | |    ∨ inl(j) = inr(i)
| . . . . . . . | . . . | . . . | |    ∨ (inl(j) ∈ s)  by ObviousFrom [-1]
| . . . . . . . | . . . | . . . | |
| . . . . . . . | . . . | . . . | .\
| . . . . . . . | . . . | . . . | . 1. the_graph : Graph
| . . . . . . . | . . . | . . . | . 2. GraphObject(the_graph)
| . . . . . . . | . . . | . . . | . 3. Vertices(the_graph) List
| . . . . . . . | . . . | . . . | . 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . . | . . . | . 5. s : traversal(the_graph)
| . . . . . . . | . . . | . . . | . 6. i : Vertices(the_graph)
| . . . . . . . | . . . | . . . | . 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . . | . . . | . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . | . . . | .    Vertices(the_graph);s)
| . . . . . . . | . . . | . . . | . 9. df-traversal(the_graph;s)
| . . . . . . . | . . . | . . . | . 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . . | . . . | .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . . | . . . | . 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . . | . . . | . 12. s2 : traversal(the_graph)
| . . . . . . . | . . . | . . . | . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . . | . . . | .     Vertices(the_graph);s2;s)
| . . . . . . . | . . . | . . . | . 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | . . . | . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . | . . . | .     Vertices(the_graph);s2)
| . . . . . . . | . . . | . . . | . 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . . | . . . | . 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . . | . . . | .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . . | . . . | . 18. ∀i@0:Vertices(the_graph)
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| . . . . . . . | . . | . . | .      , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . | . . | .      (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . | . . | .      =
| . . . . . . . | . . | . . | .      (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . | . . | . . | .      ⇒
| . . . . . . . | . . | . . | .      (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | .       (inr(j) ∈ s2@0)
| . . . . . . . | . . | . . | .       ⇒
| . . . . . . . | . . | . . | .       ¬(inl(j) ∈ s2@0)
| . . . . . . . | . . | . . | .       ⇒
| . . . . . . . | . . | . . | .       j-the_graph->*i@0)
| . . . . . . . | . . | . . | . 19. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . | . . | .      , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . | . . | .      (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | .       i@0-the_graph->*j
| . . . . . . . | . . | . . | .       ⇒
| . . . . . . . | . . | . . | .       ¬non-trivial-loop(the_graph;j))
| . . . . . . . | . . | . . | .      ⇒
| . . . . . . . | . . | . . | .      (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . | . . | .      =
| . . . . . . . | . . | . . | .      (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . | . . | . . | .      ⇒
| . . . . . . . | . . | . . | .      (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | .       i@0-the_graph->*j
| . . . . . . . | . . | . . | .       ⇒
| . . . . . . . | . . | . . | .       (inr(j) ∈ s2@0))
| . . . . . . . | . . | . . | . 20. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | . . | .      L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | . . | .      ⇒
| . . . . . . . | . . | . . | .      ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | . . | . 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | . . | .      L1 = (L1@0 @ L2)
| . . . . . . . | . . | . . | .      ⇒
| . . . . . . . | . . | . . | .      (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | . . | .       s = (s2 @ s1)
| . . . . . . . | . . | . . | .       & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | . . | .       & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | . . | .       & (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . | .          ((inl(j) ∈ s1)
| . . . . . . . | . . | . . | .           ⟺
| . . . . . . . | . . | . . | .           L1@0-the_graph->*j)
| . . . . . . . | . . | . . | .          & ((inl(j) ∈ s2)
| . . . . . . . | . . | . . | .             ⟺
| . . . . . . . | . . | . . | .             L2-the_graph->*j
| . . . . . . . | . . | . . | .             & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | . . | . 22. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | . . | . 23. L2 : Vertices(the_graph) List
| . . . . . . . | . . | . . | . 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | . . | . 25. e : Vertices(the_graph) List
| . . . . . . . | . . | . . | . 26. L1@0 = (L1 @ e)
| . . . . . . . | . . | . . | . 27. [i] = (e @ L2)
| . . . . . . . | . . | . . | . 28. ∀i:Vertices(the_graph).
| . . . . . . . | . . | . . | .      (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | . . | . 29. L2 = nil
| . . . . . . . | . . | . . | . 30. j : Vertices(the_graph)
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| . . . . . . . | . . | . . | . 31. i-the_graph->*j
| . . . . . . . | . . | . . | . 32. inr(j) = inr(i)
| . . . . . . . | . . | . . | . ⊢  inl(j) = inl(i)
| . . . . . . . | . . | . . | .     ∨ (inl(j) ∈ s2)
| . . . . . . . | . . | . . | .     ∨ inl(j) = inr(i)
| . . . . . . . | . . | . . | .     ∨ (inl(j) ∈ s)  by ObviousFrom [-1]
| . . . . . . . | . . | . . |
| . . . . . . . | . . | . . .\
| . . . . . . . | . . | . . . 1. the_graph : Graph
| . . . . . . . | . . | . . . 2. GraphObject(the_graph)
| . . . . . . . | . . | . . . 3. Vertices(the_graph) List
| . . . . . . . | . . | . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | . . . 5. s : traversal(the_graph)
| . . . . . . . | . . | . . . 6. i : Vertices(the_graph)
| . . . . . . . | . . | . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | . . . |  Vertices(the_graph);s)
| . . . . . . . | . . | . . . 9. df-traversal(the_graph;s)
| . . . . . . . | . . | . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | . . . 12. s2 : traversal(the_graph)
| . . . . . . . | . . | . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | . . . |   Vertices(the_graph);s2;s)
| . . . . . . . | . . | . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | . . . |   Vertices(the_graph);s2)
| . . . . . . . | . . | . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | . . . 18. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . | . . . |   , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . | . . . |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . | . . . |   =
| . . . . . . . | . . | . . . |   (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . | . . | . . . |   ⇒
| . . . . . . . | . . | . . . |   ...
| . . . . . . . | . . | . . . 19. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . | . . . |   , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . | . . . |   (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . . |    i@0-the_graph->*j
| . . . . . . . | . . | . . . |    ⇒
| . . . . . . . | . . | . . . |    ¬non-trivial-loop(the_graph;j))
| . . . . . . . | . . | . . . |   ⇒
| . . . . . . . | . . | . . . |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . | . . . |   =
| . . . . . . . | . . | . . . |   (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . | . . | . . . |   ⇒
| . . . . . . . | . . | . . . |   (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . . |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . | . . | . . . 20. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | . . . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | . . . |   ⇒
| . . . . . . . | . . | . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | . . . 21. ∀L1@0,L2:Vertices(the_graph) List.
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| . . . . . . . | . . | . . . |   L1 = (L1@0 @ L2)
| . . . . . . . | . . | . . . |   ⇒
| . . . . . . . | . . | . . . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | . . . |    s = (s2 @ s1)
| . . . . . . . | . . | . . . |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | . . . |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | . . . |    & (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . . |       ((inl(j) ∈ s1)
| . . . . . . . | . . | . . . |        ⟺
| . . . . . . . | . . | . . . |        L1@0-the_graph->*j)
| . . . . . . . | . . | . . . |       & ((inl(j) ∈ s2)
| . . . . . . . | . . | . . . |          ⟺
| . . . . . . . | . . | . . . |          L2-the_graph->*j
| . . . . . . . | . . | . . . |          & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | . . . 22. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | . . . 23. L2 : Vertices(the_graph) List
| . . . . . . . | . . | . . . 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | . . . 25. e : Vertices(the_graph) List
| . . . . . . . | . . | . . . 26. L1@0 = (L1 @ e)
| . . . . . . . | . . | . . . 27. [i] = (e @ L2)
| . . . . . . . | . . | . . . 28. ∀i:Vertices(the_graph).
| . . . . . . . | . . | . . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | . . . 29. L2 = nil
| . . . . . . . | . . | . . . 30. j : Vertices(the_graph)
| . . . . . . . | . . | . . . 31. i-the_graph->*j
| . . . . . . . | . . | . . . 32. (inr(j) ∈ s)
| . . . . . . . | . . | . . . ⊢  inl(j) = inl(i)
| . . . . . . . | . . | . . . |  ∨ (inl(j) ∈ s2)
| . . . . . . . | . . | . . . |  ∨ inl(j) = inr(i)
| . . . . . . . | . . | . . . |  ∨ (inl(j) ∈ s)
| . . . . . . . | . . | . . . | by FwdThru
| . . . . . . . | . . | . . . |    Thm* ∀s':(T+T) List.
| . . . . . . . | . . | . . . |         paren(T;s')
| . . . . . . . | . . | . . . |         ⇒
| . . . . . . . | . . | . . . |         (∀i:T.
| . . . . . . . | . . | . . . |          (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
| . . . . . . . | . . | . . . |    {paren_balance2}
| . . . . . . . | . . | . . . |    [7;-1]
| . . . . . . . | . . | . . . |
| . . . . . . . | . . | . . . 1. the_graph : Graph
| . . . . . . . | . . | . . . 2. GraphObject(the_graph)
| . . . . . . . | . . | . . . 3. Vertices(the_graph) List
| . . . . . . . | . . | . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . | . . . 5. s : traversal(the_graph)
| . . . . . . . | . . | . . . 6. i : Vertices(the_graph)
| . . . . . . . | . . | . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . | . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | . . .    Vertices(the_graph);s)
| . . . . . . . | . . | . . . 9. df-traversal(the_graph;s)
| . . . . . . . | . . | . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . | . . .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . | . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . | . . . 12. s2 : traversal(the_graph)
| . . . . . . . | . . | . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . | . . .     Vertices(the_graph);s2;s)
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| . . . . . . . | . . | . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . | . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . | . . .     Vertices(the_graph);s2)
| . . . . . . . | . . | . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . | . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . | . . .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . | . . . 18. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . | . . .     , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . | . . .     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . | . . .     =
| . . . . . . . | . . | . . .     (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . | . . | . . .     ⇒
| . . . . . . . | . . | . . .     ...
| . . . . . . . | . . | . . . 19. ∀i@0:Vertices(the_graph)
| . . . . . . . | . . | . . .     , s1,s2@0:traversal(the_graph).
| . . . . . . . | . . | . . .     (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . .      i@0-the_graph->*j
| . . . . . . . | . . | . . .      ⇒
| . . . . . . . | . . | . . .      ¬non-trivial-loop(the_graph;j))
| . . . . . . . | . . | . . .     ⇒
| . . . . . . . | . . | . . .     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . | . . | . . .     =
| . . . . . . . | . . | . . .     (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . | . . | . . .     ⇒
| . . . . . . . | . . | . . .     (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . .      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . | . . | . . . 20. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . | . . .     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . | . . .     ⇒
| . . . . . . . | . . | . . .     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . | . . . 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . | . . .     L1 = (L1@0 @ L2)
| . . . . . . . | . . | . . .     ⇒
| . . . . . . . | . . | . . .     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . | . . .      s = (s2 @ s1)
| . . . . . . . | . . | . . .      & paren(Vertices(the_graph);s1)
| . . . . . . . | . . | . . .      & paren(Vertices(the_graph);s2)
| . . . . . . . | . . | . . .      & (∀j:Vertices(the_graph).
| . . . . . . . | . . | . . .         ((inl(j) ∈ s1)
| . . . . . . . | . . | . . .          ⟺
| . . . . . . . | . . | . . .          L1@0-the_graph->*j)
| . . . . . . . | . . | . . .         & ((inl(j) ∈ s2)
| . . . . . . . | . . | . . .            ⟺
| . . . . . . . | . . | . . .            L2-the_graph->*j
| . . . . . . . | . . | . . .            & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . | . . . 22. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . | . . . 23. L2 : Vertices(the_graph) List

| . . . . . . . | . . | . . . 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . | . . . 25. e : Vertices(the_graph) List
| . . . . . . . | . . | . . . 26. L1@0 = (L1 @ e)
| . . . . . . . | . . | . . . 27. [i] = (e @ L2)
| . . . . . . . | . . | . . . 28. ∀i:Vertices(the_graph).
| . . . . . . . | . . | . . .     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . | . . . 29. L2 = nil
| . . . . . . . | . . | . . . 30. j : Vertices(the_graph)
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| . . . . . . . | . . | . . . 31. i-the_graph->*j
| . . . . . . . | . . | . . . 32. (inr(j) ∈ s)
| . . . . . . . | . . | . . . 33. (inl(j) ∈ s)
| . . . . . . . | . . | . . . ⊢  inl(j) = inl(i)
| . . . . . . . | . . | . . .    ∨ (inl(j) ∈ s2)
| . . . . . . . | . . | . . .    ∨ inl(j) = inr(i)
| . . . . . . . | . . | . . .    ∨ (inl(j) ∈ s)  by ObviousFrom [-1]
| . . . . . . . | . . |
| . . . . . . . | . . .\
| . . . . . . . | . . . 1. the_graph : Graph
| . . . . . . . | . . . 2. GraphObject(the_graph)
| . . . . . . . | . . . 3. Vertices(the_graph) List
| . . . . . . . | . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . . 5. s : traversal(the_graph)
| . . . . . . . | . . . 6. i : Vertices(the_graph)
| . . . . . . . | . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . |  Vertices(the_graph);s)
| . . . . . . . | . . . 9. df-traversal(the_graph;s)
| . . . . . . . | . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . . 12. s2 : traversal(the_graph)
| . . . . . . . | . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . . |   Vertices(the_graph);s2;s)
| . . . . . . . | . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . |   Vertices(the_graph);s2)
| . . . . . . . | . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . . |                             @ s2
| . . . . . . . | . . . |                             @ [inr(i)])
| . . . . . . . | . . . |   @ s)
| . . . . . . . | . . . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . . |   ⇒
| . . . . . . . | . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . . |   L1 = (L1@0 @ L2)
| . . . . . . . | . . . |   ⇒
| . . . . . . . | . . . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . . |    s = (s2 @ s1)
| . . . . . . . | . . . |    & paren(Vertices(the_graph);s1)
| . . . . . . . | . . . |    & paren(Vertices(the_graph);s2)
| . . . . . . . | . . . |    & (∀j:Vertices(the_graph).
| . . . . . . . | . . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | . . . |       & ((inl(j) ∈ s2)
| . . . . . . . | . . . |          ⟺
| . . . . . . . | . . . |          L2-the_graph->*j
| . . . . . . . | . . . |          & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . . 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . . 23. (L1 @ [i]) = (L1@0 @ L2)
```

```
| . . . . . . . | . . . 24. e : Vertices(the_graph) List
| . . . . . . . | . . . 25. L1@0 = (L1 @ e)
| . . . . . . . | . . . 26. [i] = (e @ L2)
| . . . . . . . | . . . 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . . 28. L2 = nil
| . . . . . . . | . . . 29. j : Vertices(the_graph)
| . . . . . . . | . . . ⊢  (inl(j) ∈ nil)
| . . . . . . . | . . . |  ⟺
| . . . . . . . | . . . |  nil-the_graph->*j & ¬L1 @ [i]-the_graph->*j
| . . . . . . . | . . . | by Auto
| . . . . . . . | . . . |
| . . . . . . . | . . . |\
| . . . . . . . | . . . | 1. the_graph : Graph
| . . . . . . . | . . . | 2. GraphObject(the_graph)
| . . . . . . . | . . . | 3. Vertices(the_graph) List
| . . . . . . . | . . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . . | 5. s : traversal(the_graph)
| . . . . . . . | . . . | 6. i : Vertices(the_graph)
| . . . . . . . | . . . | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . |    Vertices(the_graph);s)
| . . . . . . . | . . . | 9. df-traversal(the_graph;s)
| . . . . . . . | . . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . . | 12. s2 : traversal(the_graph)
| . . . . . . . | . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . . |     Vertices(the_graph);s2;s)
| . . . . . . . | . . . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . |     Vertices(the_graph);s2)
| . . . . . . . | . . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . . | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . . |                             @ s2

| . . . . . . . | . . . |                             @ [inr(i)])
| . . . . . . . | . . . |     @ s)
| . . . . . . . | . . . | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . . |     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . . |     ⇒
| . . . . . . . | . . . |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . . |     L1 = (L1@0 @ L2)
| . . . . . . . | . . . |     ⇒
| . . . . . . . | . . . |     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . . |      s = (s2 @ s1)
| . . . . . . . | . . . |      & paren(Vertices(the_graph);s1)
| . . . . . . . | . . . |      & paren(Vertices(the_graph);s2)
| . . . . . . . | . . . |      & (∀j:Vertices(the_graph).
| . . . . . . . | . . . |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | . . . |         & ((inl(j) ∈ s2)
| . . . . . . . | . . . |            ⟺
| . . . . . . . | . . . |            L2-the_graph->*j
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```
| . . . . . . . | . . . |            & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . . | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . . | 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . . | 24. e : Vertices(the_graph) List
| . . . . . . . | . . . | 25. L1@0 = (L1 @ e)
| . . . . . . . | . . . | 26. [i] = (e @ L2)
| . . . . . . . | . . . | 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . . |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . . | 28. L2 = nil
| . . . . . . . | . . . | 29. j : Vertices(the_graph)
| . . . . . . . | . . . | 30. (inl(j) ∈ nil)
| . . . . . . . | . . . | ⊢  nil-the_graph->*j
| . . . . . . . | . . . |   by ObviousContradiction [-1]
| . . . . . . . | . . . |
| . . . . . . . | . . . |\
| . . . . . . . | . . . | 1. the_graph : Graph
| . . . . . . . | . . . | 2. GraphObject(the_graph)
| . . . . . . . | . . . | 3. Vertices(the_graph) List
| . . . . . . . | . . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . . | 5. s : traversal(the_graph)
| . . . . . . . | . . . | 6. i : Vertices(the_graph)
| . . . . . . . | . . . | 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . |    Vertices(the_graph);s)
| . . . . . . . | . . . | 9. df-traversal(the_graph;s)
| . . . . . . . | . . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . . | 12. s2 : traversal(the_graph)
| . . . . . . . | . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . . |     Vertices(the_graph);s2;s)
| . . . . . . . | . . . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . |     Vertices(the_graph);s2)
| . . . . . . . | . . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . . | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . . |                            @ s2
| . . . . . . . | . . . |                            @ [inr(i)])
| . . . . . . . | . . . |     @ s)
| . . . . . . . | . . . | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . . |     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . . |     ⇒
| . . . . . . . | . . . |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . . |     L1 = (L1@0 @ L2)
| . . . . . . . | . . . |     ⇒
| . . . . . . . | . . . |     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . . |      s = (s2 @ s1)
| . . . . . . . | . . . |      & paren(Vertices(the_graph);s1)
| . . . . . . . | . . . |      & paren(Vertices(the_graph);s2)
| . . . . . . . | . . . |      & (∀j:Vertices(the_graph).
| . . . . . . . | . . . |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
```

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| . . . . . . | . . . |         & ((inl(j) ∈ s2)
| . . . . . . | . . . |            ⟺
| . . . . . . | . . . |            L2-the_graph->*j
| . . . . . . | . . . |            & ¬L1@0-the_graph->*j)))
| . . . . . . | . . . | 21. L1@0 : Vertices(the_graph) List
| . . . . . . | . . . | 22. L2 : Vertices(the_graph) List
| . . . . . . | . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . | . . . | 24. e : Vertices(the_graph) List
| . . . . . . | . . . | 25. L1@0 = (L1 @ e)
| . . . . . . | . . . | 26. [i] = (e @ L2)
| . . . . . . | . . . | 27. ∀i:Vertices(the_graph).
| . . . . . . | . . . |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . | . . . | 28. L2 = nil
| . . . . . . | . . . | 29. j : Vertices(the_graph)
| . . . . . . | . . . | 30. (inl(j) ∈ nil)
| . . . . . . | . . . | ⊢  ¬L1 @ [i]-the_graph->*j
| . . . . . . | . . . |   by ObviousContradiction [-1]
| . . . . . . | . . . |
| . . . . . . | . . . .\
| . . . . . . | . . . . 1. the_graph : Graph
| . . . . . . | . . . . 2. GraphObject(the_graph)
| . . . . . . | . . . . 3. Vertices(the_graph) List
| . . . . . . | . . . . 4. L1 : Vertices(the_graph) List
| . . . . . . | . . . . 5. s : traversal(the_graph)
| . . . . . . | . . . . 6. i : Vertices(the_graph)
| . . . . . . | . . . . 7. paren(Vertices(the_graph);s)
| . . . . . . | . . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . | . . . . |  Vertices(the_graph);s)
| . . . . . . | . . . . 9. df-traversal(the_graph;s)
| . . . . . . | . . . . 10. ∀i:Vertices(the_graph).
| . . . . . . | . . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . | . . . . 11. ¬(inl(i) ∈ s)
| . . . . . . | . . . . 12. s2 : traversal(the_graph)
| . . . . . . | . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . | . . . . |   Vertices(the_graph);s2;s)
| . . . . . . | . . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . | . . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . | . . . . |   Vertices(the_graph);s2)
| . . . . . . | . . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . | . . . . 17. ∀j:Vertices(the_graph).
| . . . . . . | . . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . | . . . . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . | . . . . |                           @ s2
| . . . . . . | . . . . |                           @ [inr(i)])
| . . . . . . | . . . . |   @ s)
| . . . . . . | . . . . 19. ∀i@0:Vertices(the_graph).
| . . . . . . | . . . . |   L1 @ [i]-the_graph->*i@0
| . . . . . . | . . . . |   ⇒
| . . . . . . | . . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . | . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . | . . . . |   L1 = (L1@0 @ L2)
| . . . . . . | . . . . |   ⇒
| . . . . . . | . . . . |   (∃s1,s2:traversal(the_graph).
| . . . . . . | . . . . |    s = (s2 @ s1)
| . . . . . . | . . . . |    & paren(Vertices(the_graph);s1)
```

```
| . . . . . . . | . . . . |     & paren(Vertices(the_graph);s2)
| . . . . . . . | . . . . |     & (∀j:Vertices(the_graph).
| . . . . . . . | . . . . |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | . . . . |        & ((inl(j) ∈ s2)
| . . . . . . . | . . . . |           ⟺
| . . . . . . . | . . . . |           L2-the_graph->*j
| . . . . . . . | . . . . |           & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . . . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . . . 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . . . 24. e : Vertices(the_graph) List
| . . . . . . . | . . . . 25. L1@0 = (L1 @ e)
| . . . . . . . | . . . . 26. [i] = (e @ L2)
| . . . . . . . | . . . . 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . . . 28. L2 = nil
| . . . . . . . | . . . . 29. j : Vertices(the_graph)
| . . . . . . . | . . . . 30. nil-the_graph->*j
| . . . . . . . | . . . . 31. ¬L1 @ [i]-the_graph->*j
| . . . . . . . | . . . . ⊢  (inl(j) ∈ nil)
| . . . . . . . | . . . . | by (Analyze -2) THEN (Analyze -2)
| . . . . . . . | . . . . |
| . . . . . . . | . . . . 1. the_graph : Graph
| . . . . . . . | . . . . 2. GraphObject(the_graph)
| . . . . . . . | . . . . 3. Vertices(the_graph) List
| . . . . . . . | . . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . | . . . . 5. s : traversal(the_graph)
| . . . . . . . | . . . . 6. i : Vertices(the_graph)
| . . . . . . . | . . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . | . . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . .    Vertices(the_graph);s)
| . . . . . . . | . . . . 9. df-traversal(the_graph;s)
| . . . . . . . | . . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . | . . . .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . | . . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . | . . . . 12. s2 : traversal(the_graph)
| . . . . . . . | . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . | . . . .     Vertices(the_graph);s2;s)
| . . . . . . . | . . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . | . . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . | . . . .     Vertices(the_graph);s2)
| . . . . . . . | . . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . | . . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . | . . . .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . | . . . . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . | . . . .                              @ s2
| . . . . . . . | . . . .                              @ [inr(i)])
| . . . . . . . | . . . .     @ s)
| . . . . . . . | . . . . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . | . . . .     L1 @ [i]-the_graph->*i@0
| . . . . . . . | . . . .     ⇒
| . . . . . . . | . . . .     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . | . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . | . . . .     L1 = (L1@0 @ L2)
| . . . . . . . | . . . .     ⇒
```

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| . . . . . . . | . . . .     (∃s1,s2:traversal(the_graph).
| . . . . . . . | . . . .      s = (s2 @ s1)
| . . . . . . . | . . . .      & paren(Vertices(the_graph);s1)
| . . . . . . . | . . . .      & paren(Vertices(the_graph);s2)
| . . . . . . . | . . . .      & (∀j:Vertices(the_graph).
| . . . . . . . | . . . .         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . | . . . .         & ((inl(j) ∈ s2)
| . . . . . . . | . . . .            ⟺
| . . . . . . . | . . . .            L2-the_graph->*j
| . . . . . . . | . . . .            & ¬L1@0-the_graph->*j)))
| . . . . . . . | . . . . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . | . . . . 22. L2 : Vertices(the_graph) List
| . . . . . . . | . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . | . . . . 24. e : Vertices(the_graph) List
| . . . . . . . | . . . . 25. L1@0 = (L1 @ e)
| . . . . . . . | . . . . 26. [i] = (e @ L2)
| . . . . . . . | . . . . 27. ∀i:Vertices(the_graph).
| . . . . . . . | . . . .     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . | . . . . 28. L2 = nil
| . . . . . . . | . . . . 29. j : Vertices(the_graph)
| . . . . . . . | . . . . 30. y : Vertices(the_graph)
| . . . . . . . | . . . . 31. (y ∈ nil)
| . . . . . . . | . . . . 32. y-the_graph->*j
| . . . . . . . | . . . . 33. ¬L1 @ [i]-the_graph->*j
| . . . . . . . | . . . . ⊢  (inl(j) ∈ nil)
| . . . . . . . | . . . .   by ObviousContradiction [-3]
| . . . . . . . |
| . . . . . . . .\
| . . . . . . . . 1. the_graph : Graph
| . . . . . . . . 2. GraphObject(the_graph)
| . . . . . . . . 3. Vertices(the_graph) List
| . . . . . . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . . 5. s : traversal(the_graph)
| . . . . . . . . 6. i : Vertices(the_graph)
| . . . . . . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . |  Vertices(the_graph);s)
| . . . . . . . . 9. df-traversal(the_graph;s)
| . . . . . . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . . 12. s2 : traversal(the_graph)
| . . . . . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . |   Vertices(the_graph);s2;s)
| . . . . . . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . |   Vertices(the_graph);s2)
| . . . . . . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . . . . . . |   @ s)
| . . . . . . . . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . |   ⇒
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| . . . . . . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . |   L1 = (L1@0 @ L2)
| . . . . . . . . |   ⇒
| . . . . . . . . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . |    s = (s2 @ s1)
| . . . . . . . . |    & paren(Vertices(the_graph);s1)
| . . . . . . . . |    & paren(Vertices(the_graph);s2)
| . . . . . . . . |    & (∀j:Vertices(the_graph).
| . . . . . . . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . |       & ((inl(j) ∈ s2)
| . . . . . . . . |          ⟺
| . . . . . . . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . . . . . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . 22. L2 : Vertices(the_graph) List
| . . . . . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . 24. e : Vertices(the_graph) List
| . . . . . . . . 25. L1@0 = (L1 @ e)
| . . . . . . . . 26. [i] = (e @ L2)
| . . . . . . . . 27. ∀i:Vertices(the_graph).
| . . . . . . . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . 28. e = nil
| . . . . . . . . ⊢  ∃s1,s2@0:traversal(the_graph).
| . . . . . . . . |  (([inl(i)] @ s2 @ [inr(i)]) @ s) = (s2@0 @ s1)
| . . . . . . . . |  & paren(Vertices(the_graph);s1)
| . . . . . . . . |  & paren(Vertices(the_graph);s2@0)
| . . . . . . . . |  & (∀j:Vertices(the_graph).
| . . . . . . . . |     ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . |     & ((inl(j) ∈ s2@0)
| . . . . . . . . |        ⟺
| . . . . . . . . |        L2-the_graph->*j & ¬L1@0-the_graph->*j))
| . . . . . . . . | by (HypSubst -1 -3) THEN (Reduce -3)
| . . . . . . . . |    THEN
| . . . . . . . . |    (HypSubst -1 -4)
| . . . . . . . . |    THEN
| . . . . . . . . |    (RWO
| . . . . . . . . |     Thm* ∀l:T List. (l @ nil) ~ l    {append_nil_sq}
| . . . . . . . . |     -4)
| . . . . . . . . |    THEN
| . . . . . . . . |    (HypSubst -4 0)
| . . . . . . . . |    THEN
| . . . . . . . . |    (RevHypSubst -3 0)
| . . . . . . . . |
| . . . . . . . . 1. the_graph : Graph
| . . . . . . . . 2. GraphObject(the_graph)
| . . . . . . . . 3. Vertices(the_graph) List
| . . . . . . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . . 5. s : traversal(the_graph)
| . . . . . . . . 6. i : Vertices(the_graph)
| . . . . . . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . |  Vertices(the_graph);s)
| . . . . . . . . 9. df-traversal(the_graph;s)
| . . . . . . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
```

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| . . . . . . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . . 12. s2 : traversal(the_graph)
| . . . . . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . |   Vertices(the_graph);s2;s)
| . . . . . . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . |   Vertices(the_graph);s2)
| . . . . . . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . 18. df-traversal(the_graph;([inl(i)] @ s2 @ [inr(i)])
| . . . . . . . . |   @ s)
| . . . . . . . . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . |   ⇒
| . . . . . . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . |   L1 = (L1@0 @ L2)
| . . . . . . . . |   ⇒
| . . . . . . . . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . |    s = (s2 @ s1)
| . . . . . . . . |    & paren(Vertices(the_graph);s1)
| . . . . . . . . |    & paren(Vertices(the_graph);s2)
| . . . . . . . . |    & (∀j:Vertices(the_graph).
| . . . . . . . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . |       & ((inl(j) ∈ s2)
| . . . . . . . . |          ⟺
| . . . . . . . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . . . . . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . 22. L2 : Vertices(the_graph) List
| . . . . . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . 24. e : Vertices(the_graph) List
| . . . . . . . . 25. L1@0 = L1
| . . . . . . . . 26. [i] = L2
| . . . . . . . . 27. ∀i:Vertices(the_graph).
| . . . . . . . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . 28. e = nil
| . . . . . . . . ⊢  ∃s1,s2@0:traversal(the_graph).
| . . . . . . . . |  (([inl(i)] @ s2 @ [inr(i)]) @ s) = (s2@0 @ s1)
| . . . . . . . . |  & paren(Vertices(the_graph);s1)
| . . . . . . . . |  & paren(Vertices(the_graph);s2@0)
| . . . . . . . . |  & (∀j:Vertices(the_graph).
| . . . . . . . . |     ((inl(j) ∈ s1) ⟺ L1-the_graph->*j)
| . . . . . . . . |     & ((inl(j) ∈ s2@0)
| . . . . . . . . |        ⟺
| . . . . . . . . |        [i]-the_graph->*j & ¬L1-the_graph->*j))
| . . . . . . . . | by ((InstConcl [s;[inl(i)] @ s2 @ [inr(i)]])
| . . . . . . . . |     THEN
| . . . . . . . . |     (Analyze 0))
| . . . . . . . . |    THENL
| . . . . . . . . |    [Auto;Analyze 0]
| . . . . . . . . |
| . . . . . . . . |\
| . . . . . . . . | 1. the_graph : Graph
| . . . . . . . . | 2. GraphObject(the_graph)
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| . . . . . . . . | 3. Vertices(the_graph) List
| . . . . . . . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . . | 5. s : traversal(the_graph)
| . . . . . . . . | 6. i : Vertices(the_graph)
| . . . . . . . . | 7. paren(Vertices(the_graph);s)
| . . . . . . . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . |    Vertices(the_graph);s)
| . . . . . . . . | 9. df-traversal(the_graph;s)
| . . . . . . . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . . | 12. s2 : traversal(the_graph)
| . . . . . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . |     Vertices(the_graph);s2;s)
| . . . . . . . . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . |     Vertices(the_graph);s2)
| . . . . . . . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . |                             @ s2
| . . . . . . . . |                             @ [inr(i)])
| . . . . . . . . |     @ s)
| . . . . . . . . | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . |     L1 @ [i]-the_graph->*i@0
| . . . . . . . . |     ⇒
| . . . . . . . . |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . |     L1 = (L1@0 @ L2)
| . . . . . . . . |     ⇒
| . . . . . . . . |     (∃s1,s2:traversal(the_graph).
| . . . . . . . . |      s = (s2 @ s1)
| . . . . . . . . |      & paren(Vertices(the_graph);s1)
| . . . . . . . . |      & paren(Vertices(the_graph);s2)
| . . . . . . . . |      & (∀j:Vertices(the_graph).
| . . . . . . . . |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . |         & ((inl(j) ∈ s2)
| . . . . . . . . |             ⟺
| . . . . . . . . |             L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . . . . . | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . | 22. L2 : Vertices(the_graph) List
| . . . . . . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . | 24. e : Vertices(the_graph) List
| . . . . . . . . | 25. L1@0 = L1
| . . . . . . . . | 26. [i] = L2
| . . . . . . . . | 27. ∀i:Vertices(the_graph).
| . . . . . . . . |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . | 28. e = nil
| . . . . . . . . | ⊢  paren(Vertices(the_graph);s)
| . . . . . . . . |    & paren(Vertices(the_graph);[inl(i)]
| . . . . . . . . |      @ s2

| . . . . . . . . |      @ [inr(i)])
| . . . . . . . . |   by ((Auto THEN (RecUnfold `paren` 0) THEN OrRight)
| . . . . . . . . |       THEN
```

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| . . . . . . . . |       OrLeft)
| . . . . . . . . |      THEN
| . . . . . . . . |      (InstConcl [i;s2])
| . . . . . . . . |
| . . . . . . . . .\
| . . . . . . . . . 1. the_graph : Graph
| . . . . . . . . . 2. GraphObject(the_graph)
| . . . . . . . . . 3. Vertices(the_graph) List
| . . . . . . . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . . . 5. s : traversal(the_graph)
| . . . . . . . . . 6. i : Vertices(the_graph)
| . . . . . . . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . |  Vertices(the_graph);s)
| . . . . . . . . . 9. df-traversal(the_graph;s)
| . . . . . . . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . . . 12. s2 : traversal(the_graph)
| . . . . . . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . |   Vertices(the_graph);s2;s)
| . . . . . . . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . |   Vertices(the_graph);s2)
| . . . . . . . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . |                         @ s2
| . . . . . . . . . |                         @ [inr(i)])
| . . . . . . . . . |   @ s)
| . . . . . . . . . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . |   ⇒
| . . . . . . . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . |   L1 = (L1@0 @ L2)
| . . . . . . . . . |   ⇒
| . . . . . . . . . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . |    s = (s2 @ s1)
| . . . . . . . . . |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . |    & (∀j:Vertices(the_graph).
| . . . . . . . . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . |       & ((inl(j) ∈ s2)
| . . . . . . . . . |          ⟺
| . . . . . . . . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . . . . . . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . 22. L2 : Vertices(the_graph) List
| . . . . . . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . 24. e : Vertices(the_graph) List
| . . . . . . . . . 25. L1@0 = L1
| . . . . . . . . . 26. [i] = L2
| . . . . . . . . . 27. ∀i:Vertices(the_graph).
| . . . . . . . . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
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| . . . . . . . . . 28. e = nil
| . . . . . . . . . ⊢  ∀j:Vertices(the_graph).
| . . . . . . . . . |  ((inl(j) ∈ s) ⟺ L1-the_graph->*j)
| . . . . . . . . . |  & ((inl(j) ∈ [inl(i)] @ s2 @ [inr(i)])
| . . . . . . . . . |     ⟺
| . . . . . . . . . |     [i]-the_graph->*j & ¬L1-the_graph->*j)
| . . . . . . . . . | by (Thin -1) THEN (ParallelOp -1) THEN (Analyze 0)
| . . . . . . . . . |    THEN
| . . . . . . . . . |    (Try Trivial)
| . . . . . . . . . |    THEN
| . . . . . . . . . |    (RW (SweepDnC (RevHypC -1)) 0)
| . . . . . . . . . |
| . . . . . . . . . 1. the_graph : Graph
| . . . . . . . . . 2. GraphObject(the_graph)
| . . . . . . . . . 3. Vertices(the_graph) List
| . . . . . . . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . . . 5. s : traversal(the_graph)
| . . . . . . . . . 6. i : Vertices(the_graph)
| . . . . . . . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . |  Vertices(the_graph);s)
| . . . . . . . . . 9. df-traversal(the_graph;s)
| . . . . . . . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . . . 12. s2 : traversal(the_graph)
| . . . . . . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . |   Vertices(the_graph);s2;s)
| . . . . . . . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . |   Vertices(the_graph);s2)
| . . . . . . . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . |                           @ s2
| . . . . . . . . . |                           @ [inr(i)])
| . . . . . . . . . |   @ s)
| . . . . . . . . . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . |   ⇒
| . . . . . . . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . |   L1 = (L1@0 @ L2)
| . . . . . . . . . |   ⇒
| . . . . . . . . . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . |    s = (s2 @ s1)
| . . . . . . . . . |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . |    & (∀j:Vertices(the_graph).
| . . . . . . . . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . |       & ((inl(j) ∈ s2)
| . . . . . . . . . |          ⟺
| . . . . . . . . . |          L2-the_graph->*j & ¬L1@0-the_graph->*j)))
| . . . . . . . . . 21. L1@0 : Vertices(the_graph) List
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| . . . . . . . . . 22. L2 : Vertices(the_graph) List
| . . . . . . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . 24. Vertices(the_graph) List
| . . . . . . . . . 25. L1@0 = L1
| . . . . . . . . . 26. [i] = L2
| . . . . . . . . . 27. ∀i:Vertices(the_graph).
| . . . . . . . . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . 28. j : Vertices(the_graph)
| . . . . . . . . . 29. (inl(j) ∈ s) ⟺ L1-the_graph->*j
| . . . . . . . . . ⊢  (inl(j) ∈ [inl(i)] @ s2 @ [inr(i)])
| . . . . . . . . . |  ⟺
| . . . . . . . . . |  [i]-the_graph->*j & ¬(inl(j) ∈ s)
| . . . . . . . . . | by ((RWW
| . . . . . . . . . |      Thm* ∀x:T, l1,l2:T List.
| . . . . . . . . . |           (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| . . . . . . . . . |      {member_append}
| . . . . . . . . . |      0)
| . . . . . . . . . |     THEN
| . . . . . . . . . |     (RWO
| . . . . . . . . . |      Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a
| . . . . . . . . . |      {member_singleton}
| . . . . . . . . . |      0))
| . . . . . . . . . |    THEN
| . . . . . . . . . |    (Analyze 0)
| . . . . . . . . . |    THEN
| . . . . . . . . . |    (Analyze 0)
| . . . . . . . . . |
| . . . . . . . . . |\
| . . . . . . . . . | 1. the_graph : Graph
| . . . . . . . . . | 2. GraphObject(the_graph)
| . . . . . . . . . | 3. Vertices(the_graph) List
| . . . . . . . . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . . . | 5. s : traversal(the_graph)
| . . . . . . . . . | 6. i : Vertices(the_graph)
| . . . . . . . . . | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | |  Vertices(the_graph);s)
| . . . . . . . . . | 9. df-traversal(the_graph;s)
| . . . . . . . . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . | 12. s2 : traversal(the_graph)
| . . . . . . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . | |   Vertices(the_graph);s2;s)
| . . . . . . . . . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | |   Vertices(the_graph);s2)
| . . . . . . . . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . | |                          @ s2
| . . . . . . . . . | |                          @ [inr(i)])
| . . . . . . . . . | |   @ s)
| . . . . . . . . . | 19. ∀i@0:Vertices(the_graph).
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| . . . . . . . . . | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . | |   ⇒
| . . . . . . . . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . | |   L1 = (L1@0 @ L2)
| . . . . . . . . . | |   ⇒
| . . . . . . . . . | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . | |    s = (s2 @ s1)
| . . . . . . . . . | |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . | |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . | |    & ...)
| . . . . . . . . . | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . | 22. L2 : Vertices(the_graph) List
| . . . . . . . . . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . | 24. Vertices(the_graph) List
| . . . . . . . . . | 25. L1@0 = L1
| . . . . . . . . . | 26. [i] = L2
| . . . . . . . . . | 27. ∀i:Vertices(the_graph).
| . . . . . . . . . | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . | 28. j : Vertices(the_graph)
| . . . . . . . . . | 29. (inl(j) ∈ s) ⟺ L1-the_graph->*j
| . . . . . . . . . | 30. inl(j) = inl(i)
| . . . . . . . . . | |   ∨ (inl(j) ∈ s2)
| . . . . . . . . . | |   ∨ inl(j) = inr(i)
| . . . . . . . . . | ⊢  [i]-the_graph->*j & ¬(inl(j) ∈ s)
| . . . . . . . . . | | by Analyze -1
| . . . . . . . . . | |
| . . . . . . . . . | |\
| . . . . . . . . . | | 1. the_graph : Graph
| . . . . . . . . . | | 2. GraphObject(the_graph)
| . . . . . . . . . | | 3. Vertices(the_graph) List
| . . . . . . . . . | | 4. L1 : Vertices(the_graph) List
| . . . . . . . . . | | 5. s : traversal(the_graph)
| . . . . . . . . . | | 6. i : Vertices(the_graph)
| . . . . . . . . . | | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | | |  Vertices(the_graph);s)
| . . . . . . . . . | | 9. df-traversal(the_graph;s)
| . . . . . . . . . | | 10. ∀i:Vertices(the_graph).
| . . . . . . . . . | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . | | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . | | 12. s2 : traversal(the_graph)
| . . . . . . . . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . | | |   Vertices(the_graph);s2;s)
| . . . . . . . . . | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | | |   Vertices(the_graph);s2)
| . . . . . . . . . | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . | | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . | | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . | | |                           @ s2
| . . . . . . . . . | | |                           @ [inr(i)])
| . . . . . . . . . | | |   @ s)
| . . . . . . . . . | | 19. ∀i@0:Vertices(the_graph).
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| . . . . . . . . . | | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . | | |   ⇒
| . . . . . . . . . | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . | | |   L1 = (L1@0 @ L2)
| . . . . . . . . . | | |   ⇒
| . . . . . . . . . | | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . | | |    s = (s2 @ s1)
| . . . . . . . . . | | |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . | | |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . | | |    & (∀j:Vertices(the_graph).
| . . . . . . . . . | | |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . | | |        & ((inl(j) ∈ s2)
| . . . . . . . . . | | |           ⟺
| . . . . . . . . . | | |           L2-the_graph->*j
| . . . . . . . . . | | |           & ¬L1@0-the_graph->*j)))
| . . . . . . . . . | | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . | | 22. L2 : Vertices(the_graph) List
| . . . . . . . . . | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . | | 24. Vertices(the_graph) List
| . . . . . . . . . | | 25. L1@0 = L1
| . . . . . . . . . | | 26. [i] = L2
| . . . . . . . . . | | 27. ∀i:Vertices(the_graph).
| . . . . . . . . . | | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . | | 28. j : Vertices(the_graph)
| . . . . . . . . . | | 29. (inl(j) ∈ s) ⟺ L1-the_graph->*j
| . . . . . . . . . | | 30. inl(j) = inl(i)
| . . . . . . . . . | | ⊢  [i]-the_graph->*j & ¬(inl(j) ∈ s)
| . . . . . . . . . | | | by Subst (j = i) 0
| . . . . . . . . . | | |
| . . . . . . . . . | | |\
| . . . . . . . . . | | | 1. the_graph : Graph
| . . . . . . . . . | | | 2. GraphObject(the_graph)
| . . . . . . . . . | | | 3. Vertices(the_graph) List
| . . . . . . . . . | | | 4. L1 : Vertices(the_graph) List
| . . . . . . . . . | | | 5. s : traversal(the_graph)
| . . . . . . . . . | | | 6. i : Vertices(the_graph)
| . . . . . . . . . | | | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . | | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | | |    Vertices(the_graph);s)
| . . . . . . . . . | | | 9. df-traversal(the_graph;s)
| . . . . . . . . . | | | 10. ∀i:Vertices(the_graph).
| . . . . . . . . . | | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . | | | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . | | | 12. s2 : traversal(the_graph)
| . . . . . . . . . | | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . | | |     Vertices(the_graph);s2;s)
| . . . . . . . . . | | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . | | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | | |     Vertices(the_graph);s2)
| . . . . . . . . . | | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . | | | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . | | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . | | | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . | | |                              @ s2
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| . . . . . . . . . | | |                              @ [inr(i)])
| . . . . . . . . . | | |     @ s)
| . . . . . . . . . | | | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . | | |     L1 @ [i]-the_graph->*i@0
| . . . . . . . . . | | |     ⇒
| . . . . . . . . . | | |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . | | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . | | |     L1 = (L1@0 @ L2)
| . . . . . . . . . | | |     ⇒
| . . . . . . . . . | | |     (∃s1,s2:traversal(the_graph).
| . . . . . . . . . | | |      s = (s2 @ s1)
| . . . . . . . . . | | |      & paren(Vertices(the_graph);s1)
| . . . . . . . . . | | |      & paren(Vertices(the_graph);s2)
| . . . . . . . . . | | |      & (∀j:Vertices(the_graph).
| . . . . . . . . . | | |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . | | |         & ((inl(j) ∈ s2)
| . . . . . . . . . | | |            ⟺
| . . . . . . . . . | | |            L2-the_graph->*j
| . . . . . . . . . | | |            & ¬L1@0-the_graph->*j)))
| . . . . . . . . . | | | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . | | | 22. L2 : Vertices(the_graph) List
| . . . . . . . . . | | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . | | | 24. Vertices(the_graph) List
| . . . . . . . . . | | | 25. L1@0 = L1
| . . . . . . . . . | | | 26. [i] = L2
| . . . . . . . . . | | | 27. ∀i:Vertices(the_graph).
| . . . . . . . . . | | |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . | | | 28. j : Vertices(the_graph)
| . . . . . . . . . | | | 29. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . | | | 30. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . | | | 31. inl(j) = inl(i)
| . . . . . . . . . | | | ⊢  j = i  by ObviousFrom [-1]
| . . . . . . . . . | | |
| . . . . . . . . . | | .\
| . . . . . . . . . | | . 1. the_graph : Graph
| . . . . . . . . . | | . 2. GraphObject(the_graph)
| . . . . . . . . . | | . 3. Vertices(the_graph) List
| . . . . . . . . . | | . 4. L1 : Vertices(the_graph) List
| . . . . . . . . . | | . 5. s : traversal(the_graph)
| . . . . . . . . . | | . 6. i : Vertices(the_graph)
| . . . . . . . . . | | . 7. paren(Vertices(the_graph);s)
| . . . . . . . . . | | . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | | . |  Vertices(the_graph);s)
| . . . . . . . . . | | . 9. df-traversal(the_graph;s)
| . . . . . . . . . | | . 10. ∀i:Vertices(the_graph).
| . . . . . . . . . | | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . | | . 11. ¬(inl(i) ∈ s)
| . . . . . . . . . | | . 12. s2 : traversal(the_graph)
| . . . . . . . . . | | . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . | | . |   Vertices(the_graph);s2;s)
| . . . . . . . . . | | . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . | | . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | | . |   Vertices(the_graph);s2)
| . . . . . . . . . | | . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . | | . 17. ∀j:Vertices(the_graph).
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| . . . . . . . . . | | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . | | . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . | | . |                         @ s2
| . . . . . . . . . | | . |                         @ [inr(i)])
| . . . . . . . . . | | . |   @ s)
| . . . . . . . . . | | . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . | | . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . | | . |   ⇒
| . . . . . . . . . | | . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . | | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . | | . |   L1 = (L1@0 @ L2)
| . . . . . . . . . | | . |   ⇒
| . . . . . . . . . | | . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . | | . |    s = (s2 @ s1)
| . . . . . . . . . | | . |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . | | . |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . | | . |    & (∀j:Vertices(the_graph).
| . . . . . . . . . | | . |        ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . | | . |        & ((inl(j) ∈ s2)
| . . . . . . . . . | | . |           ⟺
| . . . . . . . . . | | . |           L2-the_graph->*j
| . . . . . . . . . | | . |           & ¬L1@0-the_graph->*j)))
| . . . . . . . . . | | . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . | | . 22. L2 : Vertices(the_graph) List
| . . . . . . . . . | | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . | | . 24. Vertices(the_graph) List
| . . . . . . . . . | | . 25. L1@0 = L1
| . . . . . . . . . | | . 26. [i] = L2
| . . . . . . . . . | | . 27. ∀i:Vertices(the_graph).
| . . . . . . . . . | | . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . | | . 28. j : Vertices(the_graph)
| . . . . . . . . . | | . 29. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . | | . 30. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . | | . 31. inl(j) = inl(i)
| . . . . . . . . . | | . ⊢  [i]-the_graph->*i
| . . . . . . . . . | | . | by (Unfold ‘list-connect‘ 0)
| . . . . . . . . . | | . |    THEN
| . . . . . . . . . | | . |    (Unfold ‘l_exists‘ 0)
| . . . . . . . . . | | . |    THEN
| . . . . . . . . . | | . |    (InstConcl [i])
| . . . . . . . . . | | . |    THEN
| . . . . . . . . . | | . |    (Try Connect)
| . . . . . . . . . | | . |
| . . . . . . . . . | | . 1. the_graph : Graph
| . . . . . . . . . | | . 2. GraphObject(the_graph)
| . . . . . . . . . | | . 3. Vertices(the_graph) List
| . . . . . . . . . | | . 4. L1 : Vertices(the_graph) List
| . . . . . . . . . | | . 5. s : traversal(the_graph)
| . . . . . . . . . | | . 6. i : Vertices(the_graph)
| . . . . . . . . . | | . 7. paren(Vertices(the_graph);s)
| . . . . . . . . . | | . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | | .    Vertices(the_graph);s)
| . . . . . . . . . | | . 9. df-traversal(the_graph;s)
| . . . . . . . . . | | . 10. ∀i:Vertices(the_graph).
| . . . . . . . . . | | .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
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| . . . . . . . . . | | . 11. ¬(inl(i) ∈ s)
| . . . . . . . . . | | . 12. s2 : traversal(the_graph)
| . . . . . . . . . | | . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . | | .     Vertices(the_graph);s2;s)
| . . . . . . . . . | | . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . | | . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | | .     Vertices(the_graph);s2)
| . . . . . . . . . | | . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . | | . 17. ∀j:Vertices(the_graph).
| . . . . . . . . . | | .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . | | . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . | | .                             @ s2

| . . . . . . . . . | | .                             @ [inr(i)])
| . . . . . . . . . | | .     @ s)
| . . . . . . . . . | | . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . | | .     L1 @ [i]-the_graph->*i@0
| . . . . . . . . . | | .     ⇒
| . . . . . . . . . | | .     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . | | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . | | .     L1 = (L1@0 @ L2)
| . . . . . . . . . | | .     ⇒
| . . . . . . . . . | | .     (∃s1,s2:traversal(the_graph).
| . . . . . . . . . | | .      s = (s2 @ s1)
| . . . . . . . . . | | .      & paren(Vertices(the_graph);s1)
| . . . . . . . . . | | .      & paren(Vertices(the_graph);s2)
| . . . . . . . . . | | .      & (∀j:Vertices(the_graph).
| . . . . . . . . . | | .         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . | | .         & ((inl(j) ∈ s2)
| . . . . . . . . . | | .            ⟺
| . . . . . . . . . | | .            L2-the_graph->*j
| . . . . . . . . . | | .            & ¬L1@0-the_graph->*j)))
| . . . . . . . . . | | . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . | | . 22. L2 : Vertices(the_graph) List
| . . . . . . . . . | | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . | | . 24. Vertices(the_graph) List
| . . . . . . . . . | | . 25. L1@0 = L1
| . . . . . . . . . | | . 26. [i] = L2
| . . . . . . . . . | | . 27. ∀i:Vertices(the_graph).
| . . . . . . . . . | | .     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . | | . 28. j : Vertices(the_graph)
| . . . . . . . . . | | . 29. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . | | . 30. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . | | . 31. inl(j) = inl(i)
| . . . . . . . . . | | . ⊢  (i ∈ [i])  by ObviousConcl
| . . . . . . . . . | |
| . . . . . . . . . | .\
| . . . . . . . . . | . 1. the_graph : Graph
| . . . . . . . . . | . 2. GraphObject(the_graph)
| . . . . . . . . . | . 3. Vertices(the_graph) List
| . . . . . . . . . | . 4. L1 : Vertices(the_graph) List
| . . . . . . . . . | . 5. s : traversal(the_graph)
| . . . . . . . . . | . 6. i : Vertices(the_graph)
| . . . . . . . . . | . 7. paren(Vertices(the_graph);s)
| . . . . . . . . . | . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . |  Vertices(the_graph);s)
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| . . . . . . . . . | . 9. df-traversal(the_graph;s)
| . . . . . . . . . | . 10. ∀i:Vertices(the_graph).
| . . . . . . . . . | . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . | . 11. ¬(inl(i) ∈ s)
| . . . . . . . . . | . 12. s2 : traversal(the_graph)
| . . . . . . . . . | . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . | . |   Vertices(the_graph);s2;s)
| . . . . . . . . . | . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . | . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . |   Vertices(the_graph);s2)
| . . . . . . . . . | . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . | . 17. ∀j:Vertices(the_graph).
| . . . . . . . . . | . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . | . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . | . |                           @ s2
| . . . . . . . . . | . |                           @ [inr(i)])
| . . . . . . . . . | . |   @ s)
| . . . . . . . . . | . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . | . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . | . |   ⇒
| . . . . . . . . . | . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . | . |   L1 = (L1@0 @ L2)
| . . . . . . . . . | . |   ⇒
| . . . . . . . . . | . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . | . |    s = (s2 @ s1)
| . . . . . . . . . | . |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . | . |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . | . |    & (∀j:Vertices(the_graph).
| . . . . . . . . . | . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . | . |       & ((inl(j) ∈ s2)
| . . . . . . . . . | . |          ⟺
| . . . . . . . . . | . |          L2-the_graph->*j
| . . . . . . . . . | . |          & ¬L1@0-the_graph->*j)))
| . . . . . . . . . | . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . | . 22. L2 : Vertices(the_graph) List
| . . . . . . . . . | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . | . 24. Vertices(the_graph) List
| . . . . . . . . . | . 25. L1@0 = L1
| . . . . . . . . . | . 26. [i] = L2
| . . . . . . . . . | . 27. ∀i:Vertices(the_graph).
| . . . . . . . . . | . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . | . 28. j : Vertices(the_graph)
| . . . . . . . . . | . 29. (inl(j) ∈ s) ⟺ L1-the_graph->*j
| . . . . . . . . . | . 30. (inl(j) ∈ s2) ∨ inl(j) = inr(i)
| . . . . . . . . . | . ⊢  [i]-the_graph->*j & ¬(inl(j) ∈ s)
| . . . . . . . . . | . | by Analyze -1
| . . . . . . . . . | . |
| . . . . . . . . . | . |\
| . . . . . . . . . | . | 1. the_graph : Graph
| . . . . . . . . . | . | 2. GraphObject(the_graph)
| . . . . . . . . . | . | 3. Vertices(the_graph) List
| . . . . . . . . . | . | 4. L1 : Vertices(the_graph) List
| . . . . . . . . . | . | 5. s : traversal(the_graph)
| . . . . . . . . . | . | 6. i : Vertices(the_graph)
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| . . . . . . . . . | . | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . | . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . | |  Vertices(the_graph);s)
| . . . . . . . . . | . | 9. df-traversal(the_graph;s)
| . . . . . . . . . | . | 10. ∀i:Vertices(the_graph).
| . . . . . . . . . | . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . | . | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . | . | 12. s2 : traversal(the_graph)
| . . . . . . . . . | . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . | . | |   Vertices(the_graph);s2;s)
| . . . . . . . . . | . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . | . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . | |   Vertices(the_graph);s2)
| . . . . . . . . . | . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . | . | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . | . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . | . | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . | . | |                           @ s2
| . . . . . . . . . | . | |                           @ [inr(i)])
| . . . . . . . . . | . | |   @ s)
| . . . . . . . . . | . | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . | . | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . | . | |   ⇒
| . . . . . . . . . | . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . | . | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . | . | |   L1 = (L1@0 @ L2)
| . . . . . . . . . | . | |   ⇒
| . . . . . . . . . | . | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . | . | |    s = (s2 @ s1)
| . . . . . . . . . | . | |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . | . | |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . | . | |    & (∀j:Vertices(the_graph).
| . . . . . . . . . | . | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . | . | |       & ((inl(j) ∈ s2)
| . . . . . . . . . | . | |          ⟺
| . . . . . . . . . | . | |          L2-the_graph->*j
| . . . . . . . . . | . | |          & ¬L1@0-the_graph->*j)))
| . . . . . . . . . | . | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . | . | 22. L2 : Vertices(the_graph) List
| . . . . . . . . . | . | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . | . | 24. Vertices(the_graph) List
| . . . . . . . . . | . | 25. L1@0 = L1
| . . . . . . . . . | . | 26. [i] = L2
| . . . . . . . . . | . | 27. ∀i:Vertices(the_graph).
| . . . . . . . . . | . | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . | . | 28. j : Vertices(the_graph)
| . . . . . . . . . | . | 29. (inl(j) ∈ s) ⟺ L1-the_graph->*j
| . . . . . . . . . | . | 30. (inl(j) ∈ s2)
| . . . . . . . . . | . | ⊢  [i]-the_graph->*j & ¬(inl(j) ∈ s)  by Auto
| . . . . . . . . . | . | |
| . . . . . . . . . | . | |\
| . . . . . . . . . | . | | 1. the_graph : Graph
| . . . . . . . . . | . | | 2. GraphObject(the_graph)
| . . . . . . . . . | . | | 3. Vertices(the_graph) List
| . . . . . . . . . | . | | 4. L1 : Vertices(the_graph) List
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| . . . . . . . . . | . | | 5. s : traversal(the_graph)
| . . . . . . . . . | . | | 6. i : Vertices(the_graph)
| . . . . . . . . . | . | | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . | . | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . | | |  Vertices(the_graph);s)
| . . . . . . . . . | . | | 9. df-traversal(the_graph;s)
| . . . . . . . . . | . | | 10. ∀i:Vertices(the_graph).
| . . . . . . . . . | . | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . | . | | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . | . | | 12. s2 : traversal(the_graph)
| . . . . . . . . . | . | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . | . | | |   Vertices(the_graph);s2;s)
| . . . . . . . . . | . | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . | . | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . | | |   Vertices(the_graph);s2)
| . . . . . . . . . | . | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . | . | | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . | . | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . | . | | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . | . | | |                           @ s2
| . . . . . . . . . | . | | |                           @ [inr(i)])
| . . . . . . . . . | . | | |   @ s)
| . . . . . . . . . | . | | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . | . | | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . | . | | |   ⇒
| . . . . . . . . . | . | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . | . | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . | . | | |   L1 = (L1@0 @ L2)
| . . . . . . . . . | . | | |   ⇒
| . . . . . . . . . | . | | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . | . | | |    s = (s2 @ s1)
| . . . . . . . . . | . | | |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . | . | | |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . | . | | |    & (∀j:Vertices(the_graph).
| . . . . . . . . . | . | | |       ...
| . . . . . . . . . | . | | |       & ((inl(j) ∈ s2)
| . . . . . . . . . | . | | |          ⟺
| . . . . . . . . . | . | | |          L2-the_graph->*j
| . . . . . . . . . | . | | |          & ¬L1@0-the_graph->*j)))
| . . . . . . . . . | . | | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . | . | | 22. L2 : Vertices(the_graph) List
| . . . . . . . . . | . | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . | . | | 24. Vertices(the_graph) List
| . . . . . . . . . | . | | 25. L1@0 = L1
| . . . . . . . . . | . | | 26. [i] = L2
| . . . . . . . . . | . | | 27. ∀i:Vertices(the_graph).
| . . . . . . . . . | . | | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . | . | | 28. j : Vertices(the_graph)
| . . . . . . . . . | . | | 29. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . | . | | 30. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . | . | | 31. (inl(j) ∈ s2)
| . . . . . . . . . | . | | ⊢  [i]-the_graph->*j
| . . . . . . . . . | . | | | by (Unfold `list-connect` 0)
| . . . . . . . . . | . | | |    THEN
| . . . . . . . . . | . | | |    (Unfold `l_exists` 0)
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| . . . . . . . . . | . | | |    THEN
| . . . . . . . . . | . | | |    (InstConcl [i])
| . . . . . . . . . | . | | |
| . . . . . . . . . | . | | |\
| . . . . . . . . . | . | | | 1. the_graph : Graph
| . . . . . . . . . | . | | | 2. GraphObject(the_graph)
| . . . . . . . . . | . | | | 3. Vertices(the_graph) List
| . . . . . . . . . | . | | | 4. L1 : Vertices(the_graph) List
| . . . . . . . . . | . | | | 5. s : traversal(the_graph)
| . . . . . . . . . | . | | | 6. i : Vertices(the_graph)
| . . . . . . . . . | . | | | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . | . | | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . | | |    Vertices(the_graph);s)
| . . . . . . . . . | . | | | 9. df-traversal(the_graph;s)
| . . . . . . . . . | . | | | 10. ∀i:Vertices(the_graph).
| . . . . . . . . . | . | | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . | . | | | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . | . | | | 12. s2 : traversal(the_graph)
| . . . . . . . . . | . | | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . | . | | |     Vertices(the_graph);s2;s)
| . . . . . . . . . | . | | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . | . | | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . | | |     Vertices(the_graph);s2)
| . . . . . . . . . | . | | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . | . | | | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . | . | | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . | . | | | 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . | . | | |                             @ s2
| . . . . . . . . . | . | | |                             @ [inr(i)])
| . . . . . . . . . | . | | |     @ s)
| . . . . . . . . . | . | | | 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . | . | | |     L1 @ [i]-the_graph->*i@0
| . . . . . . . . . | . | | |     ⇒
| . . . . . . . . . | . | | |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . | . | | | 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . | . | | |     L1 = (L1@0 @ L2)
| . . . . . . . . . | . | | |     ⇒
| . . . . . . . . . | . | | |     (∃s1,s2:traversal(the_graph).
| . . . . . . . . . | . | | |      s = (s2 @ s1)
| . . . . . . . . . | . | | |      & paren(Vertices(the_graph);s1)
| . . . . . . . . . | . | | |      & paren(Vertices(the_graph);s2)
| . . . . . . . . . | . | | |      & (∀j:Vertices(the_graph).
| . . . . . . . . . | . | | |         ((inl(j) ∈ s1)
| . . . . . . . . . | . | | |          ⟺
| . . . . . . . . . | . | | |          L1@0-the_graph->*j)
| . . . . . . . . . | . | | |         & ((inl(j) ∈ s2)
| . . . . . . . . . | . | | |            ⟺
| . . . . . . . . . | . | | |            L2-the_graph->*j
| . . . . . . . . . | . | | |            & ¬L1@0-the_graph->*j)))
| . . . . . . . . . | . | | | 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . | . | | | 22. L2 : Vertices(the_graph) List
| . . . . . . . . . | . | | | 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . | . | | | 24. Vertices(the_graph) List
| . . . . . . . . . | . | | | 25. L1@0 = L1
| . . . . . . . . . | . | | | 26. [i] = L2
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| . . . . . . . . . | . | | | 27. ∀i:Vertices(the_graph).
| . . . . . . . . . | . | | |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . | . | | | 28. j : Vertices(the_graph)
| . . . . . . . . . | . | | | 29. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . | . | | | 30. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . | . | | | 31. (inl(j) ∈ s2)
| . . . . . . . . . | . | | | ⊢  (i ∈ [i])  by ObviousConcl
| . . . . . . . . . | . | | |
| . . . . . . . . . | . | | .\
| . . . . . . . . . | . | | . 1. the_graph : Graph
| . . . . . . . . . | . | | . 2. GraphObject(the_graph)
| . . . . . . . . . | . | | . 3. Vertices(the_graph) List
| . . . . . . . . . | . | | . 4. L1 : Vertices(the_graph) List
| . . . . . . . . . | . | | . 5. s : traversal(the_graph)
| . . . . . . . . . | . | | . 6. i : Vertices(the_graph)
| . . . . . . . . . | . | | . 7. paren(Vertices(the_graph);s)
| . . . . . . . . . | . | | . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . | | .    Vertices(the_graph);s)
| . . . . . . . . . | . | | . 9. df-traversal(the_graph;s)
| . . . . . . . . . | . | | . 10. ∀i:Vertices(the_graph).
| . . . . . . . . . | . | | .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . | . | | . 11. ¬(inl(i) ∈ s)
| . . . . . . . . . | . | | . 12. s2 : traversal(the_graph)
| . . . . . . . . . | . | | . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . | . | | .     Vertices(the_graph);s2;s)
| . . . . . . . . . | . | | . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . | . | | . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . | | .     Vertices(the_graph);s2)
| . . . . . . . . . | . | | . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . | . | | . 17. ∀j:Vertices(the_graph).
| . . . . . . . . . | . | | .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . | . | | . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . | . | | .                              @ s2
| . . . . . . . . . | . | | .                              @ [inr(i)])
| . . . . . . . . . | . | | .     @ s)
| . . . . . . . . . | . | | . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . | . | | .     L1 @ [i]-the_graph->*i@0
| . . . . . . . . . | . | | .     ⇒
| . . . . . . . . . | . | | .     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . | . | | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . | . | | .     L1 = (L1@0 @ L2)
| . . . . . . . . . | . | | .     ⇒
| . . . . . . . . . | . | | .     (∃s1,s2:traversal(the_graph).
| . . . . . . . . . | . | | .      s = (s2 @ s1)
| . . . . . . . . . | . | | .      & paren(Vertices(the_graph);s1)
| . . . . . . . . . | . | | .      & paren(Vertices(the_graph);s2)
| . . . . . . . . . | . | | .      & (∀j:Vertices(the_graph).
| . . . . . . . . . | . | | .         ((inl(j) ∈ s1)
| . . . . . . . . . | . | | .          ⟺
| . . . . . . . . . | . | | .          L1@0-the_graph->*j)
| . . . . . . . . . | . | | .         & ((inl(j) ∈ s2)
| . . . . . . . . . | . | | .            ⟺
| . . . . . . . . . | . | | .            L2-the_graph->*j
| . . . . . . . . . | . | | .            & ¬L1@0-the_graph->*j)))
| . . . . . . . . . | . | | . 21. L1@0 : Vertices(the_graph) List
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| . . . . . . . . . | . | | . 22. L2 : Vertices(the_graph) List
| . . . . . . . . . | . | | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . | . | | . 24. Vertices(the_graph) List
| . . . . . . . . . | . | | . 25. L1@0 = L1
| . . . . . . . . . | . | | . 26. [i] = L2
| . . . . . . . . . | . | | . 27. ∀i:Vertices(the_graph).
| . . . . . . . . . | . | | .     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . | . | | . 28. j : Vertices(the_graph)
| . . . . . . . . . | . | | . 29. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . | . | | . 30. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . | . | | . 31. (inl(j) ∈ s2)
| . . . . . . . . . | . | | . ⊢  i-the_graph->*j
| . . . . . . . . . | . | | .   by BackThruSomeHyp
| . . . . . . . . . | . | | .      THEN
| . . . . . . . . . | . | | .      (BackThru
| . . . . . . . . . | . | | .       Thm* ∀s':(T+T) List.
| . . . . . . . . . | . | | .             paren(T;s')
| . . . . . . . . . | . | | .             ⇒
| . . . . . . . . . | . | | .             ...
| . . . . . . . . . | . | | .       {paren_balance1})
| . . . . . . . . . | . | |
| . . . . . . . . . | . | .\
| . . . . . . . . . | . | . 1. the_graph : Graph
| . . . . . . . . . | . | . 2. GraphObject(the_graph)
| . . . . . . . . . | . | . 3. Vertices(the_graph) List
| . . . . . . . . . | . | . 4. L1 : Vertices(the_graph) List
| . . . . . . . . . | . | . 5. s : traversal(the_graph)
| . . . . . . . . . | . | . 6. i : Vertices(the_graph)
| . . . . . . . . . | . | . 7. paren(Vertices(the_graph);s)
| . . . . . . . . . | . | . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . | .    Vertices(the_graph);s)
| . . . . . . . . . | . | . 9. df-traversal(the_graph;s)
| . . . . . . . . . | . | . 10. ∀i:Vertices(the_graph).
| . . . . . . . . . | . | .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . | . | . 11. ¬(inl(i) ∈ s)
| . . . . . . . . . | . | . 12. s2 : traversal(the_graph)
| . . . . . . . . . | . | . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . | . | .     Vertices(the_graph);s2;s)
| . . . . . . . . . | . | . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . | . | . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . | .     Vertices(the_graph);s2)
| . . . . . . . . . | . | . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . | . | . 17. ∀j:Vertices(the_graph).
| . . . . . . . . . | . | .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . | . | . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . | . | .                              @ s2
| . . . . . . . . . | . | .                              @ [inr(i)])
| . . . . . . . . . | . | .     @ s)
| . . . . . . . . . | . | . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . | . | .     L1 @ [i]-the_graph->*i@0
| . . . . . . . . . | . | .     ⇒
| . . . . . . . . . | . | .     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . | . | . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . | . | .     L1 = (L1@0 @ L2)
| . . . . . . . . . | . | .     ⇒
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| . . . . . . . . . | . | .     (∃s1,s2:traversal(the_graph).
| . . . . . . . . . | . | .      s = (s2 @ s1)
| . . . . . . . . . | . | .      & paren(Vertices(the_graph);s1)
| . . . . . . . . . | . | .      & paren(Vertices(the_graph);s2)
| . . . . . . . . . | . | .      & (∀j:Vertices(the_graph).
| . . . . . . . . . | . | .         ...
| . . . . . . . . . | . | .         & ((inl(j) ∈ s2)
| . . . . . . . . . | . | .            ⟺
| . . . . . . . . . | . | .            L2-the_graph->*j
| . . . . . . . . . | . | .            & ¬L1@0-the_graph->*j)))
| . . . . . . . . . | . | . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . | . | . 22. L2 : Vertices(the_graph) List
| . . . . . . . . . | . | . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . | . | . 24. Vertices(the_graph) List
| . . . . . . . . . | . | . 25. L1@0 = L1
| . . . . . . . . . | . | . 26. [i] = L2
| . . . . . . . . . | . | . 27. ∀i:Vertices(the_graph).
| . . . . . . . . . | . | .     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . | . | . 28. j : Vertices(the_graph)
| . . . . . . . . . | . | . 29. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . | . | . 30. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . | . | . 31. (inl(j) ∈ s2)
| . . . . . . . . . | . | . ⊢  ¬(inl(j) ∈ s)
| . . . . . . . . . | . | .   by AllHyps
| . . . . . . . . . | . | .      (λh.
| . . . . . . . . . | . | .       (Unfold ‘l_disjoint‘ h)
| . . . . . . . . . | . | .       THEN
| . . . . . . . . . | . | .       (InstHyp [inl(j)] h)
| . . . . . . . . . | . | .       THEN
| . . . . . . . . . | . | .       (ParallelOp -1))
| . . . . . . . . . | . |
| . . . . . . . . . | . .\
| . . . . . . . . . | . . 1. the_graph : Graph
| . . . . . . . . . | . . 2. GraphObject(the_graph)
| . . . . . . . . . | . . 3. Vertices(the_graph) List
| . . . . . . . . . | . . 4. L1 : Vertices(the_graph) List
| . . . . . . . . . | . . 5. s : traversal(the_graph)
| . . . . . . . . . | . . 6. i : Vertices(the_graph)
| . . . . . . . . . | . . 7. paren(Vertices(the_graph);s)
| . . . . . . . . . | . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . .    Vertices(the_graph);s)
| . . . . . . . . . | . . 9. df-traversal(the_graph;s)
| . . . . . . . . . | . . 10. ∀i:Vertices(the_graph).
| . . . . . . . . . | . .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . | . . 11. ¬(inl(i) ∈ s)
| . . . . . . . . . | . . 12. s2 : traversal(the_graph)
| . . . . . . . . . | . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . | . .     Vertices(the_graph);s2;s)
| . . . . . . . . . | . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . | . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . | . .     Vertices(the_graph);s2)
| . . . . . . . . . | . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . | . . 17. ∀j:Vertices(the_graph).
| . . . . . . . . . | . .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . | . . 18. df-traversal(the_graph;([inl(i)]
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| . . . . . . . . . | . .                             @ s2
| . . . . . . . . . | . .                             @ [inr(i)])
| . . . . . . . . . | . .     @ s)
| . . . . . . . . . | . . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . | . .     L1 @ [i]-the_graph->*i@0
| . . . . . . . . . | . .     ⇒
| . . . . . . . . . | . .     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . | . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . | . .     L1 = (L1@0 @ L2)
| . . . . . . . . . | . .     ⇒
| . . . . . . . . . | . .     (∃s1,s2:traversal(the_graph).
| . . . . . . . . . | . .      s = (s2 @ s1)
| . . . . . . . . . | . .      & paren(Vertices(the_graph);s1)
| . . . . . . . . . | . .      & paren(Vertices(the_graph);s2)
| . . . . . . . . . | . .      & (∀j:Vertices(the_graph).
| . . . . . . . . . | . .         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . | . .         & ((inl(j) ∈ s2)
| . . . . . . . . . | . .            ⟺
| . . . . . . . . . | . .            L2-the_graph->*j
| . . . . . . . . . | . .            & ¬L1@0-the_graph->*j)))
| . . . . . . . . . | . . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . | . . 22. L2 : Vertices(the_graph) List
| . . . . . . . . . | . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . | . . 24. Vertices(the_graph) List
| . . . . . . . . . | . . 25. L1@0 = L1
| . . . . . . . . . | . . 26. [i] = L2
| . . . . . . . . . | . . 27. ∀i:Vertices(the_graph).
| . . . . . . . . . | . .     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . | . . 28. j : Vertices(the_graph)
| . . . . . . . . . | . . 29. (inl(j) ∈ s) ⟺ L1-the_graph->*j
| . . . . . . . . . | . . 30. inl(j) = inr(i)
| . . . . . . . . . | . . ⊢  [i]-the_graph->*j & ¬(inl(j) ∈ s)
| . . . . . . . . . | . .   by ObviousContradiction [-1]
| . . . . . . . . . |
| . . . . . . . . . .\
| . . . . . . . . . . 1. the_graph : Graph
| . . . . . . . . . . 2. GraphObject(the_graph)
| . . . . . . . . . . 3. Vertices(the_graph) List
| . . . . . . . . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . . . . 5. s : traversal(the_graph)
| . . . . . . . . . . 6. i : Vertices(the_graph)
| . . . . . . . . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . |  Vertices(the_graph);s)
| . . . . . . . . . . 9. df-traversal(the_graph;s)
| . . . . . . . . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . . . . 12. s2 : traversal(the_graph)
| . . . . . . . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . . |   Vertices(the_graph);s2;s)
| . . . . . . . . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . |   Vertices(the_graph);s2)
| . . . . . . . . . . 16. ¬(inr(i) ∈ s2)
```

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| . . . . . . . . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . . 18. df-traversal(the_graph;([inl(i)]
| . . . . . . . . . . |                         @ s2
| . . . . . . . . . . |                         @ [inr(i)])
| . . . . . . . . . . |   @ s)
| . . . . . . . . . . 19. ∀i@0:Vertices(the_graph).
| . . . . . . . . . . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . . |   ⇒
| . . . . . . . . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . . 20. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . . |   L1 = (L1@0 @ L2)
| . . . . . . . . . . |   ⇒
| . . . . . . . . . . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . . |    s = (s2 @ s1)
| . . . . . . . . . . |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . . |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . . |    & ...)
| . . . . . . . . . . 21. L1@0 : Vertices(the_graph) List
| . . . . . . . . . . 22. L2 : Vertices(the_graph) List
| . . . . . . . . . . 23. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . . 24. Vertices(the_graph) List
| . . . . . . . . . . 25. L1@0 = L1
| . . . . . . . . . . 26. [i] = L2
| . . . . . . . . . . 27. ∀i:Vertices(the_graph).
| . . . . . . . . . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . . 28. j : Vertices(the_graph)
| . . . . . . . . . . 29. (inl(j) ∈ s) ⟺ L1-the_graph->*j
| . . . . . . . . . . 30. [i]-the_graph->*j & ¬(inl(j) ∈ s)
| . . . . . . . . . . ⊢  inl(j) = inl(i)
| . . . . . . . . . . |  ∨ (inl(j) ∈ s2)
| . . . . . . . . . . |  ∨ inl(j) = inr(i)
| . . . . . . . . . . | by (Unfold `df-traversal` 18) THEN (Analyze 18)
| . . . . . . . . . . |    THEN
| . . . . . . . . . . |    (InstHyp [i;nil;(s2 @ [inr(i)]) @ s;j] 19)
| . . . . . . . . . . |
| . . . . . . . . . . |\
| . . . . . . . . . . | 1. the_graph : Graph
| . . . . . . . . . . | 2. GraphObject(the_graph)
| . . . . . . . . . . | 3. Vertices(the_graph) List
| . . . . . . . . . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . . . . | 5. s : traversal(the_graph)
| . . . . . . . . . . | 6. i : Vertices(the_graph)
| . . . . . . . . . . | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . | |  Vertices(the_graph);s)
| . . . . . . . . . . | 9. df-traversal(the_graph;s)
| . . . . . . . . . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . . . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . . | 12. s2 : traversal(the_graph)
| . . . . . . . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . . | |   Vertices(the_graph);s2;s)
| . . . . . . . . . . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . . | 15. no_repeats(Vertices(the_graph)+
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| . . . . . . . . . . | |   Vertices(the_graph);s2)
| . . . . . . . . . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . . | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . | |   , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . | |   =
| . . . . . . . . . . | |   (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . . . . | |   ⇒
| . . . . . . . . . . | |   (∀j:Vertices(the_graph).
| . . . . . . . . . . | |    (inr(j) ∈ s2@0)
| . . . . . . . . . . | |    ⇒
| . . . . . . . . . . | |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . . . . | 19. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . | |   , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . | |   (∀j:Vertices(the_graph).
| . . . . . . . . . . | |    i@0-the_graph->*j
| . . . . . . . . . . | |    ⇒
| . . . . . . . . . . | |    ¬non-trivial-loop(the_graph;j))
| . . . . . . . . . . | |   ⇒
| . . . . . . . . . . | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . | |   =
| . . . . . . . . . . | |   (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . . . . | |   ⇒
| . . . . . . . . . . | |   (∀j:Vertices(the_graph).
| . . . . . . . . . . | |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . . . . | 20. ∀i@0:Vertices(the_graph).
| . . . . . . . . . . | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . . | |   ⇒
| . . . . . . . . . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . . | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . . | |   L1 = (L1@0 @ L2)
| . . . . . . . . . . | |   ⇒
| . . . . . . . . . . | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . . | |    s = (s2 @ s1)
| . . . . . . . . . . | |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . . | |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . . | |    & (∀j:Vertices(the_graph).
| . . . . . . . . . . | |       ((inl(j) ∈ s1) ⇔ L1@0-the_graph->*j)
| . . . . . . . . . . | |       & ((inl(j) ∈ s2)
| . . . . . . . . . . | |          ⇔
| . . . . . . . . . . | |          L2-the_graph->*j
| . . . . . . . . . . | |          & ¬L1@0-the_graph->*j)))
| . . . . . . . . . . | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . . . . | 23. L2 : Vertices(the_graph) List
| . . . . . . . . . . | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . . | 25. Vertices(the_graph) List
| . . . . . . . . . . | 26. L1@0 = L1
| . . . . . . . . . . | 27. [i] = L2
| . . . . . . . . . . | 28. ∀i:Vertices(the_graph).
| . . . . . . . . . . | |   (inl(i) ∈ s) ⇔ L1-the_graph->*i
| . . . . . . . . . . | 29. j : Vertices(the_graph)
| . . . . . . . . . . | 30. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . . | 31. (inl(j) ∈ s) ⇐ L1-the_graph->*j
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| . . . . . . . . . . | 32. [i]-the_graph->*j
| . . . . . . . . . . | 33. ¬(inl(j) ∈ s)
| . . . . . . . . . . | 34. j1 : Vertices(the_graph)
| . . . . . . . . . . | 35. i-the_graph->*j1
| . . . . . . . . . . | ⊢  ¬non-trivial-loop(the_graph;j1)
| . . . . . . . . . . | | by BackThruSomeHyp
| . . . . . . . . . . | |    THEN
| . . . . . . . . . . | |    (Unfold `list-connect` 0)
| . . . . . . . . . . | |    THEN
| . . . . . . . . . . | |    (Unfold `l_exists` 0)
| . . . . . . . . . . | |    THEN
| . . . . . . . . . . | |    (InstConcl [i])
| . . . . . . . . . . | |
| . . . . . . . . . . | 1. the_graph : Graph
| . . . . . . . . . . | 2. GraphObject(the_graph)
| . . . . . . . . . . | 3. Vertices(the_graph) List
| . . . . . . . . . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . . . . | 5. s : traversal(the_graph)
| . . . . . . . . . . | 6. i : Vertices(the_graph)
| . . . . . . . . . . | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . |    Vertices(the_graph);s)
| . . . . . . . . . . | 9. df-traversal(the_graph;s)
| . . . . . . . . . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . . . . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . . | 12. s2 : traversal(the_graph)
| . . . . . . . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . . |     Vertices(the_graph);s2;s)
| . . . . . . . . . . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . |     Vertices(the_graph);s2)
| . . . . . . . . . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . . | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . |     , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . |     =
| . . . . . . . . . . |     (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     (∀j:Vertices(the_graph).
| . . . . . . . . . . |      (inr(j) ∈ s2@0)
| . . . . . . . . . . |      ⇒
| . . . . . . . . . . |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . . . . | 19. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . |     , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . |     (∀j:Vertices(the_graph).
| . . . . . . . . . . |      i@0-the_graph->*j
| . . . . . . . . . . |      ⇒
| . . . . . . . . . . |      ¬non-trivial-loop(the_graph;j))
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . |     =
| . . . . . . . . . . |     (s1 @ [inl(i@0)] @ s2@0)
```

```
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     (∀j:Vertices(the_graph).
| . . . . . . . . . . |      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . . . . | 20. ∀i@0:Vertices(the_graph).
| . . . . . . . . . . |     L1 @ [i]-the_graph->*i@0
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . . | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . . |     L1 = (L1@0 @ L2)
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     (∃s1,s2:traversal(the_graph).
| . . . . . . . . . . |      s = (s2 @ s1)
| . . . . . . . . . . |      & paren(Vertices(the_graph);s1)
| . . . . . . . . . . |      & paren(Vertices(the_graph);s2)
| . . . . . . . . . . |      & (∀j:Vertices(the_graph).
| . . . . . . . . . . |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . . |         & ((inl(j) ∈ s2)
| . . . . . . . . . . |            ⟺
| . . . . . . . . . . |            L2-the_graph->*j
| . . . . . . . . . . |            & ¬L1@0-the_graph->*j)))
| . . . . . . . . . . | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . . . . | 23. L2 : Vertices(the_graph) List
| . . . . . . . . . . | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . . | 25. Vertices(the_graph) List
| . . . . . . . . . . | 26. L1@0 = L1
| . . . . . . . . . . | 27. [i] = L2
| . . . . . . . . . . | 28. ∀i:Vertices(the_graph).
| . . . . . . . . . . |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . . | 29. j : Vertices(the_graph)
| . . . . . . . . . . | 30. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . . | 31. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . . | 32. [i]-the_graph->*j
| . . . . . . . . . . | 33. ¬(inl(j) ∈ s)
| . . . . . . . . . . | 34. j1 : Vertices(the_graph)
| . . . . . . . . . . | 35. i-the_graph->*j1
| . . . . . . . . . . | ⊢  (i ∈ L1 @ [i])  by ObviousConcl
| . . . . . . . . . . |
| . . . . . . . . . . |\
| . . . . . . . . . . | 1. the_graph : Graph
| . . . . . . . . . . | 2. GraphObject(the_graph)
| . . . . . . . . . . | 3. Vertices(the_graph) List
| . . . . . . . . . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . . . . | 5. s : traversal(the_graph)
| . . . . . . . . . . | 6. i : Vertices(the_graph)
| . . . . . . . . . . | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . |    Vertices(the_graph);s)
| . . . . . . . . . . | 9. df-traversal(the_graph;s)
| . . . . . . . . . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . . . . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . . | 12. s2 : traversal(the_graph)

| . . . . . . . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . . |     Vertices(the_graph);s2;s)
| . . . . . . . . . . | 14. paren(Vertices(the_graph);s2)
```

```
| . . . . . . . . . . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . |     Vertices(the_graph);s2)
| . . . . . . . . . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . . | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . |     , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . |     =
| . . . . . . . . . . |     (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     (∀j:Vertices(the_graph).
| . . . . . . . . . . |      (inr(j) ∈ s2@0)
| . . . . . . . . . . |      ⇒
| . . . . . . . . . . |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . . . . | 19. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . |     , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . |     (∀j:Vertices(the_graph).
| . . . . . . . . . . |      i@0-the_graph->*j
| . . . . . . . . . . |      ⇒
| . . . . . . . . . . |      ¬non-trivial-loop(the_graph;j))
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . |     =
| . . . . . . . . . . |     (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     (∀j:Vertices(the_graph).
| . . . . . . . . . . |      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . . . . | 20. ∀i@0:Vertices(the_graph).
| . . . . . . . . . . |     L1 @ [i]-the_graph->*i@0
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . . | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . . |     L1 = (L1@0 @ L2)
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     (∃s1,s2:traversal(the_graph).
| . . . . . . . . . . |      s = (s2 @ s1)
| . . . . . . . . . . |      & paren(Vertices(the_graph);s1)
| . . . . . . . . . . |      & paren(Vertices(the_graph);s2)
| . . . . . . . . . . |      & (∀j:Vertices(the_graph).
| . . . . . . . . . . |         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . . |         & ((inl(j) ∈ s2)
| . . . . . . . . . . |            ⟺
| . . . . . . . . . . |            L2-the_graph->*j
| . . . . . . . . . . |            & ¬L1@0-the_graph->*j)))
| . . . . . . . . . . | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . . . . | 23. L2 : Vertices(the_graph) List
| . . . . . . . . . . | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . . | 25. Vertices(the_graph) List
| . . . . . . . . . . | 26. L1@0 = L1
| . . . . . . . . . . | 27. [i] = L2
| . . . . . . . . . . | 28. ∀i:Vertices(the_graph).
| . . . . . . . . . . |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . . | 29. j : Vertices(the_graph)
| . . . . . . . . . . | 30. (inl(j) ∈ s) ⇒ L1-the_graph->*j
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| . . . . . . . . . . | 31. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . . | 32. [i]-the_graph->*j
| . . . . . . . . . . | 33. ¬(inl(j) ∈ s)
| . . . . . . . . . . | ⊢  (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . |    =
| . . . . . . . . . . |    (nil @ [inl(i)] @ (s2 @ [inr(i)]) @ s)
| . . . . . . . . . . |   by Reduce 0
| . . . . . . . . . . |
| . . . . . . . . . . |\
| . . . . . . . . . . | 1. the_graph : Graph
| . . . . . . . . . . | 2. GraphObject(the_graph)
| . . . . . . . . . . | 3. Vertices(the_graph) List
| . . . . . . . . . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . . . . | 5. s : traversal(the_graph)
| . . . . . . . . . . | 6. i : Vertices(the_graph)
| . . . . . . . . . . | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . |    Vertices(the_graph);s)
| . . . . . . . . . . | 9. df-traversal(the_graph;s)
| . . . . . . . . . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . . . . |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . . | 12. s2 : traversal(the_graph)
| . . . . . . . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . . |     Vertices(the_graph);s2;s)
| . . . . . . . . . . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . |     Vertices(the_graph);s2)
| . . . . . . . . . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . . |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . . | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . |     , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . |     =
| . . . . . . . . . . |     (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     (∀j:Vertices(the_graph).
| . . . . . . . . . . |      (inr(j) ∈ s2@0)
| . . . . . . . . . . |      ⇒
| . . . . . . . . . . |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . . . . | 19. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . |     , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . |     (∀j:Vertices(the_graph).
| . . . . . . . . . . |      i@0-the_graph->*j
| . . . . . . . . . . |      ⇒
| . . . . . . . . . . |      ¬non-trivial-loop(the_graph;j))
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . |     =
| . . . . . . . . . . |     (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     (∀j:Vertices(the_graph).
| . . . . . . . . . . |      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . . . . | 20. ∀i@0:Vertices(the_graph).
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| . . . . . . . . . . |     L1 @ [i]-the_graph->*i@0
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . . | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . . |     L1 = (L1@0 @ L2)
| . . . . . . . . . . |     ⇒
| . . . . . . . . . . |     (∃s1,s2:traversal(the_graph).
| . . . . . . . . . . |      s = (s2 @ s1)
| . . . . . . . . . . |      & paren(Vertices(the_graph);s1)
| . . . . . . . . . . |      & paren(Vertices(the_graph);s2)
| . . . . . . . . . . |      & (∀j:Vertices(the_graph).
| . . . . . . . . . . |          ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . . |         & ((inl(j) ∈ s2)
| . . . . . . . . . . |             ⟺
| . . . . . . . . . . |             L2-the_graph->*j
| . . . . . . . . . . |             & ¬L1@0-the_graph->*j)))
| . . . . . . . . . . | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . . . . | 23. L2 : Vertices(the_graph) List
| . . . . . . . . . . | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . . | 25. Vertices(the_graph) List
| . . . . . . . . . . | 26. L1@0 = L1
| . . . . . . . . . . | 27. [i] = L2
| . . . . . . . . . . | 28. ∀i:Vertices(the_graph).
| . . . . . . . . . . |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . . | 29. j : Vertices(the_graph)
| . . . . . . . . . . | 30. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . . | 31. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . . | 32. [i]-the_graph->*j
| . . . . . . . . . . | 33. ¬(inl(j) ∈ s)
| . . . . . . . . . . | ⊢  i-the_graph->*j
| . . . . . . . . . . |   by RW0
| . . . . . . . . . . |      Thm* For any graph
| . . . . . . . . . . |           ∀i,j:V. [i]-->*j ⟺ i-the_graph->*j
| . . . . . . . . . . |      {list_DASH_connect_DASH_singleton}
| . . . . . . . . . . |      -2
| . . . . . . . . . . |
| . . . . . . . . . . .\
| . . . . . . . . . . . 1. the_graph : Graph
| . . . . . . . . . . . 2. GraphObject(the_graph)
| . . . . . . . . . . . 3. Vertices(the_graph) List
| . . . . . . . . . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . . . . . 5. s : traversal(the_graph)
| . . . . . . . . . . . 6. i : Vertices(the_graph)
| . . . . . . . . . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . . . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . . |  Vertices(the_graph);s)
| . . . . . . . . . . . 9. df-traversal(the_graph;s)
| . . . . . . . . . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . . . . . |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . . . . . 12. s2 : traversal(the_graph)
| . . . . . . . . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . . . |   Vertices(the_graph);s2;s)
| . . . . . . . . . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . . . 15. no_repeats(Vertices(the_graph)+
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| . . . . . . . . . . . |   Vertices(the_graph);s2)
| . . . . . . . . . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . . . . . |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . . . 18. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . . |   , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . . |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . |   =
| . . . . . . . . . . . |   (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . . . . . |   ⇒
| . . . . . . . . . . . |   (∀j:Vertices(the_graph).
| . . . . . . . . . . . |    (inr(j) ∈ s2@0)
| . . . . . . . . . . . |    ⇒
| . . . . . . . . . . . |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . . . . . 19. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . . |   , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . . |   (∀j:Vertices(the_graph).
| . . . . . . . . . . . |    i@0-the_graph->*j
| . . . . . . . . . . . |    ⇒
| . . . . . . . . . . . |    ¬non-trivial-loop(the_graph;j))
| . . . . . . . . . . . |   ⇒
| . . . . . . . . . . . |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . |   =
| . . . . . . . . . . . |   (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . . . . . |   ⇒
| . . . . . . . . . . . |   (∀j:Vertices(the_graph).
| . . . . . . . . . . . |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . . . . . 20. ∀i@0:Vertices(the_graph).
| . . . . . . . . . . . |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . . . |   ⇒
| . . . . . . . . . . . |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . . . 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . . . |   L1 = (L1@0 @ L2)
| . . . . . . . . . . . |   ⇒
| . . . . . . . . . . . |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . . . |    s = (s2 @ s1)
| . . . . . . . . . . . |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . . . |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . . . |    & (∀j:Vertices(the_graph).
| . . . . . . . . . . . |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . . . |       & ((inl(j) ∈ s2)
| . . . . . . . . . . . |          ⟺
| . . . . . . . . . . . |          L2-the_graph->*j
| . . . . . . . . . . . |          & ¬L1@0-the_graph->*j)))
| . . . . . . . . . . . 22. L1@0 : Vertices(the_graph) List
| . . . . . . . . . . . 23. L2 : Vertices(the_graph) List
| . . . . . . . . . . . 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . . . 25. Vertices(the_graph) List
| . . . . . . . . . . . 26. L1@0 = L1
| . . . . . . . . . . . 27. [i] = L2
| . . . . . . . . . . . 28. ∀i:Vertices(the_graph).
| . . . . . . . . . . . |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . . . 29. j : Vertices(the_graph)
| . . . . . . . . . . . 30. (inl(j) ∈ s) ⟺ L1-the_graph->*j
| . . . . . . . . . . . 31. [i]-the_graph->*j & ¬(inl(j) ∈ s)
```

```
| . . . . . . . . . . . . 32. (inr(j) ∈ (s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . . ⊢  inl(j) = inl(i)
| . . . . . . . . . . . . |  ∨ (inl(j) ∈ s2)
| . . . . . . . . . . . . |  ∨ inl(j) = inr(i)
| . . . . . . . . . . . . | by (RWW
| . . . . . . . . . . . . |     Thm* ∀x:T, l1,l2:T List.
| . . . . . . . . . . . . |          (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
| . . . . . . . . . . . . |     {member_append}
| . . . . . . . . . . . . |     -1)
| . . . . . . . . . . . . |    THEN
| . . . . . . . . . . . . |    (RWO
| . . . . . . . . . . . . |     Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a
| . . . . . . . . . . . . |     {member_singleton}
| . . . . . . . . . . . . |     -1)
| . . . . . . . . . . . . |    THEN
| . . . . . . . . . . . . |    (Analyze -1)
| . . . . . . . . . . . . |
| . . . . . . . . . . . . |\
| . . . . . . . . . . . . | 1. the_graph : Graph
| . . . . . . . . . . . . | 2. GraphObject(the_graph)
| . . . . . . . . . . . . | 3. Vertices(the_graph) List
| . . . . . . . . . . . . | 4. L1 : Vertices(the_graph) List
| . . . . . . . . . . . . | 5. s : traversal(the_graph)
| . . . . . . . . . . . . | 6. i : Vertices(the_graph)
| . . . . . . . . . . . . | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . . . . | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . . . | |  Vertices(the_graph);s)
| . . . . . . . . . . . . | 9. df-traversal(the_graph;s)
| . . . . . . . . . . . . | 10. ∀i:Vertices(the_graph).
| . . . . . . . . . . . . | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . . . . | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . . . . | 12. s2 : traversal(the_graph)
| . . . . . . . . . . . . | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . . . . | |   Vertices(the_graph);s2;s)
| . . . . . . . . . . . . | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . . . . | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . . . | |   Vertices(the_graph);s2)
| . . . . . . . . . . . . | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . . . . | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . . . . | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . . . . | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . . . | |   , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . . . | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . . | |   =
| . . . . . . . . . . . . | |   (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . . . . . . | |   ⇒
| . . . . . . . . . . . . | |   (∀j:Vertices(the_graph).
| . . . . . . . . . . . . | |    (inr(j) ∈ s2@0)
| . . . . . . . . . . . . | |    ⇒
| . . . . . . . . . . . . | |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . . . . . . | 19. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . . . | |   , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . . . | |   (∀j:Vertices(the_graph).
| . . . . . . . . . . . . | |    i@0-the_graph->*j
| . . . . . . . . . . . . | |    ⇒
```

```
| . . . . . . . . . . . | |    ¬non-trivial-loop(the_graph;j))
| . . . . . . . . . . . | |   ⇒
| . . . . . . . . . . . | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . | |   =
| . . . . . . . . . . . | |   (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . . . . . | |   ⇒
| . . . . . . . . . . . | |   (∀j:Vertices(the_graph).
| . . . . . . . . . . . | |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . . . . . | 20. ∀i@0:Vertices(the_graph).
| . . . . . . . . . . . | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . . . | |   ⇒
| . . . . . . . . . . . | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . . . | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . . . | |   L1 = (L1@0 @ L2)
| . . . . . . . . . . . | |   ⇒
| . . . . . . . . . . . | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . . . | |    s = (s2 @ s1)
| . . . . . . . . . . . | |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . . . | |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . . . | |    & (∀j:Vertices(the_graph).
| . . . . . . . . . . . | |       ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . . . | |       & ((inl(j) ∈ s2)

| . . . . . . . . . . . | |          ⟺
| . . . . . . . . . . . | |          L2-the_graph->*j
| . . . . . . . . . . . | |          & ¬L1@0-the_graph->*j)))
| . . . . . . . . . . . | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . . . . . | 23. L2 : Vertices(the_graph) List
| . . . . . . . . . . . | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . . . | 25. Vertices(the_graph) List
| . . . . . . . . . . . | 26. L1@0 = L1
| . . . . . . . . . . . | 27. [i] = L2
| . . . . . . . . . . . | 28. ∀i:Vertices(the_graph).
| . . . . . . . . . . . | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . . . | 29. j : Vertices(the_graph)
| . . . . . . . . . . . | 30. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . . . | 31. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . . . | 32. [i]-the_graph->*j
| . . . . . . . . . . . | 33. ¬(inl(j) ∈ s)
| . . . . . . . . . . . | 34. (inr(j) ∈ s2) ∨ inr(j) = inr(i)
| . . . . . . . . . . . | ⊢  inl(j) = inl(i)
| . . . . . . . . . . . | |  ∨ (inl(j) ∈ s2)
| . . . . . . . . . . . | |  ∨ inl(j) = inr(i)  by Analyze -1
| . . . . . . . . . . . | |
| . . . . . . . . . . . | |\
| . . . . . . . . . . . | | 1. the_graph : Graph
| . . . . . . . . . . . | | 2. GraphObject(the_graph)
| . . . . . . . . . . . | | 3. Vertices(the_graph) List
| . . . . . . . . . . . | | 4. L1 : Vertices(the_graph) List
| . . . . . . . . . . . | | 5. s : traversal(the_graph)
| . . . . . . . . . . . | | 6. i : Vertices(the_graph)
| . . . . . . . . . . . | | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . . . | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . . | | |  Vertices(the_graph);s)
| . . . . . . . . . . . | | 9. df-traversal(the_graph;s)
| . . . . . . . . . . . | | 10. ∀i:Vertices(the_graph).
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| . . . . . . . . . . . | | |   (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . . . | | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . . . | | 12. s2 : traversal(the_graph)
| . . . . . . . . . . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . . . | | |   Vertices(the_graph);s2;s)
| . . . . . . . . . . . | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . . . | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . . | | |   Vertices(the_graph);s2)
| . . . . . . . . . . . | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . . . | | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . . . | | |   (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . . . | | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . . | | |   , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . . | | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . | | |   =
| . . . . . . . . . . . | | |   (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . . . . . | | |   ⇒
| . . . . . . . . . . . | | |   (∀j:Vertices(the_graph).
| . . . . . . . . . . . | | |    (inr(j) ∈ s2@0)
| . . . . . . . . . . . | | |    ⇒
| . . . . . . . . . . . | | |    ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . . . . . | | 19. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . . | | |   , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . . | | |   (∀j:Vertices(the_graph).
| . . . . . . . . . . . | | |    i@0-the_graph->*j
| . . . . . . . . . . . | | |    ⇒
| . . . . . . . . . . . | | |    ¬non-trivial-loop(the_graph;j))
| . . . . . . . . . . . | | |   ⇒
| . . . . . . . . . . . | | |   (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . | | |   =
| . . . . . . . . . . . | | |   (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . . . . . | | |   ⇒
| . . . . . . . . . . . | | |   (∀j:Vertices(the_graph).
| . . . . . . . . . . . | | |    i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . . . . . | | 20. ∀i@0:Vertices(the_graph).
| . . . . . . . . . . . | | |   L1 @ [i]-the_graph->*i@0
| . . . . . . . . . . . | | |   ⇒
| . . . . . . . . . . . | | |   ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . . . | | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . . . | | |   L1 = (L1@0 @ L2)
| . . . . . . . . . . . | | |   ⇒
| . . . . . . . . . . . | | |   (∃s1,s2:traversal(the_graph).
| . . . . . . . . . . . | | |    s = (s2 @ s1)
| . . . . . . . . . . . | | |    & paren(Vertices(the_graph);s1)
| . . . . . . . . . . . | | |    & paren(Vertices(the_graph);s2)
| . . . . . . . . . . . | | |    & (∀j:Vertices(the_graph).
| . . . . . . . . . . . | | |        ...
| . . . . . . . . . . . | | |       & ((inl(j) ∈ s2)
| . . . . . . . . . . . | | |           ⟺
| . . . . . . . . . . . | | |           L2-the_graph->*j
| . . . . . . . . . . . | | |           & ¬L1@0-the_graph->*j)))
| . . . . . . . . . . . | | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . . . . . | | 23. L2 : Vertices(the_graph) List
| . . . . . . . . . . . | | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . . . | | 25. Vertices(the_graph) List
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| . . . . . . . . . . . | | 26. L1@0 = L1
| . . . . . . . . . . . | | 27. [i] = L2
| . . . . . . . . . . . | | 28. ∀i:Vertices(the_graph).
| . . . . . . . . . . . | | |   (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . . . | | 29. j : Vertices(the_graph)
| . . . . . . . . . . . | | 30. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . . . | | 31. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . . . | | 32. [i]-the_graph->*j
| . . . . . . . . . . . | | 33. ¬(inl(j) ∈ s)
| . . . . . . . . . . . | | 34. (inr(j) ∈ s2)
| . . . . . . . . . . . | | ⊢  inl(j) = inl(i)
| . . . . . . . . . . . | | |  ∨ (inl(j) ∈ s2)
| . . . . . . . . . . . | | |  ∨ inl(j) = inr(i)
| . . . . . . . . . . . | | | by FwdThru
| . . . . . . . . . . . | | |    Thm* ∀s':(T+T) List.
| . . . . . . . . . . . | | |         paren(T;s')
| . . . . . . . . . . . | | |         ⇒
| . . . . . . . . . . . | | |         (∀i:T.
| . . . . . . . . . . . | | |          (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
| . . . . . . . . . . . | | |    {paren_balance2}
| . . . . . . . . . . . | | |    [14;-1]
| . . . . . . . . . . . | | |
| . . . . . . . . . . . | | 1. the_graph : Graph
| . . . . . . . . . . . | | 2. GraphObject(the_graph)
| . . . . . . . . . . . | | 3. Vertices(the_graph) List
| . . . . . . . . . . . | | 4. L1 : Vertices(the_graph) List
| . . . . . . . . . . . | | 5. s : traversal(the_graph)
| . . . . . . . . . . . | | 6. i : Vertices(the_graph)
| . . . . . . . . . . . | | 7. paren(Vertices(the_graph);s)
| . . . . . . . . . . . | | 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . . | |    Vertices(the_graph);s)
| . . . . . . . . . . . | | 9. df-traversal(the_graph;s)
| . . . . . . . . . . . | | 10. ∀i:Vertices(the_graph).
| . . . . . . . . . . . | |     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . . . | | 11. ¬(inl(i) ∈ s)
| . . . . . . . . . . . | | 12. s2 : traversal(the_graph)
| . . . . . . . . . . . | | 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . . . | |     Vertices(the_graph);s2;s)
| . . . . . . . . . . . | | 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . . . | | 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . . | |     Vertices(the_graph);s2)
| . . . . . . . . . . . | | 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . . . | | 17. ∀j:Vertices(the_graph).
| . . . . . . . . . . . | |     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . . . | | 18. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . . | |     , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . . | |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . | |     =
| . . . . . . . . . . . | |     (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . . . . . | |     ⇒
| . . . . . . . . . . . | |     (∀j:Vertices(the_graph).
| . . . . . . . . . . . | |      (inr(j) ∈ s2@0)
| . . . . . . . . . . . | |      ⇒
| . . . . . . . . . . . | |      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . . . . . | | 19. ∀i@0:Vertices(the_graph)
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| . . . . . . . . . . . . | |     , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . . . | |     (∀j:Vertices(the_graph).
| . . . . . . . . . . . . | |      i@0-the_graph->*j
| . . . . . . . . . . . . | |      ⇒
| . . . . . . . . . . . . | |      ¬non-trivial-loop(the_graph;j))
| . . . . . . . . . . . . | |     ⇒
| . . . . . . . . . . . . | |     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . . | |     =
| . . . . . . . . . . . . | |     (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . . . . . . | |     ⇒
| . . . . . . . . . . . . | |     (∀j:Vertices(the_graph).
| . . . . . . . . . . . . | |      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . . . . . . | | 20. ∀i@0:Vertices(the_graph).
| . . . . . . . . . . . . | |     L1 @ [i]-the_graph->*i@0
| . . . . . . . . . . . . | |     ⇒
| . . . . . . . . . . . . | |     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . . . . | | 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . . . . | |     L1 = (L1@0 @ L2)
| . . . . . . . . . . . . | |     ⇒
| . . . . . . . . . . . . | |     (∃s1,s2:traversal(the_graph).
| . . . . . . . . . . . . | |      s = (s2 @ s1)
| . . . . . . . . . . . . | |      & paren(Vertices(the_graph);s1)
| . . . . . . . . . . . . | |      & paren(Vertices(the_graph);s2)
| . . . . . . . . . . . . | |      & (∀j:Vertices(the_graph).
| . . . . . . . . . . . . | |         ...
| . . . . . . . . . . . . | |         & ((inl(j) ∈ s2)
| . . . . . . . . . . . . | |            ⟺
| . . . . . . . . . . . . | |            L2-the_graph->*j
| . . . . . . . . . . . . | |            & ¬L1@0-the_graph->*j)))
| . . . . . . . . . . . . | | 22. L1@0 : Vertices(the_graph) List
| . . . . . . . . . . . . | | 23. L2 : Vertices(the_graph) List
| . . . . . . . . . . . . | | 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . . . . | | 25. Vertices(the_graph) List
| . . . . . . . . . . . . | | 26. L1@0 = L1
| . . . . . . . . . . . . | | 27. [i] = L2
| . . . . . . . . . . . . | | 28. ∀i:Vertices(the_graph).
| . . . . . . . . . . . . | |     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . . . . | | 29. j : Vertices(the_graph)
| . . . . . . . . . . . . | | 30. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . . . . | | 31. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . . . . | | 32. [i]-the_graph->*j
| . . . . . . . . . . . . | | 33. ¬(inl(j) ∈ s)
| . . . . . . . . . . . . | | 34. (inr(j) ∈ s2)
| . . . . . . . . . . . . | | 35. (inl(j) ∈ s2)
| . . . . . . . . . . . . | | ⊢  inl(j) = inl(i)
| . . . . . . . . . . . . | |    ∨ (inl(j) ∈ s2)
| . . . . . . . . . . . . | |    ∨ inl(j) = inr(i)  by ObviousFrom [-1]
| . . . . . . . . . . . . | |
| . . . . . . . . . . . . | .\
| . . . . . . . . . . . . | . 1. the_graph : Graph
| . . . . . . . . . . . . | . 2. GraphObject(the_graph)
| . . . . . . . . . . . . | . 3. Vertices(the_graph) List
| . . . . . . . . . . . . | . 4. L1 : Vertices(the_graph) List
| . . . . . . . . . . . . | . 5. s : traversal(the_graph)
| . . . . . . . . . . . . | . 6. i : Vertices(the_graph)
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| . . . . . . . . . . . . | . 7. paren(Vertices(the_graph);s)
| . . . . . . . . . . . . | . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . . . | .    Vertices(the_graph);s)
| . . . . . . . . . . . . | . 9. df-traversal(the_graph;s)
| . . . . . . . . . . . . | . 10. ∀i:Vertices(the_graph).
| . . . . . . . . . . . . | .     (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . . . . | . 11. ¬(inl(i) ∈ s)
| . . . . . . . . . . . . | . 12. s2 : traversal(the_graph)
| . . . . . . . . . . . . | . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . . . . | .     Vertices(the_graph);s2;s)
| . . . . . . . . . . . . | . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . . . . | . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . . . | .     Vertices(the_graph);s2)
| . . . . . . . . . . . . | . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . . . . | . 17. ∀j:Vertices(the_graph).
| . . . . . . . . . . . . | .     (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . . . . | . 18. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . . . | .     , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . . . | .     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . . | .     =
| . . . . . . . . . . . . | .     (s1 @ [inr(i@0)] @ s2@0)
| . . . . . . . . . . . . | .     ⇒
| . . . . . . . . . . . . | .     (∀j:Vertices(the_graph).
| . . . . . . . . . . . . | .      (inr(j) ∈ s2@0)
| . . . . . . . . . . . . | .      ⇒
| . . . . . . . . . . . . | .      ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . . . . . . | . 19. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . . . | .     , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . . . | .     (∀j:Vertices(the_graph).
| . . . . . . . . . . . . | .      i@0-the_graph->*j
| . . . . . . . . . . . . | .      ⇒
| . . . . . . . . . . . . | .      ¬non-trivial-loop(the_graph;j))
| . . . . . . . . . . . . | .     ⇒
| . . . . . . . . . . . . | .     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . . | .     =
| . . . . . . . . . . . . | .     (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . . . . . . | .     ⇒
| . . . . . . . . . . . . | .     (∀j:Vertices(the_graph).
| . . . . . . . . . . . . | .      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . . . . . . | . 20. ∀i@0:Vertices(the_graph).
| . . . . . . . . . . . . | .     L1 @ [i]-the_graph->*i@0
| . . . . . . . . . . . . | .     ⇒
| . . . . . . . . . . . . | .     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . . . . | . 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . . . . | .     L1 = (L1@0 @ L2)
| . . . . . . . . . . . . | .     ⇒
| . . . . . . . . . . . . | .     (∃s1,s2:traversal(the_graph).
| . . . . . . . . . . . . | .      s = (s2 @ s1)
| . . . . . . . . . . . . | .      & paren(Vertices(the_graph);s1)
| . . . . . . . . . . . . | .      & paren(Vertices(the_graph);s2)
| . . . . . . . . . . . . | .      & (∀j:Vertices(the_graph).
| . . . . . . . . . . . . | .         ...
| . . . . . . . . . . . . | .         & ((inl(j) ∈ s2)
| . . . . . . . . . . . . | .            ⟺
| . . . . . . . . . . . . | .            L2-the_graph->*j
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| . . . . . . . . . . . . | .           & ¬L1@0-the_graph->*j)))
| . . . . . . . . . . . . | . 22. L1@0 : Vertices(the_graph) List
| . . . . . . . . . . . . | . 23. L2 : Vertices(the_graph) List
| . . . . . . . . . . . . | . 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . . . . | . 25. Vertices(the_graph) List
| . . . . . . . . . . . . | . 26. L1@0 = L1
| . . . . . . . . . . . . | . 27. [i] = L2
| . . . . . . . . . . . . | . 28. ∀i:Vertices(the_graph).
| . . . . . . . . . . . . | .     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . . . . | . 29. j : Vertices(the_graph)
| . . . . . . . . . . . . | . 30. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . . . . | . 31. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . . . . | . 32. [i]-the_graph->*j
| . . . . . . . . . . . . | . 33. ¬(inl(j) ∈ s)
| . . . . . . . . . . . . | . 34. inr(j) = inr(i)
| . . . . . . . . . . . . | . ⊢  inl(j) = inl(i)
| . . . . . . . . . . . . | .    ∨ (inl(j) ∈ s2)
| . . . . . . . . . . . . | .    ∨ inl(j) = inr(i)  by ObviousFrom [-1]
| . . . . . . . . . . . . |
| . . . . . . . . . . . . .\
| . . . . . . . . . . . . . 1. the_graph : Graph
| . . . . . . . . . . . . . 2. GraphObject(the_graph)
| . . . . . . . . . . . . . 3. Vertices(the_graph) List
| . . . . . . . . . . . . . 4. L1 : Vertices(the_graph) List
| . . . . . . . . . . . . . 5. s : traversal(the_graph)
| . . . . . . . . . . . . . 6. i : Vertices(the_graph)
| . . . . . . . . . . . . . 7. paren(Vertices(the_graph);s)
| . . . . . . . . . . . . . 8. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . . .      Vertices(the_graph);s)
| . . . . . . . . . . . . . 9. df-traversal(the_graph;s)
| . . . . . . . . . . . . . 10. ∀i:Vertices(the_graph).
| . . . . . . . . . . . .        (inl(i) ∈ s) ⇒ L1-the_graph->*i
| . . . . . . . . . . . . . 11. ¬(inl(i) ∈ s)
| . . . . . . . . . . . . . 12. s2 : traversal(the_graph)
| . . . . . . . . . . . . . 13. l_disjoint(Vertices(the_graph)+
| . . . . . . . . . . . .        Vertices(the_graph);s2;s)
| . . . . . . . . . . . . . 14. paren(Vertices(the_graph);s2)
| . . . . . . . . . . . . . 15. no_repeats(Vertices(the_graph)+
| . . . . . . . . . . . .        Vertices(the_graph);s2)
| . . . . . . . . . . . . . 16. ¬(inr(i) ∈ s2)
| . . . . . . . . . . . . . 17. ∀j:Vertices(the_graph).
| . . . . . . . . . . . .        (inr(j) ∈ s2) ⇒ i-the_graph->*j
| . . . . . . . . . . . . . 18. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . . .        , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . . .        (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . .        =
| . . . . . . . . . . . .        (s1 @ [inr(i@0)] @ s2@0)

| . . . . . . . . . . . .        ⇒
| . . . . . . . . . . . .        (∀j:Vertices(the_graph).
| . . . . . . . . . . . .         (inr(j) ∈ s2@0)
| . . . . . . . . . . . .         ⇒
| . . . . . . . . . . . .         ¬(inl(j) ∈ s2@0) ⇒ j-the_graph->*i@0)
| . . . . . . . . . . . . . 19. ∀i@0:Vertices(the_graph)
| . . . . . . . . . . . .        , s1,s2@0:traversal(the_graph).
| . . . . . . . . . . . .        (∀j:Vertices(the_graph).
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| . . . . . . . . . . . .      i@0-the_graph->*j
| . . . . . . . . . . . .      ⇒
| . . . . . . . . . . . .      ¬non-trivial-loop(the_graph;j))
| . . . . . . . . . . . .     ⇒
| . . . . . . . . . . . .     (([inl(i)] @ s2 @ [inr(i)]) @ s)
| . . . . . . . . . . . .     =
| . . . . . . . . . . . .     (s1 @ [inl(i@0)] @ s2@0)
| . . . . . . . . . . . .     ⇒
| . . . . . . . . . . . .     (∀j:Vertices(the_graph).
| . . . . . . . . . . . .      i@0-the_graph->*j ⇒ (inr(j) ∈ s2@0))
| . . . . . . . . . . . . 20. ∀i@0:Vertices(the_graph).
| . . . . . . . . . . . .     L1 @ [i]-the_graph->*i@0
| . . . . . . . . . . . .     ⇒
| . . . . . . . . . . . .     ¬non-trivial-loop(the_graph;i@0)
| . . . . . . . . . . . . 21. ∀L1@0,L2:Vertices(the_graph) List.
| . . . . . . . . . . . .     L1 = (L1@0 @ L2)
| . . . . . . . . . . . .     ⇒
| . . . . . . . . . . . .     (∃s1,s2:traversal(the_graph).
| . . . . . . . . . . . .      s = (s2 @ s1)
| . . . . . . . . . . . .      & paren(Vertices(the_graph);s1)
| . . . . . . . . . . . .      & paren(Vertices(the_graph);s2)
| . . . . . . . . . . . .      & (∀j:Vertices(the_graph).
| . . . . . . . . . . . .         ((inl(j) ∈ s1) ⟺ L1@0-the_graph->*j)
| . . . . . . . . . . . .         & ((inl(j) ∈ s2)
| . . . . . . . . . . . .            ⟺
| . . . . . . . . . . . .            L2-the_graph->*j
| . . . . . . . . . . . .            & ¬L1@0-the_graph->*j)))
| . . . . . . . . . . . . 22. L1@0 : Vertices(the_graph) List
| . . . . . . . . . . . . 23. L2 : Vertices(the_graph) List
| . . . . . . . . . . . . 24. (L1 @ [i]) = (L1@0 @ L2)
| . . . . . . . . . . . . 25. Vertices(the_graph) List
| . . . . . . . . . . . . 26. L1@0 = L1
| . . . . . . . . . . . . 27. [i] = L2
| . . . . . . . . . . . . 28. ∀i:Vertices(the_graph).
| . . . . . . . . . . . .     (inl(i) ∈ s) ⟺ L1-the_graph->*i
| . . . . . . . . . . . . 29. j : Vertices(the_graph)
| . . . . . . . . . . . . 30. (inl(j) ∈ s) ⇒ L1-the_graph->*j
| . . . . . . . . . . . . 31. (inl(j) ∈ s) ⇐ L1-the_graph->*j
| . . . . . . . . . . . . 32. [i]-the_graph->*j
| . . . . . . . . . . . . 33. ¬(inl(j) ∈ s)
| . . . . . . . . . . . . 34. (inr(j) ∈ s)
| . . . . . . . . . . . . ⊢  inl(j) = inl(i)
| . . . . . . . . . . . .    ∨ (inl(j) ∈ s2)
| . . . . . . . . . . . .    ∨ inl(j) = inr(i)
| . . . . . . . . . . . .   by (Analyze -2)
| . . . . . . . . . . . .      THEN
| . . . . . . . . . . . .      (BackThru
| . . . . . . . . . . . .       Thm* ∀s':(T+T) List.
| . . . . . . . . . . . .            paren(T;s')
| . . . . . . . . . . . .            ⇒
| . . . . . . . . . . . .            (∀i:T.
| . . . . . . . . . . . .             (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
| . . . . . . . . . . . .       {paren_balance2})
|
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. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. L : Vertices(the_graph) List
. 4. ∀L:Vertices(the_graph) List.
.    paren(Vertices(the_graph);dfsl(the_obj;L))
.    & no_repeats(Vertices(the_graph)+
.    Vertices(the_graph);dfsl(the_obj;L))
.    & dfsl-traversal(the_graph;L;dfsl(the_obj;L))
. ⊢  paren(Vertices(the_graph);dfsl(the_obj;L))
.    & no_repeats(Vertices(the_graph)+
.    Vertices(the_graph);dfsl(the_obj;L))
.    & dfsl-traversal(the_graph;L;dfsl(the_obj;L))
.    & (∀i:Vertices(the_graph).
.       (i ∈ L)
.       ⇒
.       (inr(i) ∈ dfsl(the_obj;L)) & (inl(i) ∈ dfsl(the_obj;L)))
.   by (Fold ‘l_all‘ 0)
.      THEN
.      (Inst
.       Thm* For any graph
.            ∀the_obj:GraphObject(the_graph), L:V List.
.            (∀i∈L.(inr(i) ∈ dfsl(the_obj;L))
.            & (inl(i) ∈ dfsl(the_obj;L)))
.       {dfsl_member}
.       [the_graph;the_obj;L])
.      THEN
.      (InstHyp [L] -2)
END
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