```
complete_nat_ind_with_y  THM

⊢  ∀P:(ℕ→Prop). (∀i:ℕ. (∀j:ℕi. P(j)) ⇒ P(i)) ⇒ (∀i:ℕ. P(i))
| by UseWitness (λP,g. Y(λf,x. g(x,f)))
|
⊢  (λP,g. Y(λf,x. g(x,f)))
|  ∈ (∀P:(ℕ→Prop). (∀i:ℕ. (∀j:ℕi. P(j)) ⇒ P(i)) ⇒ (∀i:ℕ. P(i)))
| by MemberEqCD
|    THEN
|    IfLab ‘subterm‘ (MemberEqCD THEN IfLab ‘subterm‘ Id Auto) Auto
|
1. P : ℕ→Prop
2. g : ∀i:ℕ. (∀j:ℕi. P(j)) ⇒ P(i)
⊢  Y(λf,x. g(x,f)) ∈ (∀i:ℕ. P(i))
| by Assert (Y(λf,x. g(x,f)) ∈ Void→Void)
|
|\
| .....assertion .....
| ⊢  Y(λf,x. g(x,f)) ∈ Void→Void
| | by Rewrite (HigherC YUnrollC) 0 THEN Reduce 0
| |
| ⊢  (λx.g(x,Y(λf,x. g(x,f)))) ∈ Void→Void  by Analyze
|
.\
. 3. Y(λf,x. g(x,f)) ∈ Void→Void
. ⊢  Y(λf,x. g(x,f)) ∈ (∀i:ℕ. P(i))
. | by Assert (∀n:ℕ. Y(λf,x. g(x,f)) ∈ (∀m:ℕn. P(m)))
. |
. |\
. | .....assertion .....
. | ⊢  ∀n:ℕ. Y(λf,x. g(x,f)) ∈ (∀m:ℕn. P(m))  by Analyze 0
. | |
. | 4. n : ℕ
. | ⊢  Y(λf,x. g(x,f)) ∈ (∀m:ℕn. P(m))
. | | by NInd 4
. | |    THEN
. | |    IfLabL
. | |    [‘wf‘,Auto
. | |    ;RULE H  ⊢ C ext t
. | |
. | |            BY arith Type{i}
. | |
. | |               Let SubGoals; t = CallLisp(ARITH)
. | |            SubGoals
. | |     {arith}
. | |     ,Auto]
. | |
. | |\
. | | .....basecase .....
. | | 3. Y(λf,x. g(x,f)) ∈ Void→Void
. | | ⊢  Y(λf,x. g(x,f)) ∈ (∀m:ℕ0. P(m))  by ExtWith [] [Void→Void]
. | | |
. | | |\
. | | | .....wf .....
. | | | ⊢  Y(λf,x. g(x,f)) ∈ Void→Void  by Auto
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. | | |\
. | | | 4. x : ℕ0
. | | | ⊢  Y((λf,x. g(x,f)),x) ∈ P(x)  by Arith
. | | |
. | | .\
. | | . .....wf .....
. | | . ⊢  ℕ0 ∈ Type  by Auto
. | |
. | .\
. | . .....upcase .....
. | . 4. n : ℤ
. | . 5. 0<n
. | . 6. Y(λf,x. g(x,f)) ∈ (∀m:ℕ(n-1). P(m))
. | . ⊢  Y(λf,x. g(x,f)) ∈ (∀m:ℕn. P(m))
. | . | by ExtWith [‘r’] [Void→Void]
. | . |
. | . 7. r : ℕn
. | . ⊢  Y((λf,x. g(x,f)),r) ∈ P(r)  by Decide (r = n-1)
. | . |
. | . |\
. | . | 8. r = n-1
. | . | ⊢  Y((λf,x. g(x,f)),r) ∈ P(r)
. | . | | by Rewrite (HigherC YUnrollC THENC ReduceC) 0
. | . | |
. | . | ⊢  g(r,Y(λf,x. g(x,f))) ∈ P(r)  by MemberEqCD
. | . | |
. | . | |\
. | . | | ⊢  g(r) ∈ (∀j:ℕr. P(j))→P(r)  by Auto
. | . | |
. | . | .\
. | . | . ⊢  Y(λf,x. g(x,f)) ∈ (∀j:ℕr. P(j))  by Inclusion 6
. | . |
. | . .\
. | . . 8. ¬r = n-1
. | . . ⊢  Y((λf,x. g(x,f)),r) ∈ P(r)  by Thin 3
. |
. .\
. . 4. ∀n:ℕ. Y(λf,x. g(x,f)) ∈ (∀m:ℕn. P(m))
. . ⊢  Y(λf,x. g(x,f)) ∈ (∀i:ℕ. P(i))  by ExtWith [‘r’] [Void→Void]
. . |
. . 5. r : ℕ
. . ⊢  Y((λf,x. g(x,f)),r) ∈ P(r)
. . | by DTerm (r+1) 4 THEN IfLab ‘main‘ (Thin 3) Auto
. . |
. . 3. r : ℕ
. . 4. Y(λf,x. g(x,f)) ∈ (∀m:ℕ(r+1). P(m))
. . ⊢  Y((λf,x. g(x,f)),r) ∈ P(r)  by MemberEqCD
. . |
. . |\
. . | ⊢  Y(λf,x. g(x,f)) ∈ m:ℕ(r+1)→P(m)  by Inclusion 4
. . |
. . .\
. . . ⊢  r ∈ ℕ(r+1)  by Auto
END
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