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all_intseg_vs_iter_and  THM

⊢  ∀a,b:ℤ, P:({a..b⁻}→Prop).
|  (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i))  by Analyze
|
1. a : ℤ
⊢  ∀b:ℤ, P:({a..b⁻}→Prop).
|  (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i))
| by ∀b:ℤ, P:({a..b⁻}→Prop).
|    b≤a ⇒ ((∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i))) |
|    ∀k:ℕ, b:ℤ.
|    b-a = k
|    ⇒
|    (∀P:({a..b⁻}→Prop). (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i)))
|    Asserted
|
|\
| .....assertion .....
| ⊢  ∀b:ℤ, P:({a..b⁻}→Prop).
| |  b≤a ⇒ ((∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i)))  by UnivCD
| |
| 2. b : ℤ
| 3. P : {a..b⁻}→Prop
| 4. b≤a
| ⊢  (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i))
|   by Rewrite by
|      Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
|            b≤a ⇒ (Iter(f;u) i:{a..b⁻}. e(i)) = u
|      {iter_via_intseg_null}
|
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| .....assertion .....
| 2. ∀b:ℤ, P:({a..b⁻}→Prop).
| |  b≤a ⇒ ((∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i)))
| ⊢  ∀k:ℕ, b:ℤ.
| |  b-a = k
| |  ⇒
| |  (∀P:({a..b⁻}→Prop). (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i)))
| | by NatInd Concl THEN UnivCD
| |
| |\
| | 3. b : ℤ
| | 4. b-a = 0
| | 5. P : {a..b⁻}→Prop
| | ⊢  (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i))
| |   by Guarding (<prop> ⟺ <prop>) BackThru: Hyp:2
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| .\
| . 3. k : ℤ
| . 4. 0<k
| . 5. ∀b:ℤ.
| . |  b-a = k-1
| . |  ⇒
| . |  (∀P:({a..b⁻}→Prop).
| . |   (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i)))
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| . 6. b : ℤ
| . 7. b-a = k
| . 8. P : {a..b⁻}→Prop
| . ⊢  (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i))
| . | by Rewrite by
| . |    Thm*  ∀a,b:ℤ, P:({a..b⁻}→Prop).
| . |          a<b
| . |          ⇒
| . |          ((∀i:{a..b⁻}. P(i)) ⟺ (∀i:{a..(b-1)⁻}. P(i)) & P(b-1))
| . |    {and_via_all_intseg_split_last}
| . |    THEN
| . |    Rewrite by
| . |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
| . |          a<b
| . |          ⇒
| . |          (Iter(f;u) i:{a..b⁻}. e(i))
| . |          =
| . |          f((Iter(f;u) i:{a..b-1⁻}. e(i)),e(b-1))
| . |    {iter_via_intseg_split_last}
| . |    THEN
| . |    Reduce Concl
| . |
| . ⊢  (∀i:{a..(b-1)⁻}. P(i)) & P(b-1)
| .    ⟺
| .    (And i:{a..b-1⁻}. P(i)) & P(b-1)  by Rewrite by Hyp:5
|
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. 2. ∀b:ℤ, P:({a..b⁻}→Prop).
. |  b≤a ⇒ ((∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i)))
. 3. ∀k:ℕ, b:ℤ.
. |  b-a = k
. |  ⇒
. |  (∀P:({a..b⁻}→Prop). (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i)))
. ⊢  ∀b:ℤ, P:({a..b⁻}→Prop).
. |  (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i))  by UnivCD
. |
. 4. b : ℤ
. 5. P : {a..b⁻}→Prop
. ⊢  (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i))  by Decide: b≤a
. |
. |\
. | 6. b≤a
. | ⊢  (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i))
. |   by Guarding (<prop> ⟺ <prop>) BackThru: Hyp:2
. |
. .\
. . 6. ¬b≤a

. . ⊢  (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i))
. .   by Guarding (<prop> ⟺ <prop>) BackThru: Hyp:3 Using:[b-a]
END
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