```
hcons_def  THM

⊢  ∀'a:S.
|  all
|  (λh:'a. all
|          (λt:hlist('a). equal
|                          (cons(h,t)
|                          ,abs_list
|                           (pair
|                            ((λm:hnum. cond
|                                       (equal(m,0)
|                                       ,h
|                                       ,fst(rep_list(t),pre(m))))
|                            ,suc(snd(rep_list(t))))))))
| by WOSimp (ioid !oid{pre_simp:s}) HN
|
1. 'a : S
2. h : 'a
3. t : hlist('a)
⊢  cons(h; t)
|  =
|  abs_list
|  (<λm:hnum. if m =_b 0 then h else 1of(rep_list('a;t))(pre(m)) fi
|   ,2of(rep_list('a;t))+1>)
| by Inst
|    Thm* ∀'a,'b:S, P:('b→𝔹), rep:('a→'b), abs:('b→'a), a:'a, r:'b.
|         iso_pair('a;'b;P;rep;abs) ⇒ rep(a) = r ⇒ a = abs(r)
|    {iso_pair_rep_to_abs}
|    ['a List
|    ;(ℕ→'a)×ℕ
|    ;is_list_rep
|    ;rep_list
|    ;abs_list
|    ;cons(h; t)
|    ;<λm:ℕ. if m =_b 0 then h else 1of(rep_list('a;t))(pre(m)) fi
|     ,2of(rep_list('a;t))+1>]
|    THEN
|    Try (Complete (Unfold `label` 0))
|
|\
| .....antecedent .....
| 1. 'a : S
| 2. 'a
| 3. hlist('a)
| ⊢  iso_pair('a List;(ℕ→'a)×ℕ;is_list_rep;rep_list;abs_list)
|   by L
|      Thm* ∀'a:S.
|           iso_pair('a List;(ℕ→'a)×ℕ;is_list_rep;rep_list;abs_list)
|      {list_iso}
|
.\
. .....antecedent .....
. 1. 'a : S
. 2. h : 'a
. 3. t : hlist('a)
```

```
. ⊢  rep_list(cons(h; t))
. |  =
. |  <λm:ℕ. if m =_b 0 then h else 1of(rep_list('a;t))(pre(m)) fi
. |  ,2of(rep_list('a;t))+1>
. | by Unab [`hrep_list`] THEN RW (NthC 1 (UnfoldTopC `rep_list`)) 0
. |    THEN
. |    Simp
. |
. 1. 'a : S
. 2. h : 'a
. 3. t : hlist('a)
. ⊢  <λn:ℕ. if n<_2||t||+1 then cons(h; t)[n] else arb('a) fi ,||t||+1>
. |  =
. |  <λm:ℕ. if m =_b 0 then h else 1of(rep_list('a;t))(pre(m)) fi
. |  ,2of(rep_list('a;t))+1>  by Analyze
. |
. |\
. | .....subterm .....
. | 1. 'a : S
. | 2. h : 'a
. | 3. t : hlist('a)
. | ⊢  (λn:ℕ. if n<_2||t||+1 then cons(h; t)[n] else arb('a) fi )
. | |  =
. | |  (λm:ℕ. if m =_b 0 then h else 1of(rep_list('a;t))(pre(m)) fi )
. | | by Analyze
. | |
. | .....subterm .....
. | 1. 'a : S
. | 2. h : 'a
. | 3. t : hlist('a)
. | 4. n : ℕ
. | ⊢  if n<_2||t||+1 then cons(h; t)[n] else arb('a) fi
. | |  =
. | |  if n =_b 0 then h else 1of(rep_list('a;t))(pre(n)) fi
. | | by BifCase THEN Simp THEN StrongAuto THEN BifCase THEN Simp
. | |    THEN
. | |    StrongAuto
. | |
. | |\
. | | 1. 'a : S
. | | 2. h : 'a
. | | 3. t : hlist('a)
. | | 4. n : ℕ
. | | 5. n<||t||+1
. | | 6. n = 0
. | | ⊢  cons(h; t)[n] = h
. | |   by L
. | |      Thm* ∀a:T, as:T List, i:ℤ. i≤0 ⇒ cons(a; as)[i] = a
. | |      {select_cons_hd}
. | |
. | |\
. | | 1. 'a : S
. | | 2. h : 'a
. | | 3. t : hlist('a)
. | | 4. n : ℕ
```

```
. | | 5. n<||t||+1
. | | 6. ¬n = 0
. | | ⊢  cons(h; t)[n] = 1of(rep_list('a;t))(pre(n))
. | | | by Assert (n>0) THEN HN THEN Simp
. | | |
. | | |\
. | | | 1. 'a : S
. | | | 2. h : 'a
. | | | 3. t : hlist('a)
. | | | 4. n : ℕ
. | | | 5. n<||t||+1
. | | | 6. ¬n = 0
. | | | 7. n>0
. | | | ⊢  n<||cons(h; t)||
. | | |   by Simpsetp [‘hol_to_nuprl‘;‘nat_to_int‘] THEN StrongAuto
. | | |      THEN
. | | |      Try (Complete (Unfold ‘label‘ 0))
. | | |
. | | .\
. | | . 1. 'a : S
. | | . 2. h : 'a
. | | . 3. t : hlist('a)
. | | . 4. n : ℕ
. | | . 5. n<||t||+1
. | | . 6. ¬n = 0
. | | . 7. n>0
. | | . ⊢  cons(h; t)[n] = 1of(rep_list('a;t))(n-1)
. | | . | by Unab [‘rep_list‘] THEN Simp THEN StrongAuto THEN BifCase
. | | . |    THEN
. | | . |    Simp
. | | . |    THEN
. | | . |    StrongAuto
. | | . |
. | | . 1. 'a : S
. | | . 2. h : 'a
. | | . 3. t : hlist('a)
. | | . 4. n : ℕ
. | | . 5. n<||t||+1
. | | . 6. ¬n = 0
. | | . 7. n>0
. | | . 8. n-1<||t||
. | | . ⊢  cons(h; t)[n] = t[(n-1)]
. | | .   by L
. | | .      Thm* ∀a:T, as:T List, i:ℤ.
. | | .           0<i ⇒ i≤||as|| ⇒ cons(a; as)[i] = as[(i-1)]
. | | .      {select_cons_tl}
. | | .      THEN
. | | .      StrongAuto
. | | .      THEN
. | | .      Try (Complete (Unfold ‘label‘ 0))
. | |
. | |\
. | | 1. 'a : S
. | | 2. h : 'a
. | | 3. t : hlist('a)
```

```
. | | 4. n : ℕ
. | | 5. ¬n<||t||+1
. | | 6. n = 0
. | | ⊢  arb('a) = h  by Simpsetp [`nat_to_int`] THEN StrongAuto
. | | |
. | | 1. 'a : S
. | | 2. h : 'a
. | | 3. t : hlist('a)
. | | 4. n : ℕ
. | | 5. ¬n<||t||+1
. | | 6. n = 0
. | | ⊢  arb('a) = h
. | |   by Inst Thm* ∀l:A List. ||l||≥0    {non_neg_length} ['a;t]
. | |      THEN
. | |      StrongAuto
. | |      THEN
. | |      Try (Complete (Unfold `label` 0))
. | |
. | .\
. | . 1. 'a : S
. | . 2. 'a
. | . 3. t : hlist('a)
. | . 4. n : ℕ
. | . 5. ¬n<||t||+1
. | . 6. ¬n = 0
. | . ⊢  arb('a) = 1of(rep_list('a;t))(pre(n))
. | . | by Simpsetp [`nat_to_int`]
. | . |
. | . 1. 'a : S
. | . 2. 'a
. | . 3. t : hlist('a)
. | . 4. n : ℕ
. | . 5. ¬n<||t||+1
. | . 6. ¬n = 0
. | . ⊢  arb('a) = 1of(rep_list('a;t))(pre(n))
. | .   by Unab [`rep_list`] THEN Simp THEN StrongAuto
. | .      THEN

. | .      BifCase THEN HN
. | .      THEN
. | .      Simp
. |
. .\
. . .....subterm .....
. . 1. 'a : S
. . 2. 'a
. . 3. t : hlist('a)
. . ⊢  ||t||+1 = 2of(rep_list('a;t))+1
. .   by Unab [`rep_list`] THEN Simp THEN StrongAuto
END
```