list_iso THM

```
⊢  ∀'a:S. iso_pair('a List;(ℕ→'a)×ℕ;is_list_rep;rep_list;abs_list)
| by Unab [`iso_pair`] THEN Simp THEN StrongAuto
|
|\
| 1. 'a : S
| 2. r : (ℕ→'a)×ℕ
| ⊢  abs_list(r) = (@a:'a List. (r = rep_list(a)))
|   by Unab [`habs_list`] THEN HN THEN Simp THEN StrongAuto
|
.\
. 1. 'a : S
. ⊢  type_definition((ℕ→'a)×ℕ;'a List;is_list_rep;rep_list)
. | by Unab [`type_definition`] THEN HN THEN StrongAuto
. |    THEN
. |    Try (Complete (Unfold `label` 0))
. |
. |\
. | 1. 'a : S
. | 2. x' : 'a List
. | 3. x'' : 'a List
. | 4. rep_list('a;x') = rep_list('a;x'')
. | ⊢  x' = x''  by Unab [`rep_list`] THEN Simp THEN StrongAuto
. | |
. | 1. 'a : S
. | 2. x' : 'a List
. | 3. x'' : 'a List
. | 4. <λn:ℕ. if n<₂||x'|| then x'[n] else arb('a) fi ,||x'||>
. | |  =
. | |  <λn:ℕ. if n<₂||x''|| then x''[n] else arb('a) fi ,||x''||>
. | ⊢  x' = x''  by Last (PrimEqHD []) THEN Simp THEN StrongAuto
. | |
. | 1. 'a : S
. | 2. x' : 'a List
. | 3. x'' : 'a List
. | 4. (λn:ℕ. if n<₂||x'|| then x'[n] else arb('a) fi )
. | |  =
. | |  (λn:ℕ. if n<₂||x''|| then x''[n] else arb('a) fi )
. | 5. ||x'|| = ||x''||
. | ⊢  x' = x''
. | | by Assert
. | |    ((λi:ℕ||x'||. x'[i]){ℕ||x'||}
. | |     =
. | |     (λi:ℕ||x''||. x''[i]){ℕ||x''||})
. | |    THEN
. | |    StrongAuto
. | |    THEN
. | |    Try (Complete (Unfold `label` 0))
. | |
. | |\
. | | .....assertion .....
. | | 1. 'a : S
. | | 2. x' : 'a List
. | | 3. x'' : 'a List
```

```
. | | 4. (λn:ℕ. if n<₂||x'|| then x'[n] else arb('a) fi )
. | | |  =
. | | |  (λn:ℕ. if n<₂||x''|| then x''[n] else arb('a) fi )
. | | 5. ||x'|| = ||x''||
. | | ⊢  (λi:ℕ||x'||. x'[i]){ℕ||x'||}
. | | |  =
. | | |  (λi:ℕ||x''||. x''[i]){ℕ||x''||}
. | | | by Analyze THEN StrongAuto
. | | |    THEN
. | | |    Try (Complete (Unfold `label` 0))
. | | |
. | | .....subterm .....
. | | 1. 'a : S
. | | 2. x' : 'a List
. | | 3. x'' : 'a List
. | | 4. (λn:ℕ. if n<₂||x'|| then x'[n] else arb('a) fi )
. | | |  =
. | | |  (λn:ℕ. if n<₂||x''|| then x''[n] else arb('a) fi )
. | | 5. ||x'|| = ||x''||
. | | ⊢  (λi:ℕ||x'||. x'[i]) = (λi:ℕ||x''||. x''[i])
. | | | by Analyze THEN StrongAuto
. | | |    THEN
. | | |    Try (Complete (Unfold `label` 0))
. | | |
. | | .....subterm .....
. | | 1. 'a : S
. | | 2. x' : 'a List
. | | 3. x'' : 'a List
. | | 4. (λn:ℕ. if n<₂||x'|| then x'[n] else arb('a) fi )
. | | |  =
. | | |  (λn:ℕ. if n<₂||x''|| then x''[n] else arb('a) fi )
. | | 5. ||x'|| = ||x''||
. | | 6. i : ℕ||x'||
. | | ⊢  x'[i] = x''[i]
. | | | by With i (PrimEqHD [] 4) THEN Simp THEN StrongAuto
. | | |
. | | 1. 'a : S
. | | 2. x' : 'a List
. | | 3. x'' : 'a List
. | | 4. ||x'|| = ||x''||
. | | 5. i : ℕ||x'||
. | | 6. if i<₂||x'|| then x'[i] else arb('a) fi
. | | |  =
. | | |  if i<₂||x''|| then x''[i] else arb('a) fi
. | | ⊢  x'[i] = x''[i]  by H 4 6 THEN Simp THEN StrongAuto
. | | |
. | | 1. 'a : S
. | | 2. x' : 'a List
. | | 3. x'' : 'a List
. | | 4. ||x'|| = ||x''||
. | | 5. i : ℕ||x'||
. | | 6. if i<₂||x''|| then x'[i] else arb('a) fi
. | |    =
. | |    if i<₂||x''|| then x''[i] else arb('a) fi
. | | ⊢  x'[i] = x''[i]
```

```
. | |   by MoveToConcl 6 THEN BifCase THEN Simp THEN StrongAuto
. | |
. | .\
. | . 1. 'a : S
. | . 2. x' : 'a List
. | . 3. x'' : 'a List
. | . 4. (λn:ℕ. if n<2||x'|| then x'[n] else arb('a) fi )
. | .    =
. | .    (λn:ℕ. if n<2||x''|| then x''[n] else arb('a) fi )
. | . 5. ||x'|| = ||x''||
. | . 6. (λi:ℕ||x'||. x'[i]){ℕ||x'||}
. | .    =
. | .    (λi:ℕ||x''||. x''[i]){ℕ||x''||}
. | . ⊢  x' = x''
. | .   by LemmaRW
. | .      Thm* ∀as:T List. (λi:ℕ||as||. as[i]){ℕ||as||} = as
. | .      {listify_select_id}
. | .      THEN
. | .      StrongAuto
. | .      THEN
. | .      Try (Complete (Unfold `label` 0))
. |
. |\
. | 1. 'a : S
. | 2. x : (ℕ→'a)×ℕ
. | 3. is_list_rep(x)
. | ⊢  ∃x':'a List. x = rep_list('a;x')
. | | by Unab [`his_list_rep`] THEN Simp THEN StrongAuto
. | |
. | 1. 'a : S
. | 2. x : (ℕ→'a)×ℕ
. | 3. ∃f:(ℕ→'a), n:ℕ.
. | |  x = <λm:ℕ. if m<2n then f(m) else @bx:'a. true2 fi ,n>
. | ⊢  ∃x':'a List. x = rep_list('a;x')
. | | by Last Analyze THEN Last Analyze THEN StrongAuto
. | |    THEN
. | |    Try (Complete (Unfold `label` 0))
. | |
. | 1. 'a : S
. | 2. x : (ℕ→'a)×ℕ
. | 3. f : ℕ→'a
. | 4. n : ℕ
. | 5. x = <λm:ℕ. if m<2n then f(m) else @bx:'a. true2 fi ,n>
. | ⊢  ∃x':'a List. x = rep_list('a;x')
. | | by Subst
. | |    (x = <λm:ℕ. if m<2n then f(m) else @bx:'a. true2 fi ,n>)
. | |    0
. | |
. | 1. 'a : S
. | 2. x : (ℕ→'a)×ℕ
. | 3. f : ℕ→'a
. | 4. n : ℕ
. | 5. x = <λm:ℕ. if m<2n then f(m) else @bx:'a. true2 fi ,n>
. | ⊢  ∃x':'a List.
. | |  <λm:ℕ. if m<2n then f(m) else @bx:'a. true2 fi ,n>
```

```
. | |  =
. | |  rep_list('a;x')  by DTerm (f{ℕn}) 0 THEN Simp THEN StrongAuto
. | |
. | |\
. | | .....subterm .....
. | | 1. 'a : S
. | | 2. x : (ℕ→'a)×ℕ
. | | 3. f : ℕ→'a
. | | 4. n : ℕ
. | | 5. x = <λm:ℕ. if m<₂n then f(m) else @_bx:'a. true₂ fi ,n>
. | | ⊢  f ∈ ℕn→'a  by Id
. | | |
. | | .....subterm .....
. | | 1. 'a : S
. | | 2. x : (ℕ→'a)×ℕ
. | | 3. f : ℕ→'a
. | | 4. n : ℕ
. | | 5. x = <λm:ℕ. if m<₂n then f(m) else @_bx:'a. true₂ fi ,n>
. | | ⊢  f ∈ ℕn→'a  by Thin 5 THEN Thin 2
. | | |
. | | 1. 'a : S
. | | 2. f : ℕ→'a
. | | 3. n : ℕ
. | | ⊢  f ∈ ℕn→'a  by ExtWith [`f'] [ℕ→'a]
. | |
. | .\
. | . 1. 'a : S
. | . 2. x : (ℕ→'a)×ℕ
. | . 3. f : ℕ→'a
. | . 4. n : ℕ
. | . 5. x = <λm:ℕ. if m<₂n then f(m) else @_bx:'a. true₂ fi ,n>
. | . ⊢  <λm:ℕ. if m<₂n then f(m) else @_bx:'a. true₂ fi ,n>
. | . |  =
. | . |  rep_list('a;f{ℕn})  by Unab [`rep_list`]
. | . |
. | . 1. 'a : S
. | . 2. x : (ℕ→'a)×ℕ
. | . 3. f : ℕ→'a
. | . 4. n : ℕ
. | . 5. x = <λm:ℕ. if m<₂n then f(m) else @_bx:'a. true₂ fi ,n>
. | . ⊢  <λm:ℕ. if m<₂n then f(m) else @_bx:'a. true₂ fi ,n>
. | . |  =
. | . |  <λn@0:ℕ. if n@0<₂||f{ℕn}|| then (f{ℕn})[n@0] else arb('a) fi
. | . |  ,||f{ℕn}||>
. | . | by Inst
. | . |    Thm* ∀m,n:ℤ, f:({m..n⁻}→T). n<m ∨ ||f{m..n⁻}|| = n-m ∈ ℤ
. | . |    {listify_length}
. | . |    ['a;0;n;f]
. | . |    THEN
. | . |    StrongAuto
. | . |    THEN
. | . |    Try (Complete (Unfold `label` 0))
. | . |    THEN
. | . |    Last Analyze
. | . |    THEN
```

```
. | . |    StrongAuto
. | . |    THEN
. | . |    Try (Complete (Unfold `label` 0))
. | . |
. | . |\
. | . | 1. 'a : S
. | . | 2. x : (ℕ→'a)×ℕ
. | . | 3. f : ℕ→'a
. | . | 4. n : ℕ
. | . | 5. x = <λm:ℕ. if m<₂n then f(m) else @_b x:'a. true₂ fi ,n>
. | . | ⊢  f ∈ ℕn→'a  by ExtWith [`f'] [ℕ→'a]
. | . |
. | . .\
. | . . 1. 'a : S
. | . . 2. x : (ℕ→'a)×ℕ
. | . . 3. f : ℕ→'a
. | . . 4. n : ℕ
. | . . 5. x = <λm:ℕ. if m<₂n then f(m) else @_b x:'a. true₂ fi ,n>
. | . . 6. ||f{ℕn}|| = n-0
. | . . ⊢  <λm:ℕ. if m<₂n then f(m) else @_b x:'a. true₂ fi ,n>
. | . . |  =
. | . . |  <λn@0:ℕ. if n@0<₂||f{ℕn}||
. | . . |           then (f{ℕn})[n@0]
. | . . |           else arb('a)
. | . . |           fi
. | . . |  ,||f{ℕn}||>
. | . . | by Analyze THEN StrongAuto
. | . . |    THEN
. | . . |    Try (Complete (Unfold `label` 0))
. | . . |
. | . . .....subterm .....
. | . . 1. 'a : S
. | . . 2. x : (ℕ→'a)×ℕ
. | . . 3. f : ℕ→'a
. | . . 4. n : ℕ
. | . . 5. x = <λm:ℕ. if m<₂n then f(m) else @_b x:'a. true₂ fi ,n>
. | . . 6. ||f{ℕn}|| = n-0
. | . . ⊢  (λm:ℕ. if m<₂n then f(m) else @_b x:'a. true₂ fi )
. | . . |  =
. | . . |  (λn@0:ℕ. if n@0<₂||f{ℕn}||
. | . . |           then (f{ℕn})[n@0]
. | . . |           else arb('a)
. | . . |           fi )
. | . . | by Analyze THEN StrongAuto
. | . . |    THEN
. | . . |    Try (Complete (Unfold `label` 0))
. | . . |
. | . . .....subterm .....
. | . . 1. 'a : S
. | . . 2. x : (ℕ→'a)×ℕ
. | . . 3. f : ℕ→'a
. | . . 4. n : ℕ
. | . . 5. x = <λm:ℕ. if m<₂n then f(m) else @_b x:'a. true₂ fi ,n>
. | . . 6. ||f{ℕn}|| = n-0
. | . . 7. m : ℕ
```

```
. | . . ⊢  if m<₂n then f(m) else @_bx:'a. true₂ fi
. | . . |  =
. | . . |  if m<₂||f{ℕn}|| then (f{ℕn})[m] else arb('a) fi
. | . . | by BifCase THEN Simp THEN StrongAuto THEN BifCase THEN Simp
. | . . |    THEN
. | . . |    StrongAuto
. | . . |
. | . . |\
. | . . | 1. 'a : S
. | . . | 2. x : (ℕ→'a)×ℕ
. | . . | 3. f : ℕ→'a
. | . . | 4. n : ℕ
. | . . | 5. x = <λm:ℕ. if m<₂n then f(m) else @_bx:'a. true₂ fi ,n>
. | . . | 6. ||f{ℕn}|| = n-0
. | . . | 7. m : ℕ
. | . . | 8. m<n
. | . . | 9. m<||f{ℕn}||
. | . . | ⊢  f(m) = (f{ℕn})[m]
. | . . | | by LemmaRW
. | . . | |    Thm* ∀n:ℕ, f:(ℕn→T), i:ℕn. (f{ℕn})[i] = f(i)
. | . . | |    {select_listify_id}
. | . . | |    THEN
. | . . | |    StrongAuto
. | . . | |    THEN
. | . . | |    Try (Complete (Unfold `label` 0))
. | . . | |
. | . . | .....wf .....

. | . . | 1. 'a : S
. | . . | 2. x : (ℕ→'a)×ℕ
. | . . | 3. f : ℕ→'a
. | . . | 4. n : ℕ
. | . . | 5. x = <λm:ℕ. if m<₂n then f(m) else @_bx:'a. true₂ fi ,n>
. | . . | 6. ||f{ℕn}|| = n-0
. | . . | 7. m : ℕ
. | . . | 8. m<n
. | . . | 9. m<||f{ℕn}||
. | . . | ⊢  f ∈ ℕn→'a  by ExtWith ['f'] [ℕ→'a]
. | . . |
. | . . .\
. | . . . 1. 'a : S
. | . . . 2. x : (ℕ→'a)×ℕ
. | . . . 3. f : ℕ→'a
. | . . . 4. n : ℕ
. | . . . 5. x = <λm:ℕ. if m<₂n then f(m) else @_bx:'a. true₂ fi ,n>
. | . . . 6. ||f{ℕn}|| = n-0
. | . . . 7. m : ℕ
. | . . . 8. ¬m<n
. | . . . 9. ¬m<||f{ℕn}||
. | . . . ⊢  (@_bx:'a. true₂) = arb('a)
. | . . . | by Inst
. | . . . |    Thm* ∀'a:S. equal(arb,select(λx:'a. t))    {harb_wd}
. | . . . |    []
. | . . . |    THEN
. | . . . |    HN
. | . . . |    THEN
```

```
. | . . . |    StrongAuto
. | . . . |    THEN
. | . . . |    Try (Complete (Unfold ‘label‘ 0))
. | . . . |
. | . . . 1. ’a : S
. | . . . 2. x : (ℕ→’a)×ℕ
. | . . . 3. f : ℕ→’a
. | . . . 4. n : ℕ
. | . . . 5. x = <λm:ℕ. if m<₂n then f(m) else @_bx:’a. true₂ fi ,n>
. | . . . 6. ||f{ℕn}|| = n-0
. | . . . 7. m : ℕ
. | . . . 8. ¬m<n
. | . . . 9. ¬m<||f{ℕn}||
. | . . . 10. ∀’a:S. arb(’a) = (@_bx:’a. true₂)
. | . . . ⊢  (@_bx:’a. true₂) = arb(’a)
. | . . .   by SwapEquands 0 THEN Last BackThru THEN StrongAuto
. | . . .      THEN
. | . . .      Try (Complete (Unfold ‘label‘ 0))
. |
. .\
. . 1. ’a : S
. . 2. x : (ℕ→’a)×ℕ
. . 3. ∃x’:’a List. x = rep_list(’a;x’)
. . ⊢  is_list_rep(x)
. . | by Last Analyze THEN Unab [‘his_list_rep‘] THEN Simp
. . |    THEN
. . |    StrongAuto
. . |
. . 1. ’a : S
. . 2. x : (ℕ→’a)×ℕ
. . 3. x’ : ’a List
. . 4. x = rep_list(’a;x’)
. . ⊢  ∃f:(ℕ→’a), n:ℕ.
. . |  x = <λm:ℕ. if m<₂n then f(m) else @_bx:’a. true₂ fi ,n>
. . | by ((H 4 0) THEN (Unab [‘rep_list‘])) THEN StrongAuto
. . |    THEN
. . |    Try (Complete (Unfold ‘label‘ 0))
. . |
. . 1. ’a : S
. . 2. x : (ℕ→’a)×ℕ
. . 3. x’ : ’a List
. . 4. x = <λn:ℕ. if n<₂||x’|| then x’[n] else arb(’a) fi ,||x’||>
. . ⊢  ∃f:(ℕ→’a), n:ℕ.
. . |  <λn:ℕ. if n<₂||x’|| then x’[n] else arb(’a) fi ,||x’||>
. . |  =
. . |  <λm:ℕ. if m<₂n then f(m) else @_bx:’a. true₂ fi ,n>
. . | by On [4;2] Thin THEN StrongAuto
. . |    THEN
. . |    Try (Complete (Unfold ‘label‘ 0))
. . |
. . 1. ’a : S
. . 2. x’ : ’a List
. . ⊢  ∃f:(ℕ→’a), n:ℕ.
. . |  <λn:ℕ. if n<₂||x’|| then x’[n] else arb(’a) fi ,||x’||>
. . |  =
```

```
. . |  <λm:ℕ. if m<₂n then f(m) else @_bx:'a. true₂ fi ,n>
. . | by ((DTerm (λn.if n<₂||x'|| then x'[n] else arb('a) fi ) 0)
. . |     THEN
. . |     (DTerm ||x'|| 0))
. . |    THEN
. . |    Simp
. . |    THEN
. . |    StrongAuto
. . |    THEN
. . |    Id
. . |    THEN
. . |    Simp
. . |    THEN
. . |    StrongAuto
. . |
. . 1. 'a : S
. . 2. x' : 'a List
. . ⊢  <λn:ℕ. if n<₂||x'|| then x'[n] else arb('a) fi ,||x'||>
. . |  =
. . |  <λm:ℕ. if m<₂||x'||
. . |         then if m<₂||x'|| then x'[m] else arb('a) fi
. . |         else @_bx:'a. true₂
. . |         fi
. . |  ,||x'||>
. . | by Analyze THEN StrongAuto
. . |    THEN
. . |    Try (Complete (Unfold `label` 0))
. . |
. . .....subterm .....
. . 1. 'a : S
. . 2. x' : 'a List
. . ⊢  (λn:ℕ. if n<₂||x'|| then x'[n] else arb('a) fi )
. . |  =
. . |  (λm:ℕ. if m<₂||x'||
. . |         then if m<₂||x'|| then x'[m] else arb('a) fi
. . |         else @_bx:'a. true₂
. . |         fi )
. . | by Analyze THEN StrongAuto
. . |    THEN
. . |    Try (Complete (Unfold `label` 0))
. . |
. . .....subterm .....
. . 1. 'a : S
. . 2. x' : 'a List
. . 3. n : ℕ
. . ⊢  if n<₂||x'|| then x'[n] else arb('a) fi
. . |  =
. . |  if n<₂||x'||
. . |  then if n<₂||x'|| then x'[n] else arb('a) fi
. . |  else @_bx:'a. true₂
. . |  fi   by BifCase THEN Simp THEN StrongAuto
. . |
. . 1. 'a : S
. . 2. x' : 'a List
. . 3. n : ℕ
```

```
. . 4. ¬n<||x'||
. . ⊢  arb('a) = (@_b x:'a. true_2)
. .   by Inst Thm* ∀'a:S. equal(arb,select(λx:'a. t))    {harb_wd} []
. .      THEN
. .      HN
. .      THEN
. .      StrongAuto
. .      THEN
. .      Try (Complete (Unfold `label` 0))
. .      THEN
. .      Last BackThru
. .      THEN
. .      StrongAuto
. .      THEN
. .      Try (Complete (Unfold `label` 0))
END
```