```
exist_intseg_vs_iter_or  THM

⊢  ∀a,b:ℤ, P:({a..b⁻}→Prop).
|  (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i))  by Analyze
|
1. a : ℤ
⊢  ∀b:ℤ, P:({a..b⁻}→Prop). (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i))
| by ∀b:ℤ, P:({a..b⁻}→Prop).
|    b≤a ⇒ ((∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i))) |
|    ∀k:ℕ, b:ℤ.
|    b-a = k
|    ⇒
|    (∀P:({a..b⁻}→Prop). (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i)))
|    Asserted
|
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| .....assertion .....
| ⊢  ∀b:ℤ, P:({a..b⁻}→Prop).
| |  b≤a ⇒ ((∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i)))  by UnivCD
| |
| 2. b : ℤ
| 3. P : {a..b⁻}→Prop
| 4. b≤a
| ⊢  (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i))
| | by Rewrite by
| |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
| |          b≤a ⇒ (Iter(f;u) i:{a..b⁻}. e(i)) = u
| |    {iter_via_intseg_null}
| |
| 5. ∃i:{a..b⁻}. P(i)
| ⊢  False
|   by FwdThru:
|      Thm*  ∀a,b:ℤ, P:({a..b⁻}→Prop). b≤a ⇒ ¬(∃i:{a..b⁻}. P(i))
|      {exist_intseg_null}
|      on [ Hyp:4 | Hyp:-1 ]
|
|\
| .....assertion .....
| 2. ∀b:ℤ, P:({a..b⁻}→Prop).
| |  b≤a ⇒ ((∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i)))
| ⊢  ∀k:ℕ, b:ℤ.
| |  b-a = k
| |  ⇒
| |  (∀P:({a..b⁻}→Prop). (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i)))
| | by NatInd Concl THEN UnivCD
| |
| |\
| | 3. b : ℤ
| | 4. b-a = 0
| | 5. P : {a..b⁻}→Prop
| | ⊢  (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i))
| |   by Guarding (<prop> ⟺ <prop>) BackThru: Hyp:2
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| . 3. k : ℤ
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| . 4. 0<k
| . 5. ∀b:ℤ.
| . |  b-a = k-1
| . |  ⇒
| . |  (∀P:({a..b⁻}→Prop).
| . |   (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i)))
| . 6. b : ℤ
| . 7. b-a = k
| . 8. P : {a..b⁻}→Prop
| . ⊢  (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i))
| . | by Rewrite by
| . |    Thm*  ∀a,b:ℤ, P:({a..b⁻}→Prop).
| . |          a<b
| . |          ⇒
| . |          ((∃i:{a..b⁻}. P(i)) ⟺ (∃i:{a..(b-1)⁻}. P(i)) ∨ P(b-1))
| . |    {or_via_exist_intseg_split_last}
| . |    THEN
| . |    Rewrite by
| . |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
| . |          a<b
| . |          ⇒
| . |          (Iter(f;u) i:{a..b⁻}. e(i))
| . |          =
| . |          f((Iter(f;u) i:{a..b-1⁻}. e(i)),e(b-1))
| . |    {iter_via_intseg_split_last}
| . |    THEN
| . |    Reduce Concl
| . |
| . ⊢  (∃i:{a..(b-1)⁻}. P(i)) ∨ P(b-1)
| .    ⟺
| .    (Or i:{a..b-1⁻}. P(i)) ∨ P(b-1)  by Rewrite by Hyp:5
|
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. 2. ∀b:ℤ, P:({a..b⁻}→Prop).
. |  b≤a ⇒ ((∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i)))
. 3. ∀k:ℕ, b:ℤ.
. |  b-a = k
. |  ⇒
. |  (∀P:({a..b⁻}→Prop). (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i)))
. ⊢  ∀b:ℤ, P:({a..b⁻}→Prop).
. |  (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i))  by UnivCD
. |
. 4. b : ℤ
. 5. P : {a..b⁻}→Prop
. ⊢  (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i))  by Decide: b≤a
. |
. |\
. | 6. b≤a
. | ⊢  (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i))
. |   by Guarding (<prop> ⟺ <prop>) BackThru: Hyp:2
. |
. .\
. . 6. ¬b≤a
. . ⊢  (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i))
. .   by Guarding (<prop> ⟺ <prop>) BackThru: Hyp:3 Using:[b-a]
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END