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div_4_to_1  THM

⊢  ∀a:ℕ, b:{...-1}. (a ÷ b) = -(a ÷ (-b))  by UnivCD
|
1. a : ℕ
2. b : {...-1}
⊢  (a ÷ b) = -(a ÷ (-b))
| by (Inst Thm* ∀a:ℤ, n:ℤ^-0. a = (a ÷ n)·n+(a rem n)    {div_rem_sum}
|      [a;-b])
|     THEN
|     (Inst
|      Thm* ∀a:ℕ, n:ℕ^+. 0≤(a rem n) & (a rem n)<n    {rem_bounds_1}
|      [a;-b])
|     THEN
|     (Inst Thm* ∀a:ℤ, n:ℤ^-0. a = (a ÷ n)·n+(a rem n)    {div_rem_sum}
|      [a;b])
|     THEN
|     (Inst
|      Thm* ∀a:ℕ, n:{...-1}. 0≤(a rem n) & (a rem n)<-n
|      {rem_bounds_4}
|      [a;b])
|
3. a = (a ÷ (-b))·(-b)+(a rem -b)
4. 0≤(a rem -b) & (a rem -b)<-b
5. a = (a ÷ b)·b+(a rem b)
6. 0≤(a rem b) & (a rem b)<-b
⊢  (a ÷ b) = -(a ÷ (-b))  by Analyze 6 THEN Analyze 4
|
4. 0≤(a rem -b)
5. (a rem -b)<-b
6. a = (a ÷ b)·b+(a rem b)
7. 0≤(a rem b)
8. (a rem b)<-b
⊢  (a ÷ b) = -(a ÷ (-b))
| by (Using [‘n’,(b·(a ÷ (-b)))]
|      (FwdThru Thm* ∀a,b,n:ℤ. a = b ⟺ a+n = b+n    {add_mono_wrt_eq}
|       [3]))
|     THEN
|     (Using [‘n’,((-b)·(a ÷ b))]
|      (FwdThru Thm* ∀a,b,n:ℤ. a = b ⟺ a+n = b+n    {add_mono_wrt_eq}
|       [6]))
|     THEN
|     (OnMHyps [-1;-2] ArithSimp)
|     THEN
|     (OnHyps [6;3] Thin)
|
3. 0≤(a rem -b)
4. (a rem -b)<-b
5. 0≤(a rem b)
6. (a rem b)<-b
7. a+b·(a ÷ (-b)) = (a rem -b)
8. a+-b·(a ÷ b) = (a rem b)
⊢  (a ÷ b) = -(a ÷ (-b))
| by (RevHypSubst 7 3) THEN (RevHypSubst 7 4) THEN (RevHypSubst 8 5)
|     THEN
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|    (RevHypSubst 8 6)
|    THEN
|    (OnHyps [8;7] Thin)
|
3. 0≤a+b·(a ÷ (-b))
4. a+b·(a ÷ (-b))<-b
5. 0≤a+-b·(a ÷ b)
6. a+-b·(a ÷ b)<-b
⊢  (a ÷ b) = -(a ÷ (-b))
| by (Using [‘n’,((-b)·(a ÷ (-b)))]
|     (FwdThru Thm* ∀a,b,n:ℤ. a≤b ⟺ a+n≤b+n    {add_mono_wrt_le}
|      [3]))
|    THEN
|    (Using [‘n’,((-b)·(a ÷ (-b)))]
|     (FwdThru Thm* ∀a,b,n:ℤ. a<b ⟺ a+n<b+n    {add_mono_wrt_lt}
|      [4]))
|    THEN
|    (Using [‘n’,(b·(a ÷ b))]
|     (FwdThru Thm* ∀a,b,n:ℤ. a≤b ⟺ a+n≤b+n    {add_mono_wrt_le}
|      [5]))
|    THEN
|    (Using [‘n’,(b·(a ÷ b))]
|     (FwdThru Thm* ∀a,b,n:ℤ. a<b ⟺ a+n<b+n    {add_mono_wrt_lt}
|      [6]))
|    THEN
|    (OnHyps [6;5;4;3] Thin)
|    THEN
|    (OnMHyps [6;5;4;3] ArithSimp)
|
3. -b·(a ÷ (-b))≤a
4. a<-b+-b·(a ÷ (-b))
5. b·(a ÷ b)≤a
6. a<-b+b·(a ÷ b)
⊢  (a ÷ b) = -(a ÷ (-b))
| by (FwdThru Thm* ∀i,j,k:ℤ. i≤j ⇒ j<k ⇒ i<k    {lt_transitivity_2}
|     [6;3])
|    THEN
|    (FwdThru Thm* ∀i,j,k:ℤ. i≤j ⇒ j<k ⇒ i<k    {lt_transitivity_2}
|     [4;5])
|    THEN
|    (OnHyps [6;5;4;3] Thin)
|
3. -b·(a ÷ (-b))<-b+b·(a ÷ b)
4. b·(a ÷ b)<-b+-b·(a ÷ (-b))
⊢  (a ÷ b) = -(a ÷ (-b))
| by AssertL
|    [(-b)·(a ÷ (-b))<(-b)·(1+-(a ÷ b))

|    ;(-b)·(-(a ÷ b))<(-b)·(1+(a ÷ (-b)))]
|    THENA
|    Arith
|
5. (-b)·(a ÷ (-b))<(-b)·(1+-(a ÷ b))
6. (-b)·(-(a ÷ b))<(-b)·(1+(a ÷ (-b)))
⊢  (a ÷ b) = -(a ÷ (-b))
| by (FwdThru Thm* ∀a,b:ℤ, n:ℕ+. n·a<n·b ⇒ a<b    {mul_cancel_in_lt}
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|      [5])
|     THEN
|     (FwdThru Thm* ∀a,b:ℤ, n:ℕ+. n·a<n·b ⇒ a<b    {mul_cancel_in_lt}
|      [6])
|     THEN
|     (OnHyps [6;5;4;3] Thin)
|
3. (a ÷ (-b))<1+-(a ÷ b)
4. -(a ÷ b)<1+(a ÷ (-b))
⊢  (a ÷ b) = -(a ÷ (-b))
| by (FwdThru
|      Thm* ∀i,j:ℤ. i>j ⇒ -i<-j    {minus_functionality_wrt_lt}
|      [3])
|     THEN
|     (FwdThru
|      Thm* ∀i,j:ℤ. i>j ⇒ -i<-j    {minus_functionality_wrt_lt}
|      [4])
|
5. -(1+-(a ÷ b))<-(a ÷ (-b))
6. -(1+(a ÷ (-b)))<-(-(a ÷ b))
⊢  (a ÷ b) = -(a ÷ (-b))  by Auto
END
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