```
rem_sym  THM

⊢  ∀a:ℤ, b:ℤ⁻⁰. (a rem -b) = (a rem b)
| by SimilarTo ∀a:ℤ, b:ℕ⁺. (a rem -b) = (a rem b)
|
|\
| 1. a : ℤ
| 2. b : ℕ⁺
| ⊢  (a rem -b) = (a rem b)  by ElaborateDivision Auto 0
| |
| 3. a = (a ÷ (-b))·(-b)+(a rem -b)
| 4. a = (a ÷ b)·b+(a rem b)
| ⊢  (a rem -b) = (a rem b)  by Decide: 0≤a THEN BoundRem Auto 0
| |
| |\
| | 5. 0≤a
| | 6. (a rem -b) ∈ ℕ(-(-b))
| | 7. (a rem b) ∈ ℕb
| | ⊢  (a rem -b) = (a rem b)
| |   by FwdThru:
| |      Thm* ∀k:ℕ⁺, r1,r2:ℕk, q1,q2:ℤ.
| |           q1·k+r1 = q2·k+r2 ⇒ q1 = q2 & r1 = r2
| |      {division1_sfa}
| |      on [ (a ÷ b)·b+(a rem b) = (-(a ÷ (-b)))·b+(a rem -b) ]
| |
| .\
| . 5. ¬0≤a
| . 6. -(a rem -b) ∈ ℕ(-(-b))
| . 7. -(a rem b) ∈ ℕb
| . ⊢  (a rem -b) = (a rem b)
| .   by FwdThru:
| .      Thm* ∀k:ℕ⁺, r1,r2:ℕk, q1,q2:ℤ.
| .           q1·k+r1 = q2·k+r2 ⇒ q1 = q2 & r1 = r2
| .      {division1_sfa}
| .      on [ (a ÷ (-b))·b+-(a rem -b) = (-(a ÷ b))·b+-(a rem b) ]
|
.\
. 1. a : ℤ
. 2. ∀b:ℕ⁺. (a rem -b) = (a rem b)
. 3. b : ℤ⁻⁰
. ⊢  (a rem -b) = (a rem b)  by Decide: 0<b
. |
. |\
. | 4. 0<b
. | ⊢  (a rem -b) = (a rem b)  by Rewrite by Hyp:2 Using:[b]
. |
. .\
. . 4. ¬0<b
. . ⊢  (a rem -b) = (a rem b)  by Rewrite by Hyp:2 Using:[-b]
END
```