```
comp_nat_ind_tp  THM

⊢  ∀P:(ℕ→Prop). (∀i:ℕ. (∀j:ℕ. j<i ⇒ P(j)) ⇒ P(i)) ⇒ (∀i:ℕ. P(i))
| by UnivCD
|
1. P : ℕ→Prop
2. ∀i:ℕ. (∀j:ℕ. j<i ⇒ P(j)) ⇒ P(i)
⊢  ∀i:ℕ. P(i)  by Analyze 0
|
3. i : ℕ
⊢  P(i)  by PushArgs [`hn`,(int_to_arg 3)]
|
.....main .....
⊢  P(i)
| by Analogy Tactic Directive: %S%
|
|    λp.
|    let i ≡ var_of_hyp (get_int_arg `hn` p) p ,
|        z ≡ get_distinct_var `zz` p in
|    Assert
|    (mk_all_term z ℕ
|     (mk_all_term i ℕ
|      (mk_implies_term (mk_less_than_term (mvt i) (mvt z))
|       (concl p))))
|    p
|
|\
| .....assertion .....
| ⊢  ∀zz,i:ℕ. i<zz ⇒ P(i)  by λp.Thin (get_int_arg `hn` p) p
| |
| .....main .....
| 2. ∀i:ℕ. (∀j:ℕ. j<i ⇒ P(j)) ⇒ P(i)
| ⊢  ∀zz,i:ℕ. i<zz ⇒ P(i)  by Analyze 0
| |
| |\
| | .....main .....
| | 3. zz : ℕ
| | ⊢  ∀i:ℕ. i<zz ⇒ P(i)  by NatInd -1
| | |
| | |\
| | | .....basecase .....
| | | 2. ∀i:ℕ. (∀j:ℕ. j<i ⇒ P(j)) ⇒ P(i)
| | | ⊢  ∀i:ℕ. i<0 ⇒ P(i)  by (Analyze 0) THEN (Analyze 0)
| | | |
| | | |\
| | | | .....main .....
| | | | 3. i : ℕ
| | | | 4. i<0
| | | | ⊢  P(i)  by Arith
| | | |
| | | |\
| | | | .....wf .....
| | | | 3. i : ℕ
| | | | ⊢  (i<0) ∈ Prop  by MemEqCD THEN MemEqCD
| | | |
```

```
| | | .\
| | | . .....wf .....
| | | . ⊢  ℕ ∈ Type  by MemEqCD
| | |
| | .\
| | . .....upcase .....
| | . 3. zz : ℤ
| | . 4. 0<zz
| | . 5. ∀i:ℕ. i<zz-1 ⇒ P(i)
| | . ⊢  ∀i:ℕ. i<zz ⇒ P(i)  by (Analyze 0) THEN (Analyze 0)
| | . |
| | . |\
| | . | .....main .....
| | . | 6. i : ℕ
| | . | 7. i<zz
| | . | ⊢  P(i)
| | . | | by λp.
| | . | |    let i ≡ var_of_hyp -2 p ,
| | . | |        j ≡ get_distinct_var i p ,
| | . | |        Pi ≡ concl p ,
| | . | |        Pj ≡ subst [i,(mvt j)] Pi in
| | . | |    Assert
| | . | |    (mk_all_term j ℕ
| | . | |     (mk_implies_term (mk_less_than_term (mvt j) (mvt i))
| | . | |      Pj))
| | . | |    p
| | . | |
| | . | |\
| | . | | .....assertion .....
| | . | | ⊢  ∀i1:ℕ. i1<i ⇒ P(i1)  by (Analyze 0) THEN (Analyze 0)
| | . | | |
| | . | | |\
| | . | | | .....main .....
| | . | | | 8. i1 : ℕ
| | . | | | 9. i1<i
| | . | | | ⊢  P(i1)
| | . | | | | by λp.let i1 ≡ var_of_hyp -2 p in DTerm (mvt i1) -5 p
| | . | | | |
| | . | | | |\
| | . | | | | .....wf .....
| | . | | | | 5. i : ℕ
| | . | | | | 6. i<zz
| | . | | | | 7. i1 : ℕ
| | . | | | | 8. i1<i
| | . | | | | ⊢  i1 ∈ ℕ  by MemEqCD
| | . | | | |
| | . | | | .\
| | . | | | . .....main .....
| | . | | | . 5. i : ℕ
| | . | | | . 6. i<zz
| | . | | | . 7. i1 : ℕ
| | . | | | . 8. i1<i
| | . | | | . 9. i1<zz-1 ⇒ P(i1)
| | . | | | . ⊢  P(i1)
| | . | | | .   by Analyze -1
```

```
| | . | | | .      THEN
| | . | | | .      IfLab `main` Hypothesis
| | . | | | .      (Arith THEN Analyze THEN Analyze)
| | . | | |
| | . | | |\
| | . | | | .....wf .....
| | . | | | 8. i1 : ℕ
| | . | | | ⊢  (i1<i) ∈ Prop  by MemEqCD THEN MemEqCD
| | . | | |
| | . | | .\
| | . | | . .....wf .....
| | . | | . ⊢  ℕ ∈ Type  by MemEqCD
| | . | |
| | . | .\
| | . | . .....main .....
| | . | . 8. ∀i1:ℕ. i1<i ⇒ P(i1)
| | . | . ⊢  P(i)  by OnHyps [-2;-3;-3;-3] Thin
| | . | . |
| | . | . .....main .....
| | . | . 3. i : ℕ
| | . | . 4. ∀i1:ℕ. i1<i ⇒ P(i1)
| | . | . ⊢  P(i)
| | . | .   by Analogy Tactic Directive: %E%
| | . | .
| | . | .      FwdThru 2 [4]
| | . |
| | . |\
| | . | .....wf .....
| | . | 6. i : ℕ
| | . | ⊢  (i<zz) ∈ Prop  by MemEqCD THEN MemEqCD
| | . |
| | . .\
| | . . .....wf .....
| | . . ⊢  ℕ ∈ Type  by MemEqCD
| |
| .\
| . .....wf .....
| . ⊢  ℕ ∈ Type  by MemEqCD
|
.\
. .....main .....
. 4. ∀zz,i:ℕ. i<zz ⇒ P(i)
. ⊢  P(i)
. | by λp.
. |    let i ≡ mvt (var_of_hyp -2 p) ,
. |        ip1 ≡ mk_add_term i 1 in
. |    ((DTerm ip1 -1) THEN (DTerm i -1)) p
. |
. |\
. | .....wf .....
. | 3. i : ℕ
. | ⊢  i+1 ∈ ℕ  by ArithMemberReflEqTypeCD
. | |
. | |\
. | | .....main .....
```

```
. | | ⊢  i+1 ∈ ℤ  by MemEqCD THEN MemEqCD
. | |
. | |\
. | | .....set predicate .....
. | | ⊢  0≤i+1  by Arith
. | | |
. | | |\
. | | | .....wf .....
. | | | 3. i : ℤ
. | | | 4. 0≤i
. | | | ⊢  i+1 ∈ ℤ  by Analyze THEN Analyze
. | | |
. | | .\
. | | . .....wf .....
. | | . 3. i : ℤ
. | | . 4. 0≤i
. | | . ⊢  0 ∈ ℤ  by Analyze
. | |
. | .\
. | . .....wf .....
. | . 4. i1 : ℤ
. | . ⊢  (0≤i1) ∈ Type  by MemEqCD THEN MemEqCD
. |
. |\
. | .....wf .....
. | 3. i : ℕ
. | ⊢  i ∈ ℕ  by MemEqCD
. |
. .\
. . .....main .....
. . 4. i<i+1 ⇒ P(i)
. . ⊢  P(i)  by Analyze -1
. . |
. . |\
. . | .....antecedent .....
. . | 3. i : ℕ
. . | ⊢  i<i+1  by Arith THEN Analyze THEN Analyze

. . |
. . .\
. . . .....main .....
. . . 4. P(i)
. . . ⊢  P(i)  by Trivial
END
```