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div_3_to_1  THM

⊢  ∀a:{...0}, b:{...-1}. (a ÷ b) = ((-a) ÷ (-b))  by UnivCD
|
1. a : {...0}
2. b : {...-1}
⊢  (a ÷ b) = ((-a) ÷ (-b))
| by (Inst Thm* ∀a:ℤ, n:ℤ^-0. a = (a ÷ n)·n+(a rem n)    {div_rem_sum}
|      [-a;-b])
|    THEN
|    (Inst
|     Thm* ∀a:ℕ, n:ℕ+. 0≤(a rem n) & (a rem n)<n    {rem_bounds_1}
|     [-a;-b])
|    THEN
|    (Inst Thm* ∀a:ℤ, n:ℤ^-0. a = (a ÷ n)·n+(a rem n)    {div_rem_sum}
|     [a;b])
|    THEN
|    (Inst
|     Thm* ∀a:{...0}, n:{...-1}. 0≥(a rem n) & (a rem n)>n
|     {rem_bounds_3}
|     [a;b])
|
3. -a = ((-a) ÷ (-b))·(-b)+((-a) rem -b)
4. 0≤((-a) rem -b) & ((-a) rem -b)<-b
5. a = (a ÷ b)·b+(a rem b)
6. 0≥(a rem b) & (a rem b)>b
⊢  (a ÷ b) = ((-a) ÷ (-b))  by Analyze 6 THEN Analyze 4
|
4. 0≤((-a) rem -b)
5. ((-a) rem -b)<-b
6. a = (a ÷ b)·b+(a rem b)
7. 0≥(a rem b)
8. (a rem b)>b
⊢  (a ÷ b) = ((-a) ÷ (-b))
| by (Using [‘n’,(b·((-a) ÷ (-b)))]
|      (FwdThru Thm* ∀a,b,n:ℤ. a = b ⟺ a+n = b+n    {add_mono_wrt_eq}
|       [3]))
|    THEN
|    (Using [‘n’,((-b)·(a ÷ b))]
|     (FwdThru Thm* ∀a,b,n:ℤ. a = b ⟺ a+n = b+n    {add_mono_wrt_eq}
|      [6]))
|    THEN
|    (OnMHyps [-1;-2] ArithSimp)
|    THEN
|    (OnHyps [6;3] Thin)
|
3. 0≤((-a) rem -b)
4. ((-a) rem -b)<-b
5. 0≥(a rem b)
6. (a rem b)>b
7. -a+b·((-a) ÷ (-b)) = ((-a) rem -b)
8. a+-b·(a ÷ b) = (a rem b)
⊢  (a ÷ b) = ((-a) ÷ (-b))
| by (RevHypSubst 7 3) THEN (RevHypSubst 7 4) THEN (RevHypSubst 8 5)
|    THEN
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|    (RevHypSubst 8 6)
|    THEN
|    (OnHyps [8;7] Thin)
|
3. 0≤-a+b·((-a) ÷ (-b))
4. -a+b·((-a) ÷ (-b))<-b
5. 0≥a+-b·(a ÷ b)
6. a+-b·(a ÷ b)>b
⊢  (a ÷ b) = ((-a) ÷ (-b))
| by (Using [‘n’,a]
|     (FwdThru Thm* ∀a,b,n:ℤ. a≤b ⟺ a+n≤b+n    {add_mono_wrt_le}
|      [3]))
|    THEN
|    (Using [‘n’,(a+b)]
|     (FwdThru Thm* ∀a,b,n:ℤ. a<b ⟺ a+n<b+n    {add_mono_wrt_lt}
|      [4]))
|    THEN
|    (Using [‘n’,(b·(a ÷ b))]
|     (FwdThru Thm* ∀a,b,n:ℤ. a≤b ⟺ a+n≤b+n    {add_mono_wrt_le}
|      [5]))
|    THEN
|    (Using [‘n’,(b·(a ÷ b))]
|     (FwdThru Thm* ∀a,b,n:ℤ. a<b ⟺ a+n<b+n    {add_mono_wrt_lt}
|      [6]))
|    THEN
|    (OnHyps [6;5;4;3] Thin)
|    THEN
|    (OnMHyps [6;5;4;3] ArithSimp)
|
3. a≤b·((-a) ÷ (-b))
4. b+b·((-a) ÷ (-b))<a
5. a≤b·(a ÷ b)
6. b+b·(a ÷ b)<a
⊢  (a ÷ b) = ((-a) ÷ (-b))
| by (FwdThru Thm* ∀i,j,k:ℤ. i<j ⇒ j≤k ⇒ i<k    {lt_transitivity_1}
|     [6;3])
|    THEN
|    (FwdThru Thm* ∀i,j,k:ℤ. i<j ⇒ j≤k ⇒ i<k    {lt_transitivity_1}
|     [4;5])
|    THEN
|    (OnHyps [6;5;4;3] Thin)
|
3. b+b·(a ÷ b)<b·((-a) ÷ (-b))
4. b+b·((-a) ÷ (-b))<b·(a ÷ b)
⊢  (a ÷ b) = ((-a) ÷ (-b))
| by AssertL
|    [(-b)·(-((a ÷ b)+1))<(-b)·(-((-a) ÷ (-b)))
|    ;(-b)·(-(1+((-a) ÷ (-b))))<(-b)·(-(a ÷ b))]
|    THENA
|    Try Arith
|
5. (-b)·(-((a ÷ b)+1))<(-b)·(-((-a) ÷ (-b)))
6. (-b)·(-(1+((-a) ÷ (-b))))<(-b)·(-(a ÷ b))
⊢  (a ÷ b) = ((-a) ÷ (-b))
| by (FwdThru Thm* ∀a,b:ℤ, n:ℕ+ . n·a<n·b ⇒ a<b    {mul_cancel_in_lt}
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|     [5])
|    THEN
|    (FwdThru Thm* ∀a,b:ℤ, n:ℕ+ . n·a<n·b ⇒ a<b    {mul_cancel_in_lt}
|     [6])
|    THEN
|    (OnHyps [6;5;4;3] Thin)
|
3. -((a ÷ b)+1)<-((-a) ÷ (-b))
4. -(1+((-a) ÷ (-b)))<-(a ÷ b)
⊢  (a ÷ b) = ((-a) ÷ (-b))
| by (FwdThru
|     Thm* ∀i,j:ℤ. i>j ⇒ -i<-j    {minus_functionality_wrt_lt}
|     [3])
|    THEN
|    (FwdThru
|     Thm* ∀i,j:ℤ. i>j ⇒ -i<-j    {minus_functionality_wrt_lt}
|     [4])
|    THEN
|    (ArithSimp 5)
|    THEN
|    (ArithSimp 6)
|    THEN
|    (OnHyps [4;3] Thin)
|
3. ((-a) ÷ (-b))<1+(a ÷ b)
4. (a ÷ b)<1+((-a) ÷ (-b))
⊢  (a ÷ b) = ((-a) ÷ (-b))  by Arith
END
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