```
vertex-count-le  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), P,Q:(V→𝔹).
|  (∀x:V. P(x) ⇒ Q(x))
|  ⇒
|  vertex-count(the_obj;x.P(x))≤vertex-count(the_obj;x.Q(x))
| by Auto THEN (Unfold `vertex-count` 0)
|    THEN
|    (Inst
|     Thm* For any graph
|          ∀the_obj:GraphObject(the_graph), P:(V→𝔹).
|          no_repeats(V;vertex-subset(the_obj;x.P(x)))
|          & (∀x:V. (x ∈ vertex-subset(the_obj;x.P(x))) ⟺ P(x))
|     {vertex_DASH_subset_DASH_properties}
|     [the_graph;the_obj;P])
|    THEN
|    (MoveToConcl -1)
|    THEN
|    (GenConclAtAddr [1;1;2])
|    THEN
|    (Thin -1)
|    THEN
|    (Inst
|     Thm* For any graph
|          ∀the_obj:GraphObject(the_graph), P:(V→𝔹).
|          no_repeats(V;vertex-subset(the_obj;x.P(x)))
|          & (∀x:V. (x ∈ vertex-subset(the_obj;x.P(x))) ⟺ P(x))
|     {vertex_DASH_subset_DASH_properties}
|     [the_graph;the_obj;Q])
|    THEN
|    (MoveToConcl -1)
|    THEN
|    (GenConclAtAddr [1;1;2])
|    THEN
|    (Thin -1)
|    THEN
|    (BackThru
|     Thm* ∀A,B:T List.
|          no_repeats(T;A)
|          ⇒
|          (∀x:T. (x ∈ A) ⇒ (x ∈ B)) ⇒ ||A||≤||B||
|     {length_le})
|
1. the_graph : Graph
2. GraphObject(the_graph)
3. P : Vertices(the_graph)→𝔹
4. Q : Vertices(the_graph)→𝔹
5. ∀x:Vertices(the_graph). P(x) ⇒ Q(x)
6. z : Vertices(the_graph) List
7. z1 : Vertices(the_graph) List
8. no_repeats(Vertices(the_graph);z1)
9. ∀x:Vertices(the_graph). (x ∈ z1) ⟺ Q(x)
10. no_repeats(Vertices(the_graph);z)
11. ∀x:Vertices(the_graph). (x ∈ z) ⟺ P(x)
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12. x : Vertices(the_graph)
13. (x ∈ z)
⊢  (x ∈ z1)  by Obvious
END
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