```
divide_wf  THM

⊢  ∀a:ℕ, n:ℕ+. (a ÷ n) ∈ ℕ  by UnivFmlaCD []
|
1. a : ℕ
2. n : ℕ+
⊢  (a ÷ n) ∈ ℕ  by UnfoldTopAb 0 THEN AbSetEqTypeCD
|
|\
| ⊢  (a ÷ n) = (a ÷ n)  by PrimEqCD
|
|\
| .....set predicate .....
| ⊢  0≤(a ÷ n)  by BackThru Thm* ∀i,j:ℤ. i≤j ⟺ i<j+1    {le_to_lt}
| |
| ⊢  0<(a ÷ n)+1
| | by Using [‘n’,n]
| |    (BackThru
| |     Thm* ∀a,b:ℤ, n:ℕ+. n·a<n·b ⇒ a<b    {mul_cancel_in_lt})
| |
| ⊢  n·0<n·((a ÷ n)+1)  by ArithSimp 0
| |
| ⊢  0<n+n·(a ÷ n)
| | by Assert
| |    (0≤n·(a ÷ n)+(a rem n) & n·(a ÷ n)+(a rem n)<n+n·(a ÷ n))
| |
| |\
| | .....assertion .....
| | ⊢  0≤n·(a ÷ n)+(a rem n) & n·(a ÷ n)+(a rem n)<n+n·(a ÷ n)
| | | by Analyze 0
| | |
| | |\
| | | ⊢  0≤n·(a ÷ n)+(a rem n)
| | |   by Inst
| | |      Thm* ∀a:ℤ, n:ℤ⁻⁰. a = (a ÷ n)·n+(a rem n)    {div_rem_sum}
| | |      [a;n]
| | |
| | .\
| | . ⊢  n·(a ÷ n)+(a rem n)<n+n·(a ÷ n)
| | . | by Assert ((a rem n)+n·(a ÷ n)<n+n·(a ÷ n))
| | . |
| | . .....assertion .....
| | . ⊢  (a rem n)+n·(a ÷ n)<n+n·(a ÷ n)
| | . | by BackThru
| | . |    Thm* ∀i1,i2,j1,j2:ℤ. i1<j1 ⇒ i2≤j2 ⇒ i1+i2<j1+j2
| | . |    {add_functionality_wrt_lt}
| | . |
| | . |\
| | . | ⊢  (a rem n)<n
| | . |   by BackThru
| | . |      Thm* ∀a:ℕ, n:ℕ+. 0≤(a rem n) & (a rem n)<n
| | . |      {rem_bounds_1}
| | . |
| | . .\
| | . . ⊢  n·(a ÷ n)≤n·(a ÷ n)  by Auto
```

```
| |
| .\
| . 3. 0≤n·(a ÷ n)+(a rem n) & n·(a ÷ n)+(a rem n)<n+n·(a ÷ n)
| . ⊢  0<n+n·(a ÷ n)  by Auto
|
.\
. .....wf .....
. 3. i : ℤ
. ⊢  (0≤i) = (0≤i)  by Auto
END
```