```
hsum_iso_def  THM

⊢  ∀'a,'b:S.
|  and
|  (all(λa:hsum('a; 'b). equal(abs_sum(rep_sum(a)),a))
|  ,all
|   (λr:hbool → 'a → 'b → hbool. equal
|                                  (is_sum_rep(r)
|                                  ,equal(rep_sum(abs_sum(r)),r))))
| by Simpsetp [‘hol_to_nuprl‘]
|
⊢  ∀'a,'b:S.
|  (∀a:'a+'b. abs_sum(rep_sum(a)) = a)
|  & (∀r:(𝔹→'a→'b→𝔹). is_sum_rep(r) = ((rep_sum(abs_sum(r))) =_b r))
| by ((RepeatMFor 2 (Analyze 0))
|     THEN
|     (L
|      Thm* ∀'a,'b:S, P:('b→𝔹), rep:('a→'b), abs:('b→'a).
|           iso_pair('a;'b;P;rep;abs)
|           ⇒
|           (∀a:'a. abs(rep(a)) = a)
|           & (∀r:'b. P(r) = ((rep(abs(r))) =_b r))
|      {iso_pair_char}))
|    THENA
|    Try (Complete Auto)
|
|\
| .....wf .....
| 1. 'a : S
| 2. 'b : S
| ⊢  𝔹→'a→'b→𝔹 ∈ S
|   by RepeatFor 3
|      (L Thm* ∀'a,'b:S. 'a→'b ∈ S    {function_wf_stype}
|       THEN
|       Try (Complete Auto))
|
.\
. 1. 'a : S
. 2. 'b : S
. ⊢  iso_pair('a+'b;𝔹→'a→'b→𝔹;is_sum_rep;rep_sum;abs_sum)
. | by Unab [‘iso_pair‘;‘habs_sum‘;‘his_sum_rep‘] THEN HN
. |    THEN
. |    StrongAuto
. |    THEN
. |    Try (Complete (Unfold ‘label‘ 0))
. |
. |\
. | 1. 'a : S
. | 2. 'b : S
. | 3. r : 𝔹→'a→'b→𝔹
. | ⊢  (@u:'a+'b. (rep_sum(u) = r)) = (@a:'a+'b. (r = rep_sum(a)))
. |   by L
. |      Thm* ∀T:S, P,Q:(T→Type).
. |           (∀x:T. P(x) ⟺ Q(x)) ⇒ (@x:T. P(x)) = (@x:T. Q(x))
. |      {choose_functionality_axiom}
```

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. |      THEN
. |      StrongAuto
. |      THEN
. |      Try (Complete (Unfold `label` 0))
. |
. .\
. . 1. 'a : S
. . 2. 'b : S
. . ⊢  type_definition(𝔹→'a→'b→𝔹;'a+'b;λf:𝔹→'a→'b→
. . |  𝔹. ∃_b u:'a+'b. (f =_b (rep_sum(u)));rep_sum)
. . | by Unab [`type_definition`] THEN Try (Complete Auto)
. . |
. . 1. 'a : S
. . 2. 'b : S
. . ⊢  (∀x',x'':'a+'b. rep_sum(x') = rep_sum(x'') ⇒ x' = x'')
. . |  & (∀x:(𝔹→'a→'b→𝔹).
. . |     (λf:𝔹→'a→'b→𝔹. ∃_b u:'a+'b. (f =_b (rep_sum(u))))(x)
. . |     ⟺
. . |     (∃x':'a+'b. x = rep_sum(x')))
. . | by Analyze 0 THEN Try (Complete Auto)
. . |
. . |\
. . | 1. 'a : S
. . | 2. 'b : S
. . | ⊢  ∀x',x'':'a+'b. rep_sum(x') = rep_sum(x'') ⇒ x' = x''
. . | | by Auto THEN Analyze 4 THEN Analyze 3 THEN Unab [`hrep_sum`]
. . | |
. . | |\
. . | | 1. 'a : S
. . | | 2. 'b : S
. . | | 3. x1 : 'a
. . | | 4. x : 'a
. . | | 5. (λu:'a+'b. InjCase(u
. . | | |                    ; p. λb:𝔹. λx:'a. λy:'b. (x =_b p)∧_2 b
. . | | |                    ; q. λb:𝔹. λx:'a. λy:'b. (y =_b q)∧_2 ¬_2 b))
. . | | |  (inl(x1))
. . | | |  =
. . | | |  (λu:'a+'b. InjCase(u
. . | | |                    ; p. λb:𝔹. λx:'a. λy:'b. (x =_b p)∧_2 b
. . | | |                    ; q. λb:𝔹. λx:'a. λy:'b. (y =_b q)∧_2 ¬_2 b))
. . | | |  (inl(x))
. . | | ⊢  inl(x1) = inl(x)
. . | | | by ((EquandMap (λf.f(true_2,x1,arb('b))) 𝔹 5) THEN (Thin 5)
. . | | |     THEN
. . | | |     Simp)
. . | | |    THEN
. . | | |    StrongAuto
. . | | |    THEN
. . | | |    Try (Complete (Unfold `label` 0))
. . | | |
. . | | 1. 'a : S
. . | | 2. 'b : S
. . | | 3. x1 : 'a
. . | | 4. x : 'a
. . | | 5. x1 = x
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. . | | ⊢  inl(x1) = inl(x)  by Analyze
. . | |
. . | |\
. . | | 1. 'a : S
. . | | 2. 'b : S
. . | | 3. y : 'b
. . | | 4. x : 'a
. . | | 5. (λu:'a+'b. InjCase(u
. . | | |                      ; p. λb:𝔹. λx:'a. λy:'b. (x =_b p)∧_2b
. . | | |                      ; q. λb:𝔹. λx:'a. λy:'b. (y =_b q)∧_2¬_2b))
. . | | |  (inr(y))
. . | | |  =
. . | | |  (λu:'a+'b. InjCase(u
. . | | |                      ; p. λb:𝔹. λx:'a. λy:'b. (x =_b p)∧_2b
. . | | |                      ; q. λb:𝔹. λx:'a. λy:'b. (y =_b q)∧_2¬_2b))
. . | | |  (inl(x))
. . | | ⊢  inr(y) = inl(x)
. . | | | by ((EquandMap (λf.f(true_2,x,arb('b))) 𝔹 5) THEN (Thin 5))
. . | | |    THEN
. . | | |    Try (Complete Auto)
. . | | |    THEN
. . | | |    Simp
. . | | |
. . | | 1. 'a : S
. . | | 2. 'b : S
. . | | 3. y : 'b
. . | | 4. x : 'a
. . | | 5. ¬True
. . | | ⊢  inr(y) = inl(x)  by Simpsetp [`nlogic`]
. . | |
. . | |\
. . | | 1. 'a : S
. . | | 2. 'b : S
. . | | 3. x : 'a
. . | | 4. y : 'b
. . | | 5. (λu:'a+'b. InjCase(u
. . | | |                      ; p. λb:𝔹. λx:'a. λy:'b. (x =_b p)∧_2b
. . | | |                      ; q. λb:𝔹. λx:'a. λy:'b. (y =_b q)∧_2¬_2b))
. . | | |  (inl(x))
. . | | |  =
. . | | |  (λu:'a+'b. InjCase(u
. . | | |                      ; p. λb:𝔹. λx:'a. λy:'b. (x =_b p)∧_2b
. . | | |                      ; q. λb:𝔹. λx:'a. λy:'b. (y =_b q)∧_2¬_2b))
. . | | |  (inr(y))
. . | | ⊢  inl(x) = inr(y)
. . | | | by ((EquandMap (λf.f(true_2,x,arb('b))) 𝔹 5) THEN (Thin 5))
. . | | |    THEN
. . | | |    Simp
. . | | |
. . | | 1. 'a : S
. . | | 2. 'b : S
. . | | 3. x : 'a
. . | | 4. y : 'b
. . | | 5. ¬True
. . | | ⊢  inl(x) = inr(y)  by Simpsetp [`nlogic`]
```

```
. . | |
. . | .\
. . | . 1. 'a : S
. . | . 2. 'b : S
. . | . 3. y1 : 'b
. . | . 4. y : 'b
. . | . 5. (λu:'a+'b. InjCase(u
. . | . |                     ; p. λb:𝔹. λx:'a. λy:'b. (x =_b p)∧₂b
. . | . |                     ; q. λb:𝔹. λx:'a. λy:'b. (y =_b q)∧₂¬₂b))
. . | . |  (inr(y1))
. . | . |  =
. . | . |  (λu:'a+'b. InjCase(u
. . | . |                     ; p. λb:𝔹. λx:'a. λy:'b. (x =_b p)∧₂b
. . | . |                     ; q. λb:𝔹. λx:'a. λy:'b. (y =_b q)∧₂¬₂b))
. . | . |  (inr(y))
. . | . ⊢  inr(y1) = inr(y)
. . | . | by ((EquandMap (λf.f(false₂,arb('a),y1)) 𝔹 5) THEN (Thin 5)
. . | . |     THEN
. . | . |     Simp)
. . | . |    THEN
. . | . |    StrongAuto
. . | . |    THEN
. . | . |    Try (Complete (Unfold `label` 0))
. . | . |
. . | . 1. 'a : S
. . | . 2. 'b : S
. . | . 3. y1 : 'b
. . | . 4. y : 'b
. . | . 5. y1 = y
. . | . ⊢  inr(y1) = inr(y)  by Analyze
. . |
. . .\
. . . 1. 'a : S
. . . 2. 'b : S
. . . ⊢  ∀x:(𝔹→'a→'b→𝔹).
. . .    (λf:𝔹→'a→'b→𝔹. ∃_b u:'a+'b. (f =_b (rep_sum(u))))(x)
. . .    ⟺

. . .    (∃x':'a+'b. x = rep_sum(x'))  by Auto THEN Simp
END
```