```
division4_sfa  THM

⊢  ∀k:ℕ⁺ , r1,r2:ℕk, q1,q2:ℤ.
|  q1·k+r1 = q2·k-r2
|  ⇒
|  q1 = q2 & r1 = 0 & r2 = 0 ∨ q2 = q1+1 & r2 = k-r1  by Auto
|
1. k : ℕ⁺
2. r1 : ℕk
3. r2 : ℕk
4. q1 : ℤ
5. q2 : ℤ
6. q1·k+r1 = q2·k-r2
⊢  q1 = q2 & r1 = 0 & r2 = 0 ∨ q2 = q1+1 & r2 = k-r1
| by Decide: r2 = 0 THENL [Select: 1;Select: 2]
|
|\
| 7. r2 = 0
| ⊢  q1 = q2 & r1 = 0 & r2 = 0
|   by FwdThru:
|      Thm* ∀k:ℕ⁺ , r1,r2:ℕk, q1,q2:ℤ.
|           q1·k+r1 = q2·k+r2 ⇒ q1 = q2 & r1 = r2
|      {division1_sfa}
|      on [ q1·k+r1 = q2·k+0 ]
|
.\
. 7. ¬r2 = 0
. ⊢  q2 = q1+1 & r2 = k-r1
.   by FwdThru:
.      Thm* ∀k:ℕ⁺ , r1,r2:ℕk, q1,q2:ℤ.
.           q1·k+r1 = q2·k+r2 ⇒ q1 = q2 & r1 = r2
.      {division1_sfa}
.      on [ q1·k+r1 = (q2-1)·k+k-r2 ]
END
```