```
rem_addition  THM

⊢  ∀i,j:ℕ, n:ℕ+. (((i rem n)+(j rem n)) rem n) = ((i+j) rem n)
| by Auto
|
1. i : ℕ
2. j : ℕ
3. n : ℕ+
⊢  (((i rem n)+(j rem n)) rem n) = ((i+j) rem n)
| by ElaborateDivision Auto 0
|
4. (i rem n)+(j rem n)
|  =
|  (((i rem n)+(j rem n)) ÷ n)·n+(((i rem n)+(j rem n)) rem n)
5. i = (i ÷ n)·n+(i rem n)
6. j = (j ÷ n)·n+(j rem n)
7. i+j = ((i+j) ÷ n)·n+((i+j) rem n)
8. (i rem n) ∈ ℕn
9. (j rem n) ∈ ℕn
10. (((i rem n)+(j rem n)) rem n) ∈ ℕn
11. ((i+j) rem n) ∈ ℕn
⊢  (((i rem n)+(j rem n)) rem n) = ((i+j) rem n)
| by FwdThru:
|    Thm* ∀k:ℕ+, r1,r2:ℕk, q1,q2:ℤ.
|         q1·k+r1 = q2·k+r2 ⇒ q1 = q2 & r1 = r2
|    {division1_sfa}
|    on [ ((i ÷ n)+(j ÷ n)+(((i rem n)+(j rem n)) ÷ n))·n
|         +(((i rem n)+(j rem n)) rem n)
|         =
|         ((i+j) ÷ n)·n+((i+j) rem n) ]
|
|    {Auto would finish this now, but there would be too many steps
|     for easy reading. }
|
|
.....assertion .....
⊢  ((i ÷ n)+(j ÷ n)+(((i rem n)+(j rem n)) ÷ n))·n
   +(((i rem n)+(j rem n)) rem n)
   =
   ((i+j) ÷ n)·n+((i+j) rem n)
  by i+j = ((i ÷ n)+(j ÷ n))·n+(i rem n)+(j rem n)  Asserted
     {Hyp:5 Hyp:6 then rearrange }

     THEN
     ((i ÷ n)+(j ÷ n))·n+(i rem n)+(j rem n)
     =
     ((i ÷ n)+(j ÷ n)+(((i rem n)+(j rem n)) ÷ n))·n
     +(((i rem n)+(j rem n)) rem n)
     Asserted

     {Hyp:4 then rearrange }

END
```