```
vertex-count-less  THM

⊢  For any graph
|  ∀the_obj:GraphObject(the_graph), P,Q:(V⟶𝔹).
|  (∀x:V. P(x) ⇒ Q(x))
|  ⇒
|  (∃x:V. ¬P(x) & Q(x))
|  ⇒
|  vertex-count(the_obj;x.P(x))<vertex-count(the_obj;x.Q(x))
| by Auto THEN (Unfold ‘vertex-count‘ 0)
|    THEN
|    (Inst
|     Thm* For any graph
|          ∀the_obj:GraphObject(the_graph), P:(V⟶𝔹).
|          no_repeats(V;vertex-subset(the_obj;x.P(x)))
|          & (∀x:V. (x ∈ vertex-subset(the_obj;x.P(x))) ⟺ P(x))
|     {vertex_DASH_subset_DASH_properties}
|     [the_graph;the_obj;P])
|    THEN
|    (MoveToConcl -1)
|    THEN
|    (GenConclAtAddr [1;1;2])
|    THEN
|    (Thin -1)
|    THEN
|    (Inst
|     Thm* For any graph
|          ∀the_obj:GraphObject(the_graph), P:(V⟶𝔹).
|          no_repeats(V;vertex-subset(the_obj;x.P(x)))
|          & (∀x:V. (x ∈ vertex-subset(the_obj;x.P(x))) ⟺ P(x))
|     {vertex_DASH_subset_DASH_properties}
|     [the_graph;the_obj;Q])
|    THEN
|    (MoveToConcl -1)
|    THEN
|    (GenConclAtAddr [1;1;2])
|    THEN
|    (Thin -1)
|    THEN
|    (BackThru
|     Thm* ∀A,B:T List.
|          no_repeats(T;A)
|          ⇒
|          (∀x:T. (x ∈ A) ⇒ (x ∈ B))
|          ⇒
|          (∃x:T. ¬(x ∈ A) & (x ∈ B)) ⇒ ||A||<||B||
|     {length_less})
|
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| 1. the_graph : Graph
| 2. GraphObject(the_graph)
| 3. P : Vertices(the_graph)⟶𝔹
| 4. Q : Vertices(the_graph)⟶𝔹
| 5. ∀x:Vertices(the_graph). P(x) ⇒ Q(x)
| 6. ∃x:Vertices(the_graph). ¬P(x) & Q(x)
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| 7. z : Vertices(the_graph) List
| 8. z1 : Vertices(the_graph) List
| 9. no_repeats(Vertices(the_graph);z1)
| 10. ∀x:Vertices(the_graph). (x ∈ z1) ⟺ Q(x)
| 11. no_repeats(Vertices(the_graph);z)
| 12. ∀x:Vertices(the_graph). (x ∈ z) ⟺ P(x)
| 13. x : Vertices(the_graph)
| 14. (x ∈ z)
| ⊢  (x ∈ z1)  by Obvious
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. 1. the_graph : Graph
. 2. GraphObject(the_graph)
. 3. P : Vertices(the_graph)→𝔹
. 4. Q : Vertices(the_graph)→𝔹
. 5. ∀x:Vertices(the_graph). P(x) ⇒ Q(x)
. 6. ∃x:Vertices(the_graph). ¬P(x) & Q(x)
. 7. z : Vertices(the_graph) List
. 8. z1 : Vertices(the_graph) List
. 9. no_repeats(Vertices(the_graph);z1)
. 10. ∀x:Vertices(the_graph). (x ∈ z1) ⟺ Q(x)
. 11. no_repeats(Vertices(the_graph);z)
. 12. ∀x:Vertices(the_graph). (x ∈ z) ⟺ P(x)
. ⊢  ∃x:Vertices(the_graph). ¬(x ∈ z) & (x ∈ z1)
.   by (ParallelOp 6) THEN (ParallelOp 7) THEN EasyHyp
.      THEN
.      (ParallelOp 7)
.      THEN
.      EasyHyp
END
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