```
div_fun_sat_div_nrel  THM

⊢  ∀a:ℕ, n:ℕ+. Div(a;n;a ÷ n)
| by Unfold ‘div_nrel‘ 0 THEN UnivFmlaCD []
|
1. a : ℕ
2. n : ℕ+
⊢  n·(a ÷ n) ≤ a < n·((a ÷ n)+1)
| by Inst Thm* ∀a:ℕ, n:ℕ+. 0≤(a rem n) & (a rem n)<n    {rem_bounds_1}
|    [a;n]
|
3. 0≤(a rem n) & (a rem n)<n
⊢  n·(a ÷ n) ≤ a < n·((a ÷ n)+1)  by Analyze 3
|
3. 0≤(a rem n)
4. (a rem n)<n
⊢  n·(a ÷ n) ≤ a < n·((a ÷ n)+1)
| by Rewrite
|    (GenLemmaWithThenLC 1 [‘n’,((a ÷ n)·n)] []
|     Thm* ∀a,b,n:ℤ. a≤b ⟺ a+n≤b+n    {add_mono_wrt_le_rw})
|    3
|
3. 0+(a ÷ n)·n≤(a rem n)+(a ÷ n)·n
4. (a rem n)<n
⊢  n·(a ÷ n) ≤ a < n·((a ÷ n)+1)
| by Rewrite
|    (GenLemmaWithThenLC 1 [‘n’,((a ÷ n)·n)] []
|     Thm* ∀a,b,n:ℤ. a<b ⟺ a+n<b+n    {add_mono_wrt_lt_rw})
|    4
|
4. (a rem n)+(a ÷ n)·n<n+(a ÷ n)·n
⊢  n·(a ÷ n) ≤ a < n·((a ÷ n)+1)
| by Rewrite
|    (NthC 2 (LemmaC Thm* ∀a,b:ℤ. a+b = b+a    {add_com})
|     THENC
|     HigherC
|     (RevLemmaC
|      Thm* ∀a:ℤ, n:ℤ-0. a = (a ÷ n)·n+(a rem n)    {div_rem_sum}))
|    3
|
3. 0+(a ÷ n)·n≤a
4. (a rem n)+(a ÷ n)·n<n+(a ÷ n)·n
⊢  n·(a ÷ n) ≤ a < n·((a ÷ n)+1)
| by Rewrite
|    (NthC 1 (LemmaC Thm* ∀a,b:ℤ. a+b = b+a    {add_com})
|     THENC
|     HigherC
|     (RevLemmaC
|      Thm* ∀a:ℤ, n:ℤ-0. a = (a ÷ n)·n+(a rem n)    {div_rem_sum}))
|    4
|
4. a<n+(a ÷ n)·n
⊢  n·(a ÷ n) ≤ a < n·((a ÷ n)+1)  by Analyze 0
END
```