```
pos_mul_arg_bounds  THM

⊢  ∀a,b:ℤ. a·b>0 ⟺ a>0 & b>0 ∨ a<0 & b<0  by Auto
|
|\
| 1. a : ℤ
| 2. b : ℤ
| 3. a·b>0
| ⊢  a>0 & b>0 ∨ a<0 & b<0
| | by Inst: Thm* ∀i,j:ℤ. i<j ∨ i = j ∨ i>j    {int_trichot}
| |    Using:[a | 0]
| |    THEN
| |    CaseSplitNWay 3 Hyp:-1
| |
| |\
| | 4. a<0
| | ⊢  a>0 & b>0 ∨ a<0 & b<0
| | | by Inst: Thm* ∀a,b:ℤ, n:ℕ+. n·a<n·b ⇒ a<b    {mul_cancel_in_lt}
| | |    Using:[b | 0 | -a]
| | |
| | |\
| | | .....antecedent .....
| | | ⊢  (-a)·b<(-a)·0  by Auto
| | |
| | .\
| | . 5. b<0
| | . ⊢  a>0 & b>0 ∨ a<0 & b<0  by Auto
| |
| |\
| | 4. a = 0
| | ⊢  a>0 & b>0 ∨ a<0 & b<0
| |   by False  Asserted THEN Rewrite3 by a = 0
| |
| .\
| . 4. a>0
| . ⊢  a>0 & b>0 ∨ a<0 & b<0
| . | by Inst: Thm* ∀a,b:ℤ, n:ℕ+. n·a<n·b ⇒ a<b    {mul_cancel_in_lt}
| . |    Using:[0 | b | a]
| . |
| . |\
| . | .....antecedent .....
| . | ⊢  a·0<a·b  by Auto
| . |
| . .\
| . . 5. 0<b
| . . ⊢  a>0 & b>0 ∨ a<0 & b<0  by Auto
|
.\
. 1. a : ℤ
. 2. b : ℤ
. 3. a>0 & b>0 ∨ a<0 & b<0
. ⊢  a·b>0  by Analyze-1
. |
. |\
. | 3. a>0 & b>0
```

```
. | ⊢  a·b>0
. |   by Inst: Thm* ∀a,b:ℕ⁺ . 0<a·b    {mul_bounds_1b} Using:[a | b]
. |
. .\
. . 3. a<0 & b<0
. . ⊢  a·b>0
. .   by Inst: Thm* ∀a,b:ℕ⁺ . 0<a·b    {mul_bounds_1b} Using:[-a | -b]
END
```