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iter_nat_prod_one_iff_factors_one  THM

⊢  ∀e:({a..b⁻}→ℕ). (Π i:{a..b⁻}. e(i)) = 1 ⟺ (∀i:{a..b⁻}. e(i) = 1)
| by Thm*  ∀a,b:ℤ, P:({a..b⁻}→Prop).
|          (∀i:{a..b⁻}. P(i)) ⟺ (And i:{a..b⁻}. P(i))
|    {all_intseg_vs_iter_and}
|    Asserted
|    THEN
|    Guarding (∀i:<type>. <prop>) Rewrite by Hyp:-1 THEN Thin Hyp:-1
|    THEN
|    Analyze
|
1. a : ℤ
⊢  ∀b:ℤ, e:({a..b⁻}→ℕ).
|  (Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1)
| by ∀b:ℤ, e:({a..b⁻}→ℕ).
|    a≤b ⇒ ((Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1))
|    Asserted
|    THEN
|    UnivCD
|
|\
| .....assertion .....
| ⊢  ∀b:ℤ, e:({a..b⁻}→ℕ).
| |  a≤b ⇒ ((Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1))
| | by ∀k:ℕ, b:ℤ, e:({a..b⁻}→ℕ).
| |    b-a = k
| |    ⇒
| |    ((Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1))
| |    Asserted
| |    THEN
| |    UnivCD
| |
| |\
| | .....assertion .....
| | ⊢  ∀k:ℕ, b:ℤ, e:({a..b⁻}→ℕ).
| | |  b-a = k
| | |  ⇒
| | |  ((Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1))
| | | by NatInd Concl THEN UnivCD
| | |
| | |\
| | | 2. b : ℤ
| | | 3. e : {a..b⁻}→ℕ
| | | 4. b-a = 0
| | | ⊢  (Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1)
| | | | by Rewrite by
| | | |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
| | | |          b≤a ⇒ (Iter(f;u) i:{a..b⁻}. e(i)) = u
| | | |    {iter_via_intseg_null}
| | | |
| | | ⊢  1 = 1 ⟺ True  by Auto
| | |
| | .\
| | . 2. k : ℤ
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| | . 3. 0<k
| | . 4. ∀b:ℤ, e:({a..b⁻}→ℕ).
| | . |  b-a = k-1
| | . |  ⇒
| | . |  ((Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1))
| | . 5. b : ℤ
| | . 6. e : {a..b⁻}→ℕ
| | . 7. b-a = k
| | . ⊢  (Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1)
| | . | by Rewrite by
| | . |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
| | . |          a<b
| | . |          ⇒
| | . |          (Iter(f;u) i:{a..b⁻}. e(i))
| | . |          =
| | . |          f((Iter(f;u) i:{a..b-1⁻}. e(i)),e(b-1))
| | . |    {iter_via_intseg_split_last}
| | . |    THEN
| | . |    Reduce Concl
| | . |
| | . ⊢  (Π i:{a..b-1⁻}. e(i))•e(b-1) = 1
| | . |  ⟺
| | . |  (And i:{a..b-1⁻}. e(i) = 1) & e(b-1) = 1
| | . | by Rewrite by
| | . |    Thm*  ∀x,y:ℕ. x•y = 1 ⟺ x = 1 & y = 1
| | . |    {nat_prod_one_iff_factors_one} ...wℕ
| | . |
| | . ⊢  (Π i:{a..b-1⁻}. e(i)) = 1 & e(b-1) = 1
| | .    ⟺
| | .    (And i:{a..b-1⁻}. e(i) = 1) & e(b-1) = 1
| | .   by Rewrite by Hyp:4 ...ℕ
| |
| .\
| . 2. ∀k:ℕ, b:ℤ, e:({a..b⁻}→ℕ).
| .    b-a = k
| .    ⇒
| .    ((Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1))
| . 3. b : ℤ
| . 4. e : {a..b⁻}→ℕ
| . 5. a≤b
| . ⊢  (Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1)
| .   by Guarding (<prop> ⟺ <prop>) BackThru: Hyp:2 Using:[b-a]
|
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. 2. ∀b:ℤ, e:({a..b⁻}→ℕ).
. |  a≤b ⇒ ((Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1))
. 3. b : ℤ

. 4. e : {a..b⁻}→ℕ
. ⊢  (Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1)
. | by Decide: a≤b
. |
. |\
. | 5. a≤b
. | ⊢  (Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1)
. |   by Guarding (<prop> ⟺ <prop>) BackThru: Hyp:2
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. . 5. ¬a≤b
. . ⊢  (Π i:{a..b⁻}. e(i)) = 1 ⟺ (And i:{a..b⁻}. e(i) = 1)
. . | by Rewrite by
. . |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
. . |          b≤a
. . |          ⇒
. . |          (Iter(f;u) i:{a..b⁻}. e(i))
. . |          =
. . |          (Iter(f;u) i:{a..a⁻}. e(i))
. . |    {iter_via_intseg_nullnorm}
. . |
. . ⊢  (Π i:{a..a⁻}. e(i)) = 1 ⟺ (And i:{a..a⁻}. e(i) = 1)
. .   by Guarding (<prop> ⟺ <prop>) BackThru: Hyp:2
END
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