```
dfsl_member  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), L:V List.
|  (∀i∈L.(inr(i) ∈ dfsl(the_obj;L)) & (inl(i) ∈ dfsl(the_obj;L)))
| by Auto
|    THEN
|    (Inst Thm: dfsl_DASH_induction2
|     [the_graph
|     ;the_obj
|     ;λ2L,s. (∀i∈L.(inr(i) ∈ s) & (inl(i) ∈ s))
|     ;L])
|
|\
| .....antecedent .....
| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. Vertices(the_graph) List
| ⊢  (∀i∈nil.(inr(i) ∈ nil) & (inl(i) ∈ nil))  by Obvious
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.\
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. Vertices(the_graph) List
. 4. L1 : Vertices(the_graph) List
. 5. s : traversal(the_graph)
. 6. i : Vertices(the_graph)
. 7. paren(Vertices(the_graph);s)
. 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
. 9. (∀i∈L1.(inr(i) ∈ s) & (inl(i) ∈ s))
. ⊢  (∀i@0∈L1 @ [i].(inr(i@0) ∈ dfs(the_obj;s;i))
. |  & (inl(i@0) ∈ dfs(the_obj;s;i)))
. | by (ParallelOp -1)
. |    THEN
. |    (Inst Thm: dfs_member [the_graph;the_obj;s;i])
. |    THEN
. |    ExRepDHyps
. |    THEN
. |    (HypSubst -1 0)
. |
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. Vertices(the_graph) List
. 4. L1 : Vertices(the_graph) List
. 5. s : traversal(the_graph)
. 6. i : Vertices(the_graph)
. 7. paren(Vertices(the_graph);s)
. 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
. 9. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inr(i) ∈ s)
. 10. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inl(i) ∈ s)
. 11. s' : traversal(the_graph)
. 12. (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. 15. paren(Vertices(the_graph);s')
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. 16. dfs(the_obj;s;i) = (s' @ s)
. ⊢  ∀i@0:Vertices(the_graph).
. |  (i@0 ∈ L1 @ [i]) ⇒ (inr(i@0) ∈ s' @ s) & (inl(i@0) ∈ s' @ s)
. | by ((RWO
. |      Thm* ∀x:T, l1,l2:T List. (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
. |      {member_append}
. |      0)
. |     THEN
. |     (RWO Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a    {member_singleton} 0))
. |    THEN
. |    (RepeatFor 2 (Analyze 0))
. |
. 1. the_graph : Graph
. 2. the_obj : GraphObject(the_graph)
. 3. Vertices(the_graph) List
. 4. L1 : Vertices(the_graph) List
. 5. s : traversal(the_graph)
. 6. i : Vertices(the_graph)
. 7. paren(Vertices(the_graph);s)
. 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
. 9. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inr(i) ∈ s)
. 10. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inl(i) ∈ s)
. 11. s' : traversal(the_graph)
. 12. (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. 15. paren(Vertices(the_graph);s')
. 16. dfs(the_obj;s;i) = (s' @ s)
. 17. i@0 : Vertices(the_graph)
. 18. (i@0 ∈ L1) ∨ i@0 = i
. ⊢  (inr(i@0) ∈ s') ∨ (inr(i@0) ∈ s)
. |  & (inl(i@0) ∈ s') ∨ (inl(i@0) ∈ s)  by Analyze -1
. |
. |\
. | 1. the_graph : Graph
. | 2. the_obj : GraphObject(the_graph)
. | 3. Vertices(the_graph) List
. | 4. L1 : Vertices(the_graph) List
. | 5. s : traversal(the_graph)
. | 6. i : Vertices(the_graph)
. | 7. paren(Vertices(the_graph);s)
. | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
. | 9. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inr(i) ∈ s)
. | 10. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inl(i) ∈ s)
. | 11. s' : traversal(the_graph)
. | 12. (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. | 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. | 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. | 15. paren(Vertices(the_graph);s')
. | 16. dfs(the_obj;s;i) = (s' @ s)
. | 17. i@0 : Vertices(the_graph)
. | 18. (i@0 ∈ L1)
. | ⊢  (inr(i@0) ∈ s') ∨ (inr(i@0) ∈ s)
. |    & (inl(i@0) ∈ s') ∨ (inl(i@0) ∈ s)  by Obvious
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. .\
. . 1. the_graph : Graph
. . 2. the_obj : GraphObject(the_graph)
. . 3. Vertices(the_graph) List
. . 4. L1 : Vertices(the_graph) List
. . 5. s : traversal(the_graph)
. . 6. i : Vertices(the_graph)
. . 7. paren(Vertices(the_graph);s)
. . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
. . 9. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inr(i) ∈ s)
. . 10. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inl(i) ∈ s)
. . 11. s' : traversal(the_graph)
. . 12. (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. . 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. . 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. . 15. paren(Vertices(the_graph);s')
. . 16. dfs(the_obj;s;i) = (s' @ s)
. . 17. i@0 : Vertices(the_graph)
. . 18. i@0 = i
. . ⊢  (inr(i@0) ∈ s') ∨ (inr(i@0) ∈ s)
. . |  & (inl(i@0) ∈ s') ∨ (inl(i@0) ∈ s)
. . | by (HypSubst -1 0) THEN (Thin -1)
. . |
. . 1. the_graph : Graph
. . 2. the_obj : GraphObject(the_graph)
. . 3. Vertices(the_graph) List
. . 4. L1 : Vertices(the_graph) List
. . 5. s : traversal(the_graph)
. . 6. i : Vertices(the_graph)
. . 7. paren(Vertices(the_graph);s)
. . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
. . 9. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inr(i) ∈ s)
. . 10. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inl(i) ∈ s)
. . 11. s' : traversal(the_graph)
. . 12. (inl(i) ∈ s') ∨ (inl(i) ∈ s) ∨ (inr(i) ∈ s)
. . 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. . 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. . 15. paren(Vertices(the_graph);s')
. . 16. dfs(the_obj;s;i) = (s' @ s)
. . 17. Vertices(the_graph)
. . ⊢  (inr(i) ∈ s') ∨ (inr(i) ∈ s) & (inl(i) ∈ s') ∨ (inl(i) ∈ s)
. . | by SplitOrHyps
. . |
. . |\
. . | 1. the_graph : Graph
. . | 2. the_obj : GraphObject(the_graph)
. . | 3. Vertices(the_graph) List
. . | 4. L1 : Vertices(the_graph) List
. . | 5. s : traversal(the_graph)
. . | 6. i : Vertices(the_graph)
. . | 7. paren(Vertices(the_graph);s)
. . | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
. . | 9. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inr(i) ∈ s)
. . | 10. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inl(i) ∈ s)
. . | 11. s' : traversal(the_graph)
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. . | 12. (inl(i) ∈ s')
. . | 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. . | 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. . | 15. paren(Vertices(the_graph);s')
. . | 16. dfs(the_obj;s;i) = (s' @ s)
. . | 17. Vertices(the_graph)
. . | ⊢  (inr(i) ∈ s') ∨ (inr(i) ∈ s) & (inl(i) ∈ s') ∨ (inl(i) ∈ s)
. . |   by (FwdThru
. . |       Thm* ∀s':(T+T) List.
. . |            paren(T;s') ⇒ (∀i:T. (inl(i) ∈ s') ⇒ (inr(i) ∈ s'))
. . |       {paren_balance1}
. . |       [-3;-6])
. . |      THEN
. . |      Obvious
. . |
. . |\
. . | 1. the_graph : Graph
. . | 2. the_obj : GraphObject(the_graph)
. . | 3. Vertices(the_graph) List
. . | 4. L1 : Vertices(the_graph) List
. . | 5. s : traversal(the_graph)
. . | 6. i : Vertices(the_graph)
. . | 7. paren(Vertices(the_graph);s)
. . | 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
. . | 9. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inr(i) ∈ s)
. . | 10. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inl(i) ∈ s)
. . | 11. s' : traversal(the_graph)
. . | 12. (inl(i) ∈ s)
. . | 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. . | 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. . | 15. paren(Vertices(the_graph);s')
. . | 16. dfs(the_obj;s;i) = (s' @ s)
. . | 17. Vertices(the_graph)
. . | ⊢  (inr(i) ∈ s') ∨ (inr(i) ∈ s) & (inl(i) ∈ s') ∨ (inl(i) ∈ s)
. . |   by (FwdThru
. . |       Thm* ∀s':(T+T) List.
. . |            paren(T;s') ⇒ (∀i:T. (inl(i) ∈ s') ⇒ (inr(i) ∈ s'))
. . |       {paren_balance1}
. . |       [-11;-6])
. . |      THEN
. . |      Obvious
. . |
. . .\
. . . 1. the_graph : Graph
. . . 2. the_obj : GraphObject(the_graph)
. . . 3. Vertices(the_graph) List
. . . 4. L1 : Vertices(the_graph) List
. . . 5. s : traversal(the_graph)
. . . 6. i : Vertices(the_graph)
. . . 7. paren(Vertices(the_graph);s)
. . . 8. no_repeats(Vertices(the_graph)+Vertices(the_graph);s)
. . . 9. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inr(i) ∈ s)
. . . 10. ∀i:Vertices(the_graph). (i ∈ L1) ⇒ (inl(i) ∈ s)
. . . 11. s' : traversal(the_graph)
. . . 12. (inr(i) ∈ s)
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. . . 13. l_disjoint(Vertices(the_graph)+Vertices(the_graph);s';s)
. . . 14. no_repeats(Vertices(the_graph)+Vertices(the_graph);s')
. . . 15. paren(Vertices(the_graph);s')
. . . 16. dfs(the_obj;s;i) = (s' @ s)
. . . 17. Vertices(the_graph)
. . . ⊢  (inr(i) ∈ s') ∨ (inr(i) ∈ s) & (inl(i) ∈ s') ∨ (inl(i) ∈ s)
. . .   by (FwdThru
. . .       Thm* ∀s':(T+T) List.
. . .            paren(T;s') ⇒ (∀i:T. (inr(i) ∈ s') ⇒ (inl(i) ∈ s'))
. . .       {paren_balance2}
. . .       [-11;-6])
. . .      THEN
. . .      Obvious
END
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