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iter_via_intseg_all_units  THM

⊢  ∀A:Type, f:(A→A→A), u:A.
|  is_ident(A; f; u)
|  ⇒
|  (∀a,b:ℤ, e:({a..b⁻}→A).
|   (∀i:{a..b⁻}. e(i) = u) ⇒ (Iter(f;u) i:{a..b⁻}. e(i)) = u)
| by Guarding (∀a:<type>. <prop>) Auto
|
1. A : Type
2. f : A→A→A
3. u : A
4. is_ident(A; f; u)
⊢  ∀a,b:ℤ, e:({a..b⁻}→A).
|  (∀i:{a..b⁻}. e(i) = u) ⇒ (Iter(f;u) i:{a..b⁻}. e(i)) = u
| by BackThru:
|    Thm*  ∀P:(ℤ→ℤ→Prop).
|          (∀a:ℤ, b:{...a}. P(a,b))
|          ⇒
|          (∀a:ℤ, b:{a+1...}. (∀c:{a..b⁻}. P(a,c)) ⇒ P(a,b))
|          ⇒
|          (∀a,b:ℤ. P(a,b))
|    {all_int_segs_by_upper_ind}
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| 5. a : ℤ
| 6. b : {...a}
| 7. e : {a..b⁻}→A
| 8. ∀i:{a..b⁻}. e(i) = u
| ⊢  (Iter(f;u) i:{a..b⁻}. e(i)) = u
|   by Rewrite by
|      Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
|            b≤a ⇒ (Iter(f;u) i:{a..b⁻}. e(i)) = u
|      {iter_via_intseg_null}
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. 5. a : ℤ
. 6. b : {a+1...}
. 7. ∀c:{a..b⁻}, e:({a..c⁻}→A).
. |  (∀i:{a..c⁻}. e(i) = u) ⇒ (Iter(f;u) i:{a..c⁻}. e(i)) = u
. 8. e : {a..b⁻}→A
. 9. ∀i:{a..b⁻}. e(i) = u
. ⊢  (Iter(f;u) i:{a..b⁻}. e(i)) = u
. | by Rewrite by
. |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
. |          a<b
. |          ⇒
. |          (Iter(f;u) i:{a..b⁻}. e(i))
. |          =
. |          f((Iter(f;u) i:{a..b-1⁻}. e(i)),e(b-1))
. |    {iter_via_intseg_split_last}
. |
. ⊢  f((Iter(f;u) i:{a..b-1⁻}. e(i)),e(b-1)) = u  by Rewrite by Hyp:9
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. ⊢  f((Iter(f;u) i:{a..b-1⁻}. u),u) = u  by Rewrite by Hyp:7
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. ⊢  f(u,u) = u  by BackThru: is_ident(A; f; u)
END
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