```
iso_pair_char  THM

⊢  ∀'a,'b:S, P:('b→𝔹), rep:('a→'b), abs:('b→'a).
|  iso_pair('a;'b;P;rep;abs)
|  ⇒
|  (∀a:'a. abs(rep(a)) = a) & (∀r:'b. P(r) = ((rep(abs(r))) =_b r))
| by Unab [‘guard‘]
|
⊢  ∀'a,'b:S, P:('b→𝔹), rep:('a→'b), abs:('b→'a).
|  iso_pair('a;'b;P;rep;abs)
|  ⇒
|  (∀a:'a. abs(rep(a)) = a) & (∀r:'b. P(r) = ((rep(abs(r))) =_b r))
| by Id THEN StrongAuto THEN Unab [‘iso_pair‘] THEN StrongAuto
|
|\
| 1. 'a : S
| 2. 'b : S
| 3. P : 'b→𝔹
| 4. rep : 'a→'b
| 5. abs : 'b→'a
| 6. ∀r:'b. abs(r) = (@a:'a. (r = rep(a)))
| 7. type_definition('b;'a;P;rep)
| 8. a : 'a
| ⊢  abs(rep(a)) = a  by WithSomeHypSimps [6] [] Simp THEN StrongAuto
| |
| 1. 'a : S
| 2. 'b : S
| 3. P : 'b→𝔹
| 4. rep : 'a→'b
| 5. abs : 'b→'a
| 6. ∀r:'b. abs(r) = (@a:'a. (r = rep(a)))
| 7. type_definition('b;'a;P;rep)
| 8. a : 'a
| ⊢  (@a1:'a. (rep(a) = rep(a1))) = a
| | by L
| |    Thm* ∀P:('a→Type), x:'a.
| |         P(x) ⇒ (∀y:'a. P(y) ⇒ x = y) ⇒ (@y:'a. P(y)) = x
| |    {choose_unique}
| |    THEN
| |    StrongAuto
| |
| 1. 'a : S
| 2. 'b : S
| 3. P : 'b→𝔹
| 4. rep : 'a→'b
| 5. abs : 'b→'a
| 6. ∀r:'b. abs(r) = (@a:'a. (r = rep(a)))
| 7. type_definition('b;'a;P;rep)
| 8. a : 'a
| 9. a1 : 'a
| 10. rep(a) = rep(a1)
| ⊢  a = a1  by Unab [‘type_definition‘] THEN StrongAuto
| |
| 1. 'a : S
| 2. 'b : S
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| 3. P : 'b→𝔹
| 4. rep : 'a→'b
| 5. abs : 'b→'a
| 6. ∀r:'b. abs(r) = (@a:'a. (r = rep(a)))
| 7. ∀x',x'':'a. rep(x') = rep(x'') ⇒ x' = x''
| 8. ∀x:'b. P(x) ⟺ (∃x':'a. x = rep(x'))
| 9. a : 'a
| 10. a1 : 'a
| 11. rep(a) = rep(a1)
| ⊢  a = a1  by BackThru 7 THEN StrongAuto
|
.\
. 1. 'a : S
. 2. 'b : S
. 3. P : 'b→𝔹
. 4. rep : 'a→'b
. 5. abs : 'b→'a
. 6. ∀r:'b. abs(r) = (@a:'a. (r = rep(a)))
. 7. type_definition('b;'a;P;rep)
. 8. r : 'b
. ⊢  P(r) = ((rep(abs(r))) =b r)
. | by Unab [‘type_definition‘] THEN Simpsetp [‘bequal‘]
. |    THEN
. |    StrongAuto
. |
. |\
. | 1. 'a : S
. | 2. 'b : S
. | 3. P : 'b→𝔹
. | 4. rep : 'a→'b
. | 5. abs : 'b→'a
. | 6. ∀r:'b. abs(r) = (@a:'a. (r = rep(a)))
. | 7. ∀x',x'':'a. rep(x') = rep(x'') ⇒ x' = x''
. | 8. ∀x:'b. P(x) ⟺ (∃x':'a. x = rep(x'))
. | 9. r : 'b
. | 10. P(r)
. | ⊢  rep(abs(r)) = r  by WithSomeHypSimps [6] [] Simp
. | |
. | 1. 'a : S
. | 2. 'b : S
. | 3. P : 'b→𝔹
. | 4. rep : 'a→'b
. | 5. abs : 'b→'a
. | 6. ∀r:'b. abs(r) = (@a:'a. (r = rep(a)))
. | 7. ∀x',x'':'a. rep(x') = rep(x'') ⇒ x' = x''
. | 8. ∀x:'b. P(x) ⟺ (∃x':'a. x = rep(x'))
. | 9. r : 'b
. | 10. P(r)
. | ⊢  rep(@a:'a. (r = rep(a))) = r  by ChooseDC
. | |
. | 1. 'a : S
. | 2. 'b : S
. | 3. P : 'b→𝔹
. | 4. rep : 'a→'b
. | 5. abs : 'b→'a
```

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. | 6. ∀r:'b. abs(r) = (@a:'a. (r = rep(a)))
. | 7. ∀x',x'':'a. rep(x') = rep(x'') ⇒ x' = x''
. | 8. ∀x:'b. P(x) ⟺ (∃x':'a. x = rep(x'))
. | 9. r : 'b
. | 10. P(r)
. | ⊢  ∃a:'a. r = rep(a)  by InstHyp [r] 8
. | |
. | 1. 'a : S
. | 2. 'b : S
. | 3. P : 'b→𝔹
. | 4. rep : 'a→'b
. | 5. abs : 'b→'a
. | 6. ∀r:'b. abs(r) = (@a:'a. (r = rep(a)))
. | 7. ∀x',x'':'a. rep(x') = rep(x'') ⇒ x' = x''
. | 8. ∀x:'b. P(x) ⟺ (∃x':'a. x = rep(x'))
. | 9. r : 'b
. | 10. P(r)
. | 11. P(r) ⇒ (∃x':'a. r = rep(x'))
. | 12. (P(r)) ⇐ (∃x':'a. r = rep(x'))
. | ⊢  ∃a:'a. r = rep(a)  by Analyze 11
. |
. .\
. . 1. 'a : S
. . 2. 'b : S
. . 3. P : 'b→𝔹
. . 4. rep : 'a→'b
. . 5. abs : 'b→'a
. . 6. ∀r:'b. abs(r) = (@a:'a. (r = rep(a)))
. . 7. ∀x',x'':'a. rep(x') = rep(x'') ⇒ x' = x''
. . 8. ∀x:'b. P(x) ⟺ (∃x':'a. x = rep(x'))
. . 9. r : 'b
. . 10. rep(abs(r)) = r
. . ⊢  P(r)  by InstHyp [r] 8 THEN StrongAuto
. . |
. . 1. 'a : S
. . 2. 'b : S
. . 3. P : 'b→𝔹
. . 4. rep : 'a→'b
. . 5. abs : 'b→'a
. . 6. ∀r:'b. abs(r) = (@a:'a. (r = rep(a)))
. . 7. ∀x',x'':'a. rep(x') = rep(x'') ⇒ x' = x''
. . 8. ∀x:'b. P(x) ⟺ (∃x':'a. x = rep(x'))
. . 9. r : 'b
. . 10. rep(abs(r)) = r
. . 11. P(r) ⇒ (∃x':'a. r = rep(x'))
. . 12. (P(r)) ⇐ (∃x':'a. r = rep(x'))
. . ⊢  P(r)  by Analyze 12 THEN StrongAuto
. . |
. . .....antecedent .....
. . 1. 'a : S
. . 2. 'b : S
. . 3. P : 'b→𝔹
. . 4. rep : 'a→'b
. . 5. abs : 'b→'a
. . 6. ∀r:'b. abs(r) = (@a:'a. (r = rep(a)))
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. . 7. ∀x',x'':'a. rep(x') = rep(x'') ⇒ x' = x''
. . 8. ∀x:'b. P(x) ⟺ (∃x':'a. x = rep(x'))
. . 9. r : 'b
. . 10. rep(abs(r)) = r
. . 11. P(r) ⇒ (∃x':'a. r = rep(x'))
. . ⊢  ∃x':'a. r = rep(x')  by DTerm (abs(r)) 0 THEN StrongAuto
END
```