```
decidable__ex_int_seg  THM

⊢  ∀i,j:ℤ, F:({i..j⁻}→Prop).
|  (∀k:{i..j⁻}. Dec(F(k))) ⇒ Dec(∃k:{i..j⁻}. F(k))  by UnivCD
|
1. i : ℤ
2. j : ℤ
3. F : {i..j⁻}→Prop
4. ∀k:{i..j⁻}. Dec(F(k))
⊢  Dec(∃k:{i..j⁻}. F(k))  by Decide: i≤j
|
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| 5. i≤j
| ⊢  Dec(∃k:{i..j⁻}. F(k))
| | by Inst:
| |    Thm* ∀i:ℤ, E:({...i}→Prop).
| |         E(i) ⇒ (∀k:{...i-1}. E(k+1) ⇒ E(k)) ⇒ (∀k:{...i}. E(k))
| |    {int_lower_ind}
| |    Using:[j | E(n):= i≤n ⇒ Dec(∃k:{n..j⁻}. F(k))]
| |
| |\
| | 6. i≤j
| | ⊢  Dec(∃k:{j..j⁻}. F(k))  by Select: 2
| | |
| | ⊢  ¬(∃k:{j..j⁻}. F(k))  by Analyze THEN Analyze-1
| |
| |\
| | 6. k : {...j-1}
| | 7. i≤k
| | 8. Dec(∃k:{(k+1)..j⁻}. F(k))
| | ⊢  Dec(∃k:{k..j⁻}. F(k))
| |   by Rewrite by
| |      Thm* ∀i,j:ℤ.
| |           i<j
| |           ⇒
| |           (∀F:({i..j⁻}→Prop).
| |            (∃k:{i..j⁻}. F(k)) ⟺ F(i) ∨ (∃k:{(i+1)..j⁻}. F(k)))
| |      {split_int_seg_exist_dom_sfa}
| |      THEN
| |      BackThru:
| |      Thm* Dec(P) ⇒ Dec(Q) ⇒ Dec(P ∨ Q)    {decidable__or}
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| .\
| . 6. ∀k:{...j}. i≤k ⇒ Dec(∃k:{k..j⁻}. F(k))
| . ⊢  Dec(∃k:{i..j⁻}. F(k))  by BackThru: Hyp:-1
|
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. 5. ¬i≤j
. ⊢  Dec(∃k:{i..j⁻}. F(k))  by Select: 2
. |
. ⊢  ¬(∃k:{i..j⁻}. F(k))  by SimilarTo: Hyp:5 THEN Analyze-1
END
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