```
hel_nuprl  THM

⊢  ∀'a:S, n:ℕ, l:'a List. n<||l|| ⇒ el(n,l) = l[n]
| by RepeatMFor 3 (Try (Analyze 0)) THEN CAuto
|
1. 'a : S
2. n : ℕ
3. l : 'a List
4. n<||l||
⊢  el(n,l) = l[n]
| by MoveToConcl 2 THEN Last ListInd THEN Simp THEN StrongAuto
|
1. 'a : S
2. 'a List
3. u : 'a
4. v : 'a List
5. ∀n:ℕ. n<||v|| ⇒ el(n,v) = v[n]
6. n : ℕ
7. n<||v||+1
⊢  el(n,cons(u; v)) = cons(u; v)[n]
| by RecUnfold `hel` 0 THEN Simp THEN StrongAuto
|    THEN
|    (BifCase THEN HN)
|    THEN
|    Simp
|    THEN
|    StrongAuto
|
|\
| 1. 'a : S
| 2. 'a List
| 3. 'a
| 4. v : 'a List
| 5. ∀n:ℕ. n<||v|| ⇒ el(n,v) = v[n]
| 6. n : ℕ
| 7. n<||v||+1
| 8. n = 0
| ⊢  ¬False  by Analyze 0
|
|\
| 1. 'a : S
| 2. 'a List
| 3. u : 'a
| 4. v : 'a List
| 5. ∀n:ℕ. n<||v|| ⇒ el(n,v) = v[n]
| 6. n : ℕ
| 7. n<||v||+1
| 8. (n=₂0) = true₂
| ⊢  n<||cons(u; v)||  by RW HNC -1
| |
| 1. 'a : S
| 2. 'a List
| 3. u : 'a
| 4. v : 'a List
| 5. ∀n:ℕ. n<||v|| ⇒ el(n,v) = v[n]
```

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| 6. n : ℕ
| 7. n<||v||+1
| 8. n = 0
| ⊢  n<||cons(u; v)||  by Reduce 0
|
|\
| 1. 'a : S
| 2. 'a List
| 3. u : 'a
| 4. v : 'a List
| 5. ∀n:ℕ. n<||v|| ⇒ el(n,v) = v[n]
| 6. n : ℕ
| 7. n<||v||+1
| 8. n = 0
| ⊢  u = cons(u; v)[n]
|   by LemmaRW
|      Thm* ∀a:T, as:T List, i:ℤ. i≤0 ⇒ cons(a; as)[i] = a
|      {select_cons_hd}
|      THEN
|      StrongAuto
|      THEN
|      Try (Complete (Unfold `label` 0))
|
.\
. 1. 'a : S
. 2. 'a List
. 3. u : 'a
. 4. v : 'a List
. 5. ∀n:ℕ. n<||v|| ⇒ el(n,v) = v[n]
. 6. n : ℕ
. 7. n<||v||+1
. 8. ¬n = 0
. ⊢  el(n-1,v) = cons(u; v)[n]
. | by LemmaRW
. |    Thm* ∀a:T, as:T List, i:ℤ.
. |         0<i ⇒ i≤||as|| ⇒ cons(a; as)[i] = as[(i-1)]
. |    {select_cons_tl}
. |    THEN
. |    StrongAuto
. |    THEN
. |    Try (Complete (Unfold `label` 0))
. |
. |\
. | .....wf .....
. | 1. 'a : S
. | 2. 'a List
. | 3. 'a

. | 4. v : 'a List
. | 5. ∀n:ℕ. n<||v|| ⇒ el(n,v) = v[n]
. | 6. n : ℕ
. | 7. n<||v||+1
. | 8. ¬n = 0
. | ⊢  el(n-1,v) = el(n-1,v)  by InstHyp [n-1] 5
. |
. .\
```

```
. . 1. 'a : S
. . 2. 'a List
. . 3. 'a
. . 4. v : 'a List
. . 5. ∀n:ℕ. n<||v|| ⇒ el(n,v) = v[n]
. . 6. n : ℕ
. . 7. n<||v||+1
. . 8. ¬n = 0
. . ⊢  el(n-1,v) = v[(n-1)]
. .   by BackThru 5 THEN StrongAuto
. .      THEN
. .      Try (Complete (Unfold `label` 0))
END
```