```
pos_mul_arg_bounds  THM

⊢  ∀a,b:ℤ. a·b>0 ⟺ a>0 & b>0 ∨ a<0 & b<0  by GenRepD
|
|\
| 1. a : ℤ
| 2. b : ℤ
| 3. a·b>0
| ⊢  a>0 & b>0 ∨ a<0 & b<0
| | by (Inst Thm* ∀i,j:ℤ. i<j ∨ i = j ∨ i>j    {int_trichot} [a;0])
| |    THEN
| |    GenExRepD
| |
| |\
| | 4. a<0
| | ⊢  a>0 & b>0 ∨ a<0 & b<0  by Sel 2 (Analyze 0)
| | |
| | ⊢  b<0
| | | by ReplaceWithEqv (TryC (HigherC IntSimpC)) ((-a)·(-b)>(-a)·0)
| | |    3
| | |
| | 3. (-a)·(-b)>(-a)·0
| | 4. a<0
| | ⊢  b<0
| |   by FwdThru
| |      Thm* ∀a,b:ℤ, n:ℕ+. n·a<n·b ⇒ a<b    {mul_cancel_in_lt}
| |      [3]
| |
| |\
| | 4. a = 0
| | ⊢  a>0 & b>0 ∨ a<0 & b<0  by HypSubst 4 3
| |
| .\
| . 4. a>0
| . ⊢  a>0 & b>0 ∨ a<0 & b<0  by Sel 1 (Analyze 0)
| . |
| . ⊢  b>0  by ReplaceWithEqv (TryC (HigherC IntSimpC)) (a·b>a·0) 3
| . |
| . 3. a·b>a·0
| . 4. a>0
| . ⊢  b>0
| .   by FwdThru
| .      Thm* ∀a,b:ℤ, n:ℕ+. n·a<n·b ⇒ a<b    {mul_cancel_in_lt}
| .      [3]
|
.\
. 1. a : ℤ
. 2. b : ℤ
. 3. a>0 & b>0 ∨ a<0 & b<0
. ⊢  a·b>0  by GenExRepD
. |
. |\
. | 3. a>0
. | 4. b>0
. | ⊢  a·b>0  by BackThru Thm* ∀a,b:ℕ+. 0<a·b    {mul_bounds_1b}
```

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. |
. .\
. . 3. a<0
. . 4. b<0
. . ⊢  a·b>0
. . | by ReplaceWithEqv (TryC (HigherC IntSimpC)) ((-a)·(-b)>0) 0
. . |
. . ⊢  (-a)·(-b)>0
. .   by BackThru Thm* ∀a,b:ℕ+. 0<a·b    {mul_bounds_1b}
END
```