```
hsimp_rec_wd  THM

⊢  ∀'a:S.
|  all
|  (λx:'a. all
|          (λf:'a → 'a. all
|                       (λn:hnum. equal
|                                  (simp_rec(x,f,n)
|                                  ,simp_rec_fun(x,f,suc(n),n)))))
| by HN THEN StrongAuto THEN Try (Complete (Unfold `label` 0))
|    THEN
|    Unab [`hsimp_rec`]
|    THEN
|    StrongAuto
|    THEN
|    Try (Complete (Unfold `label` 0))
|    THEN
|    Simpsetp [`hol_to_nuprl`;`bequal`]
|    THEN
|    StrongAuto
|    THEN
|    Try (Complete (Unfold `label` 0))
|
1. 'a : S
2. x : 'a
3. f : 'a → 'a
4. n : ℕ
⊢  ncompose(f;n;x) = simp_rec_fun(x,f,n+1,n)
| by Unab [`hsimp_rec_fun`] THEN Simp THEN StrongAuto
|
1. 'a : S
2. x : 'a
3. f : 'a → 'a
4. n : ℕ
⊢  ncompose(f;n;x) = (@_b fun:ℕ→'a. (simp_rec_rel(fun,x,f,n+1)))(n)
| by Unab [`bchoose`] THEN ChooseDC THEN Simp THEN StrongAuto THEN HN
|    THEN
|    StrongAuto
|    THEN
|    Try (Complete (Unfold `label` 0))
|
|\
| 1. 'a : S
| 2. x : 'a
| 3. f : 'a → 'a
| 4. n : ℕ
| 5. fun : ℕ→'a
| 6. simp_rec_rel(fun,x,f,n+1)
| ⊢  ncompose(f;n;x) = fun(n)
| | by Unab [`hsimp_rec_rel`] THEN Simp THEN StrongAuto
| |
| 1. 'a : S
| 2. x : 'a
| 3. f : 'a → 'a
| 4. n : ℕ
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| 5. fun : ℕ→'a
| 6. fun(0) = x
| 7. ∀m:ℕ. m<n+1 ⇒ fun(m+1) = f(fun(m))
| ⊢  ncompose(f;n;x) = fun(n)
| | by MoveDepHypsToConclFor NInd 4 THEN Simp THEN StrongAuto
| |
| 1. 'a : S
| 2. x : 'a
| 3. f : 'a → 'a
| 4. fun : ℕ→'a
| 5. fun(0) = x
| 6. n : ℤ
| 7. 0<n
| 8. (∀m:ℕ. m<n-1+1 ⇒ fun(m+1) = f(fun(m)))
| |  ⇒
| |  ncompose(f;n-1;x) = fun(n-1)
| 9. ∀m:ℕ. m<n+1 ⇒ fun(m+1) = f(fun(m))
| ⊢  f(ncompose(f;n-1;x)) = fun(n)
| | by InstHyp [n-1] 9 THEN StrongAuto
| |    THEN
| |    Try (Complete (Unfold `label` 0))
| |
| 1. 'a : S
| 2. x : 'a
| 3. f : 'a → 'a
| 4. fun : ℕ→'a
| 5. fun(0) = x
| 6. n : ℤ
| 7. 0<n
| 8. (∀m:ℕ. m<n-1+1 ⇒ fun(m+1) = f(fun(m)))
| |  ⇒
| |  ncompose(f;n-1;x) = fun(n-1)
| 9. ∀m:ℕ. m<n+1 ⇒ fun(m+1) = f(fun(m))
| 10. fun(n-1+1) = f(fun(n-1))
| ⊢  f(ncompose(f;n-1;x)) = fun(n)
| | by Analyze 8 THEN StrongAuto
| |    THEN
| |    Try (Complete (Unfold `label` 0))
| |
| |\
| | 1. 'a : S
| | 2. x : 'a
| | 3. f : 'a → 'a
| | 4. fun : ℕ→'a
| | 5. fun(0) = x
| | 6. n : ℤ
| | 7. 0<n
| | 8. ∀m:ℕ. m<n+1 ⇒ fun(m+1) = f(fun(m))
| | 9. fun(n-1+1) = f(fun(n-1))
| | 10. m : ℕ
| | 11. m<n-1+1
| | ⊢  fun(m+1) = f(fun(m))
| |   by BackThru 8 THEN StrongAuto
| |      THEN
| |      Try (Complete (Unfold `label` 0))
```

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| |
| .\
| . 1. 'a : S
| . 2. x : 'a
| . 3. f : 'a → 'a
| . 4. fun : ℕ→'a
| . 5. fun(0) = x
| . 6. n : ℤ
| . 7. 0<n
| . 8. ∀m:ℕ. m<n+1 ⇒ fun(m+1) = f(fun(m))
| . 9. fun(n-1+1) = f(fun(n-1))
| . 10. ncompose(f;n-1;x) = fun(n-1)
| . ⊢  f(ncompose(f;n-1;x)) = fun(n)
| . | by On [0;9;10] ArithSimp THEN StrongAuto
| . |    THEN
| . |    Try (Complete (Unfold `label` 0))
| . |
| . 1. 'a : S
| . 2. x : 'a
| . 3. f : 'a → 'a
| . 4. fun : ℕ→'a
| . 5. fun(0) = x
| . 6. n : ℤ
| . 7. 0<n
| . 8. ∀m:ℕ. m<n+1 ⇒ fun(m+1) = f(fun(m))
| . 9. fun(n) = f(fun(-1+n))
| . 10. ncompose(f;-1+n;x) = fun(-1+n)
| . ⊢  f(ncompose(f;-1+n;x)) = fun(n)
| . | by H 9 0 THEN StrongAuto THEN Try (Complete (Unfold `label` 0))
| . |
| . 1. 'a : S
| . 2. x : 'a
| . 3. f : 'a → 'a
| . 4. fun : ℕ→'a
| . 5. fun(0) = x
| . 6. n : ℤ
| . 7. 0<n
| . 8. ∀m:ℕ. m<n+1 ⇒ fun(m+1) = f(fun(m))
| . 9. fun(n) = f(fun(-1+n))
| . 10. ncompose(f;-1+n;x) = fun(-1+n)
| . ⊢  f(ncompose(f;-1+n;x)) = f(fun(-1+n))
| . | by Analyze THEN StrongAuto
| . |    THEN
| . |    Try (Complete (Unfold `label` 0))
| . |
| . .....subterm .....
| . 1. 'a : S
| . 2. x : 'a
| . 3. f : 'a → 'a
| . 4. fun : ℕ→'a
| . 5. fun(0) = x
| . 6. n : ℤ
| . 7. 0<n
| . 8. ∀m:ℕ. m<n+1 ⇒ fun(m+1) = f(fun(m))
| . 9. fun(n) = f(fun(-1+n))
```

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| . 10. ncompose(f;-1+n;x) = fun(-1+n)
| . ⊢  ncompose(f;-1+n;x) = fun(-1+n)
| .   by Unab [`guard`] THEN StrongAuto
| .      THEN
| .      Try (Complete (Unfold `label` 0))
|
.\
. 1. 'a : S
. 2. x : 'a
. 3. f : 'a → 'a
. 4. n : ℕ
. ⊢  ∃fun:(ℕ→'a). simp_rec_rel(fun,x,f,n+1)
. | by ((DTerm (λm:ℕ. ncompose(f;m;x)) 0)
. |     THEN
. |     (Unab [`hsimp_rec_rel`]))
. |    THEN
. |    Simp
. |    THEN
. |    StrongAuto
. |    THEN
. |    HN
. |    THEN
. |    StrongAuto
. |    THEN
. |    Try (Complete (Unfold `label` 0))
. |
. 1. 'a : S
. 2. x : 'a
. 3. f : 'a → 'a
. 4. n : ℕ
. 5. m : ℕ
. 6. m<n+1
. ⊢  ncompose(f;m+1;x) = f(ncompose(f;m;x))
.   by Assert (0<m+1) THEN Simp THEN StrongAuto
END
```