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rem_antisym  THM

⊢  ∀a:ℤ, b:ℤ⁻⁰. ((-a) rem b) = -(a rem b)
| by SimilarTo ∀a:ℤ, b:ℕ⁺. ((-a) rem b) = -(a rem b)
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| 1. a : ℤ
| 2. b : ℕ⁺
| ⊢  ((-a) rem b) = -(a rem b)
| | by Decide: 0≤a THEN ElaborateDivision Auto 0
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| | 3. 0≤a
| | 4. -a = ((-a) ÷ b)·b+((-a) rem b)
| | 5. a = (a ÷ b)·b+(a rem b)
| | 6. -((-a) rem b) ∈ ℕb
| | 7. (a rem b) ∈ ℕb
| | ⊢  ((-a) rem b) = -(a rem b)
| |   by FwdThru:
| |      Thm* ∀k:ℕ⁺, r1,r2:ℕk, q1,q2:ℤ.
| |           q1·k+r1 = q2·k+r2 ⇒ q1 = q2 & r1 = r2
| |      {division1_sfa}
| |      on [ (-((-a) ÷ b))·b+-((-a) rem b) = (a ÷ b)·b+(a rem b) ]
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| .\
| . 3. ¬0≤a
| . 4. -a = ((-a) ÷ b)·b+((-a) rem b)
| . 5. a = (a ÷ b)·b+(a rem b)
| . 6. ((-a) rem b) ∈ ℕb
| . 7. -(a rem b) ∈ ℕb
| . ⊢  ((-a) rem b) = -(a rem b)
| .   by FwdThru:
| .      Thm* ∀k:ℕ⁺, r1,r2:ℕk, q1,q2:ℤ.
| .           q1·k+r1 = q2·k+r2 ⇒ q1 = q2 & r1 = r2
| .      {division1_sfa}
| .      on [ ((-a) ÷ b)·b+((-a) rem b) = (-(a ÷ b))·b+-(a rem b) ]
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. 1. a : ℤ
. 2. ∀b:ℕ⁺. ((-a) rem b) = -(a rem b)
. 3. b : ℤ⁻⁰
. ⊢  ((-a) rem b) = -(a rem b)  by Decide: 0<b
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. | 4. 0<b
. | ⊢  ((-a) rem b) = -(a rem b)  by BackThru: Hyp:2
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. . 4. ¬0<b
. . ⊢  ((-a) rem b) = -(a rem b)
. .   by Rewrite by
. .      Thm* ∀a:ℤ, b:ℤ⁻⁰. (a rem -b) = (a rem b)    {rem_sym}
. .      THEN
. .      BackThru: Hyp:2
END
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