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hprod_ty_def  THM

⊢  ∀'a,'b:S.
|  exists
|  (λrep:hprod('a; 'b) → 'a → 'b → hbool. type_definition
|                                          (is_pair
|                                          ,rep))
| by HN THEN StrongAuto
|
1. 'a : S
2. 'b : S
⊢  ∃rep:(hprod('a; 'b)→'a→'b→hbool).
|  type_definition('a→'b→hbool;hprod('a; 'b);is_pair;rep)
| by ((DTerm rep_prod 0) THEN (Unab [‘type_definition‘])) THEN Simp
|    THEN
|    StrongAuto
|
|\
| 1. 'a : S
| 2. 'b : S
| 3. x' : hprod('a; 'b)
| 4. x'' : hprod('a; 'b)
| 5. rep_prod(x') = rep_prod(x'')
| ⊢  x' = x''
| | by Unab [‘hrep_prod‘;‘hmk_pair‘] THEN Simp THEN StrongAuto
| |
| 1. 'a : S
| 2. 'b : S
| 3. x' : hprod('a; 'b)
| 4. x'' : hprod('a; 'b)
| 5. (λa:'a. λb:'b. (a =_b 1of(x'))∧_2(b =_b 2of(x')))
| |  =
| |  (λa:'a. λb:'b. (a =_b 1of(x''))∧_2(b =_b 2of(x'')))
| ⊢  x' = x''
| | by EquandMap (λf.f(1of(x'),2of(x'))) 𝔹 -1 THEN Simp
| |    THEN
| |    StrongAuto
| |
| 1. 'a : S
| 2. 'b : S
| 3. x' : hprod('a; 'b)
| 4. x'' : hprod('a; 'b)
| 5. (λa:'a. λb:'b. (a =_b 1of(x'))∧_2(b =_b 2of(x')))
|    =
|    (λa:'a. λb:'b. (a =_b 1of(x''))∧_2(b =_b 2of(x'')))
| 6. 1of(x') = 1of(x'')
| 7. 2of(x') = 2of(x'')
| ⊢  x' = x''
|   by Analyze 4 THEN Analyze 3 THEN Simp THEN StrongAuto THEN Analyze
|      THEN
|      StrongAuto
|
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| 1. 'a : S
| 2. 'b : S
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| 3. x : 'a→'b→hbool
| 4. is_pair(x)
| ⊢  ∃x':hprod('a; 'b). x = rep_prod(x')
| | by Unab [‘his_pair‘;‘hrep_prod‘] THEN Simp THEN StrongAuto
| |
| 1. 'a : S
| 2. 'b : S
| 3. x : 'a→'b→hbool
| 4. ∃x@0:'a, y:'b. x = mk_pair(x@0,y)
| ⊢  ∃x':hprod('a; 'b). x = mk_pair(1of(x'),2of(x'))
| | by Lasts 2 Analyze THEN Unab [‘hmk_pair‘] THEN Simp
| |    THEN
| |    StrongAuto
| |
| 1. 'a : S
| 2. 'b : S
| 3. x : 'a→'b→hbool
| 4. x@0 : 'a
| 5. y : 'b
| 6. x = (λa:'a. λb:'b. (a =b x@0)∧2(b =b y))
| ⊢  ∃x':hprod('a; 'b).
|    x = (λa:'a. λb:'b. (a =b 1of(x'))∧2(b =b 2of(x')))
|   by ((DTerm <x@0,y> 0) THEN (H 6 0)) THEN Simp THEN StrongAuto
|
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. 1. 'a : S
. 2. 'b : S
. 3. x : 'a→'b→hbool
. 4. ∃x':hprod('a; 'b). x = rep_prod(x')
. ⊢  is_pair(x)
. | by Last Analyze THEN Unab [‘hrep_prod‘;‘his_pair‘] THEN Simp
. |    THEN
. |    StrongAuto
. |
. 1. 'a : S
. 2. 'b : S
. 3. x : 'a→'b→hbool
. 4. x' : hprod('a; 'b)
. 5. x = mk_pair(1of(x'),2of(x'))
. ⊢  ∃x@0:'a, y:'b. x = mk_pair(x@0,y)
.   by ((DTerm 1of(x') 0) THEN (DTerm 2of(x') 0)) THEN StrongAuto
END
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