```
iter_prod_zero_iff_factor_zero  THM

⊢  (Π i:{a..b⁻}. e(i)) = 0 ⟺ (∃i:{a..b⁻}. e(i) = 0)
| by Rewrite by
|    Thm*  ∀a,b:ℤ, P:({a..b⁻}→Prop).
|          (∃i:{a..b⁻}. P(i)) ⟺ (Or i:{a..b⁻}. P(i))
|    {exist_intseg_vs_iter_or}
|    THEN
|    Analyze
|
1. a : ℤ
⊢  ∀b:ℤ, e:({a..b⁻}→ℤ).
|  (Π i:{a..b⁻}. e(i)) = 0 ⟺ (Or i:{a..b⁻}. e(i) = 0)
| by ∀b:ℤ, e:({a..b⁻}→ℤ).
|    a≤b ⇒ ((Π i:{a..b⁻}. e(i)) = 0 ⟺ (Or i:{a..b⁻}. e(i) = 0))
|    Asserted
|    THEN
|    UnivCD
|
|\
| .....assertion .....
| ⊢  ∀b:ℤ, e:({a..b⁻}→ℤ).
| |  a≤b ⇒ ((Π i:{a..b⁻}. e(i)) = 0 ⟺ (Or i:{a..b⁻}. e(i) = 0))
| | by ∀k:ℕ, b:ℤ, e:({a..b⁻}→ℤ).
| |    b-a = k
| |    ⇒
| |    ((Π i:{a..b⁻}. e(i)) = 0 ⟺ (Or i:{a..b⁻}. e(i) = 0))
| |    Asserted
| |    THEN
| |    (UnivCD
| |     THEN
| |     Guarding (<prop> ⟺ <prop>) BackThru: Hyp:2 Using:[b-a])
| |
| .....assertion .....
| ⊢  ∀k:ℕ, b:ℤ, e:({a..b⁻}→ℤ).
| |  b-a = k ⇒ ((Π i:{a..b⁻}. e(i)) = 0 ⟺ (Or i:{a..b⁻}. e(i) = 0))
| | by Guarding (<prop> ⟺ <prop>) (NatInd Concl)
| |
| |\
| | 2. b : ℤ
| | 3. e : {a..b⁻}→ℤ
| | 4. b-a = 0
| | ⊢  (Π i:{a..b⁻}. e(i)) = 0 ⟺ (Or i:{a..b⁻}. e(i) = 0)
| | | by Rewrite by
| | |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
| | |          b≤a ⇒ (Iter(f;u) i:{a..b⁻}. e(i)) = u
| | |    {iter_via_intseg_null}
| | |
| | ⊢  1 = 0 ⟺ False  by Auto
| |
| .\
| . 2. k : ℤ
| . 3. 0<k
| . 4. ∀b:ℤ, e:({a..b⁻}→ℤ).
| . |  b-a = k-1
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| . |  ⇒
| . |  ((Π i:{a..b⁻}. e(i)) = 0 ⟺ (Or i:{a..b⁻}. e(i) = 0))
| . 5. b : ℤ
| . 6. e : {a..b⁻}→ℤ
| . 7. b-a = k
| . ⊢  (Π i:{a..b⁻}. e(i)) = 0 ⟺ (Or i:{a..b⁻}. e(i) = 0)
| . | by Rewrite by
| . |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
| . |          a<b
| . |          ⇒
| . |          (Iter(f;u) i:{a..b⁻}. e(i))
| . |          =
| . |          f((Iter(f;u) i:{a..b-1⁻}. e(i)),e(b-1))
| . |    {iter_via_intseg_split_last}
| . |    THEN
| . |    Reduce Concl
| . |
| . ⊢  (Π i:{a..b-1⁻}. e(i))•e(b-1) = 0
| . |  ⟺
| . |  (Or i:{a..b-1⁻}. e(i) = 0) ∨ e(b-1) = 0
| . | by Rewrite by
| . |    Thm*  ∀a,b:ℤ. a•b = 0 ⟺ a = 0 ∨ b = 0
| . |    {prod_zero_iff_factor_zero}
| . |
| . ⊢  (Π i:{a..b-1⁻}. e(i)) = 0 ∨ e(b-1) = 0
| .    ⟺
| .    (Or i:{a..b-1⁻}. e(i) = 0) ∨ e(b-1) = 0  by Rewrite by Hyp:4
|
.\
. 2. ∀b:ℤ, e:({a..b⁻}→ℤ).
. |  a≤b ⇒ ((Π i:{a..b⁻}. e(i)) = 0 ⟺ (Or i:{a..b⁻}. e(i) = 0))
. 3. b : ℤ
. 4. e : {a..b⁻}→ℤ
. ⊢  (Π i:{a..b⁻}. e(i)) = 0 ⟺ (Or i:{a..b⁻}. e(i) = 0)
. | by Decide: a≤b
. |
. |\
. | 5. a≤b
. | ⊢  (Π i:{a..b⁻}. e(i)) = 0 ⟺ (Or i:{a..b⁻}. e(i) = 0)
. |   by Guarding (<prop> ⟺ <prop>) BackThru: Hyp:2
. |
. .\
. . 5. ¬a≤b
. . ⊢  (Π i:{a..b⁻}. e(i)) = 0 ⟺ (Or i:{a..b⁻}. e(i) = 0)
. . | by Rewrite by
. . |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
. . |          b≤a
. . |          ⇒
. . |          (Iter(f;u) i:{a..b⁻}. e(i))
. . |          =
. . |          (Iter(f;u) i:{a..a⁻}. e(i))
. . |    {iter_via_intseg_nullnorm}
. . |
. . ⊢  (Π i:{a..a⁻}. e(i)) = 0 ⟺ (Or i:{a..a⁻}. e(i) = 0)
. .   by Guarding (<prop> ⟺ <prop>) BackThru: Hyp:2
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END